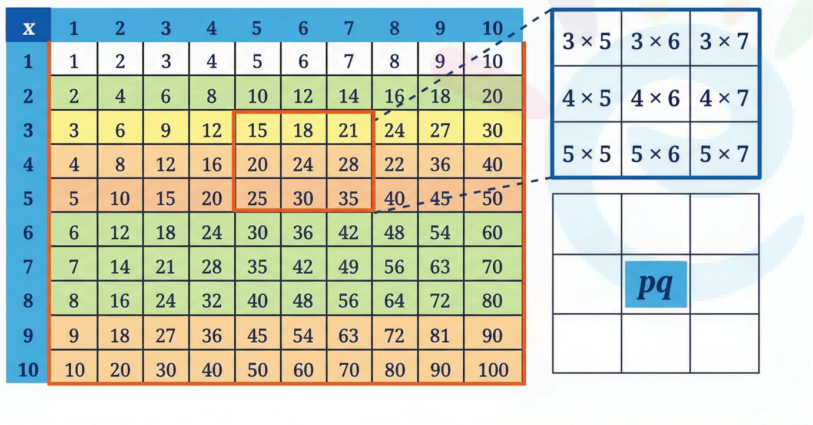
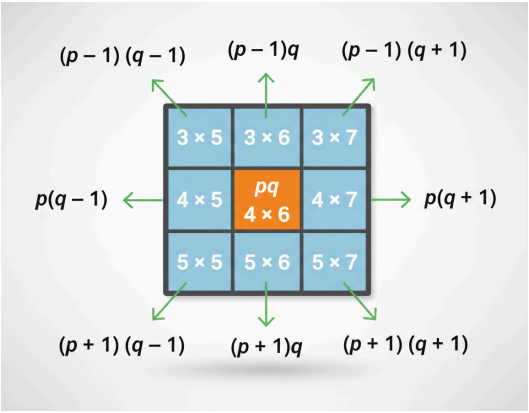
NCERT Solutions For Class 8 Maths
Chapter 6 - We Distribute, Yet Things Multiply

1. Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression *pq*, as shown in the figure, write the expressions for the other numbers in the grid.



Answer:



2. Expand the following products.

(i) (3 + u) (v – 3)

(ii) 2/3(15 + 6a)

(iii) (10a + b) (10c + d)

(iv) (3 – x)(x – 6)

(v) (-5a + b) (c + d)

(vi) (5 + z) (y + 9)

Answer:

(i)

We have, (3 + u)(v – 3)

= 3(v – 3) + u(v – 3)

= 3v – 9 + uv – 3u

= 3v – 3u + uv – 9

(ii)

Here, (2/3)(15 + 6a)

= (2/3) × 15 + (2/3) × 6a

= 10 + 4a

(iii)

Here, (10a + b)(10c + d)

= 10a × 10c + 10a × d + b × 10c + b × d

= 100ac + 10ad + 10bc + bd

(iv)

Here, $(3 - x)(x - 6)$

$= 3(x - 6) - x(x - 6)$

$= 3x - 18 - x^2 + 6x$

$= -x^2 + 9x - 18$

(v)

We have, $(-5a + b)(c + d)$

$= -5a(c + d) + b(c + d)$

$= -5ac - 5ad + bc + bd$

(vi)

We have, $(5 + z)(y + 9)$

$= 5(y + 9) + z(y + 9)$

$= 5y + 45 + yz + 9z$

3. Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

Answer:

Let the numbers be *a* and *b*.

Then, $ab = (a + 2)(b - 4)$

$\Rightarrow ab = ab - 4a + 2b - 8$

$\Rightarrow ab - ab + 4a + 8 = 2b$

$\Rightarrow 4a + 8 = 2b$

Dividing both sides by 2,

$\Rightarrow 2a + 4 = b$

$\Rightarrow b = 2a + 4$

For $a = 1, b = 6$

$ab = 6$ and $(a + 2)(b - 4) = 6$

Hence, $ab = (a + 2)(b - 4)$

For $a = 2, b = 8$

For $a = 3, b = 10$

Thus, the three pairs are (1, 6), (2, 8), and (3, 10).

4. Expand (i) $(a + ab - 3b^2)(4 + b)$, and (ii) $(4y + 7)(y + 11z - 3)$.

Answer:

(i)

Here, $(a + ab - 3b^2)(4 + b)$

$= 4(a + ab - 3b^2) + b(a + ab - 3b^2)$

$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$

$= 4a + 5ab - 12b^2 + ab^2 - 3b^3$

(ii)

Here, $(4y + 7)(y + 11z - 3)$

$= 4y(y + 11z - 3) + 7(y + 11z - 3)$

$= 4y^2 + 44yz - 12y + 7y + 77z - 21$

$= 4y^2 + 44yz - 5y + 77z - 21$

5. Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$, and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$. Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

Answer:

(i)

$(a - b)(a + b)$

$= a^2 + ab - ab - b^2$

$= a^2 - b^2$

(ii)

$(a - b)(a^2 + ab + b^2)$

$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$

$= a^3 - b^3$

(iii)

$(a - b)(a^3 + a^2b + ab^2 + b^3)$

$= a^4 + a^3b + a^2b^2 + ab^3 - a^3b - a^2b^2 - ab^3 - b^4$

$= a^4 - b^4$

We observe the pattern:

$(a - b)(a^n + a^{n-1}b + \ldots + b^n) = a^{n+1} - b^{n+1}$

Thus, the next identity would be:

$(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$

Figure It Out (Page 149)

1. Which is greater: $(a - b)^2$ or $(b - a)^2$? Justify your answer.

(i)

$(a - b)^2 = a^2 + b^2 - 2ab$ .....(1)

$(b - a)^2 = b^2 + a^2 - 2ba = a^2 + b^2 - 2ab$ .....(2)

Comparing (1) and (2):
$(a - b)^2 = (b - a)^2$

2. Express 100 as the difference of two squares.

Answer:

$a^2 - b^2 = 100$
$(a + b)(a - b) = 100$
Let $(a + b) = 50$ and $(a - b) = 2$
Adding: $2a = 52 \Rightarrow a = 26$
Substitute in (1): $26 + b = 50 \Rightarrow b = 24$
Checking: $26^2 - 24^2 = 676 - 576 = 100$
Hence, $26^2 - 24^2 = 100$

3. Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learned so far.

Answer:

(i)
$406^2 = (400 + 6)^2$
$= 400^2 + 2 \times 400 \times 6 + 6^2$
$= 160000 + 4800 + 36$
$= 164836$

(ii)
$72^2 = (50 + 22)^2$

$= 50^2 + 2 \times 50 \times 22 + 22^2$

$= 2500 + 2200 + 484$

$= 5184$

(iii)

$145^2 = (150 - 5)^2$

$= 150^2 - 2 \times 150 \times 5 + 5^2$

$= 22500 - 1500 + 25$

$= 21025$

(iv)

$1097^2 = (1100 - 3)^2$

$= 1100^2 - 2 \times 1100 \times 3 + 3^2$

$= 1210000 - 6600 + 9$

$= 1203409$

(v)

$124^2 = (100 + 24)^2$

$= 100^2 + 2 \times 100 \times 24 + 24^2$

$= 10000 + 4800 + 576$

$= 15376$

4. Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.

Pattern 1:

$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

Case I: a = 4, b = 2

LHS = 2(16 + 4) = 40

RHS = $6^2 + 2^2$ = 36 + 4 = 40

Case II: a = -4, b = -2

LHS = 2(16 + 4) = 40

RHS = $(-6)^2 + (-2)^2$ = 36 + 4 = 40

Case III: a = 1/2, b = 1/3

LHS = $2[(1/2)^2 + (1/3)^2] = 2 \times 13/36 = 13/18$

RHS = $(5/6)^2 + (1/6)^2 = 25/36 + 1/36 = 13/18$

Thus, Pattern 1 holds for all numbers.

Pattern 2:

$a^2 - b^2 = (a + b)(a - b)$

Case I: a = 5, b = 3

LHS = 16, RHS = 8 × 2 = 16

Case II: a = -5, b = -3

LHS = 25 − 9 = 16, RHS = (-8)(-2) = 16

Case III: a = 1/2, b = 1/3

LHS = 1/4 − 1/9 = 5/36

RHS = (5/6)(1/6) = 5/36

Hence, Pattern 2 is valid for fractions and integers.

Figure It Out (Pages 154-156)

1. Compute these products using the suggested identity.

(i) 46^2 using Identity 1A for $(a + b)^2$

(ii) 397×403 using Identity 1C for $(a + b)(a - b)$

(iii) 91^2 using Identity 1B for $(a - b)^2$

(iv) 43×45 using Identity 1C for $(a + b)(a - b)$

(i)

$46^2 = (40 + 6)^2$

$= 40^2 + 2 \times 40 \times 6 + 6^2$

$= 1600 + 480 + 36$

$= 2116$

(ii)

$397 \times 403 = (400 - 3)(400 + 3)$

$= 400^2 - 3^2$

$= 160000 - 9$

$= 159991$

(iii)

$91^2 = (100 - 9)^2$

$= 10000 - 1800 + 81$

$= 8281$

(iv)

$43 \times 45 = (44 - 1)(44 + 1)$

$= 44^2 - 1^2$

= 1936 – 1

= 1935

2. Use either a suitable identity or the distributive property to find each of the following products.

(i) (p – 1) (p + 11)

(ii) (3a – 9b) (3a + 9b)

(iii) -(2y + 5)(3y + 4)

(iv) $(6x + 5y)^2$

(v) $(2x – 1/2)^2$

(vi) (7p) × (3r) × (p + 2)

(i) $(p – 1)(p + 11) = p^2 + 11p – p – 11 = p^2 + 10p – 11$

(ii) $(3a – 9b)(3a + 9b) = (3a)^2 – (9b)^2 = 9a^2 – 81b^2$

(iii) $-(2y + 5)(3y + 4) = -2y(3y + 4) – 5(3y + 4)$

$= -6y^2 – 8y – 15y – 20 = -6y^2 – 23y – 20$

(iv) $(6x + 5y)^2 = 36x^2 + 60xy + 25y^2$

(v) $(2x – 1/2)^2 = 4x^2 – 2x + 1/4$

(vi) $(7p)(3r)(p + 2) = 21pr(p + 2) = 21p^2r + 42pr$

3. For each statement, identify the appropriate algebraic expression(s).

(i) Two more than a square number.

- **2 + s**
- **$(s + 2)^2$**
- **$s^2 + 2$**
- **$s^2 + 4$**
- **$2s^2$**

- **2^2s**

(ii) The sum of the squares of two consecutive numbers

(ii) The sum of the squares of two consecutive numbers

- **$m^2 + n^2$**
- **$(m + n)^2$**
- **$m^2 + 1$**
- **$m^2 + (m + 1)^2$**
- **$m^2 + (m - 1)^2$**
- **$(m + (m + 1))^2$**
- **$(2m)^2 + (2m + 1)^2$**

Solution:

(i) "Two more than a square number" = $s^2 + 2$

(ii) "The sum of the squares of two consecutive numbers" = $m^2 + (m + 1)^2$

4. Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

February						
Su	M	Tu	W	Th	F	Sa
						1
2	3	4	5	6	6	8
9	10	11	11	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Find products of numbers lying along each diagonal – 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.

Hint: Label the numbers in each 2 by 2 square as

a	**(a+1)**
a+7	**(a+8)**

Answer:

Case-I

6	7
13	14

Here, 6 × 14 = 84

13 × 7 = 91

Difference = 91 – 84 = 7

Case-II

9	10

16	17

Here, 9×17=1539 × 17 = 1539×17=153 and 16×10=16016 × 10 = 16016×10=160.

The difference between these two products is 160–153=7160 – 153 = 7160–153=7.

Thus, we can see that the difference between the diagonal products is always 7.

5. Verify which of the following statements are true.
(i) (k + 1)(k + 2) - (k + 3) is always a multiple of 2.
(ii) (2q + 1)(2q - 3) is a multiple of 4.
(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.
(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

Answer:
(i) (k + 1)(k + 2) - (k + 3) is always a multiple of 2.
Let k = 5, then
(5 + 1)(5 + 2) - (5 + 3) = 6 × 7 - 8 = 42 - 8 = 34.
Since 34 is divisible by 2, the statement is true.

(ii) (2q + 1)(2q - 3) is a multiple of 4.
Let q = 3, then

$(6 + 1)(6 - 3) = 7 \times 3 = 21$.

Since 21 is not divisible by 4, the statement is false.

(iii)

The square of an even number is a multiple of 4.

$2^2 = 4 = 4 \times 1$

$4^2 = 16 = 4 \times 4$

$6^2 = 36 = 4 \times 9$

$\therefore$ The statement is true.

The square of an odd number is 1 more than a multiple of 8.

$3^2 = 9 = 8 \times 1 + 1$

$5^2 = 25 = 8 \times 3 + 1$

$7^2 = 49 = 8 \times 6 + 1$

$\therefore$ The statement is true.

(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

Let n = 2,

$(6 \times 2 + 2)^2 - (4 \times 2 + 3)^2 = 14^2 - 11^2 = 196 - 121 = 75 = 80 - 5$.

Since 80 is not a perfect square, the statement is false.

6. A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

Answer:

Let the numbers be x and y.

Then $x = 7a + 3$ and $y = 7b + 5$.

Sum:

$x + y = 7a + 3 + 7b + 5 = 7(a + b) + 8 = 7(a + b + 1) + 1$

$\therefore$ Remainder when divided by 7 is 1.

Difference:

$x - y = (7a + 3) - (7b + 5) = 7(a - b) - 2 = 7(a - b - 1) + 5$

$\therefore$ Remainder when divided by 7 is 5.

Product:

$xy = (7a + 3)(7b + 5) = 49ab + 35a + 21b + 15 = 7(7ab + 5a + 3b + 2) + 1$

$\therefore$ Remainder when divided by 7 is 1.

7. Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

Answer:

Take the numbers 7, 8, 9:

$8^2 - 7 \times 9 = 64 - 63 = 1$.

Take another set, 10, 11, 12:

$11^2 - 10 \times 12 = 121 - 120 = 1$.

Generalizing:

Let the numbers be $(a - 1)$, a, $(a + 1)$.

Then,

$a^2 - (a + 1)(a - 1) = a^2 - (a^2 - 1) = 1$.

Thus, LHS = RHS = 1, and the identity is verified:

$a^2 - (a + 1)(a - 1) = 1$

8. What is the algebraic expression describing the following steps: add any two numbers? Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

Answer:

Let the two numbers be a and b.

Step 1: Add the numbers → a + b

Step 2: Multiply this by half of their sum → (a + b) × (1/2)(a + b)

Therefore,

$(a + b) \times (1/2)(a + b) = (1/2)(a + b)^2$.

Hence, the result is half the square of the sum of the two numbers.

9. Which is larger? Find out without fully computing the product.

(i) 14 × 26 or 16 × 24

(ii) 25 × 75 or 26 × 74

(i) Let p = 14 × 26 and p' = 16 × 24.

Now,

p' = (14 + 2)(26 - 2)

= 14 × 26 + 2 × 26 - 14 × 2 - 2 × 2

= 14 × 26 + 2(26 - 14 - 2)

= 14 × 26 + 2 × 10

= p + 20

∴ p' > p, hence 16 × 24 > 14 × 26.

(ii) Let p = 25 × 75 and p' = 26 × 74.

Then,

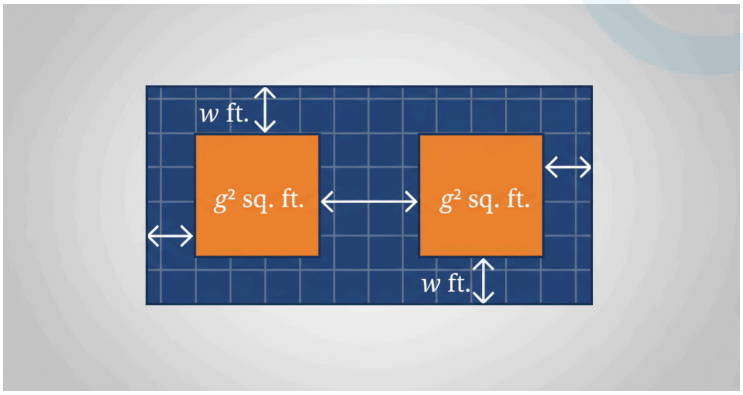
p' = (25 + 1)(75 - 1)

= 25 × 75 + 75 - 25 - 1

= p + 49

∴ p' > p, hence 26 × 74 > 25 × 75.

10. A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g^2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to he tiled. Write an expression for the area that needs to be tiled.



Answer:

Length = w + g + 2w + g + w = 4w + 2g

Breadth = w + g + w = 2w + g

Area of park = (4w + 2g)(2w + g)

= $8w^2$ + 4wg + 4wg + $2g^2$

= $8w^2$ + 8wg + $2g^2$

Area of path = Area of park - Area of green cover

$= (8w^2 + 8wg + 2g^2) - 2g^2$

$= 8w^2 + 8wg$

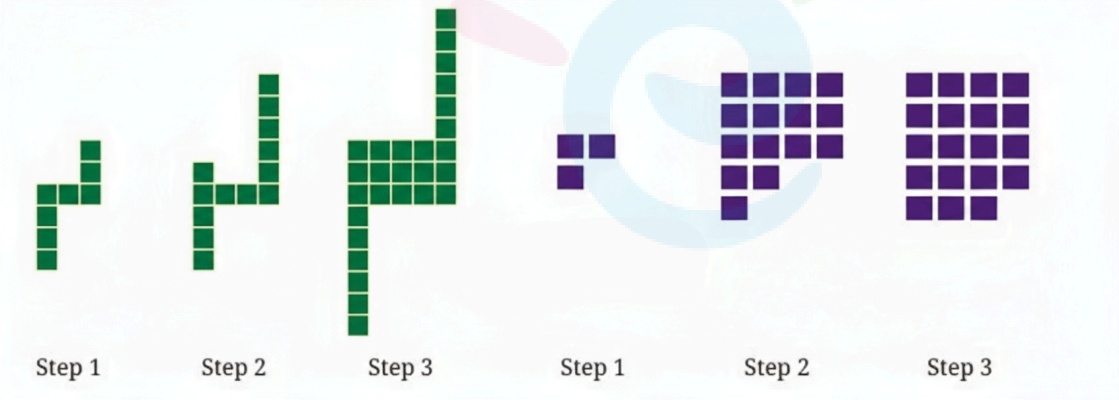
$\therefore$ $(8w^2 + 8wg)$ square feet of area needs to be tiled.

11. For each pattern shown below,

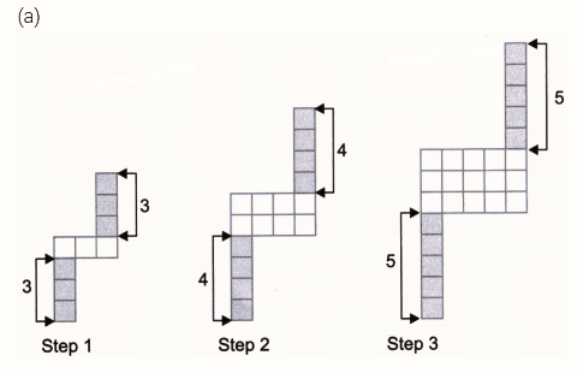
(i) Draw the next figure in the sequence.

(ii) How many basic units are there in Step 10?

(iii) Write an expression to describe the number of basic units in Step y.



Answer:



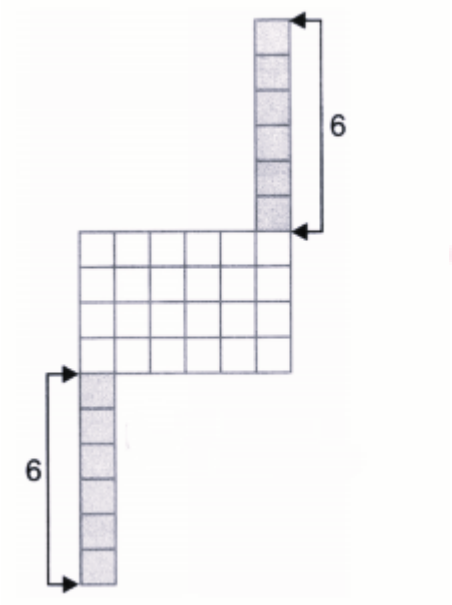
Step 1:

2 vertical strips of 3 units each + 1 vertical strip of 3 units

= 3 strips of 3 units each

= 9 unit squares

= $(1 + 2)^2$ unit squares



Step 2:

4 strips of 4 units each

= 16 unit squares

= $(2 + 2)^2$ unit squares

Step 3:

5 strips of 5 units each

= 25 unit squares

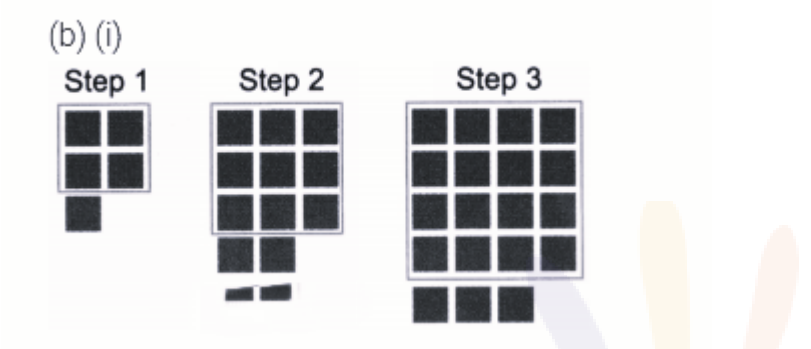
= $(3 + 2)^2$ unit squares

Step 4: (i) 6 strips of 6 units each = 2 are vertical and 4 are horizontal

(ii) Number of unit squares in step 10 = $(10 + 2)^2 = 144$

(iii) Number of unit squares in step y = $(y + 2)^2$

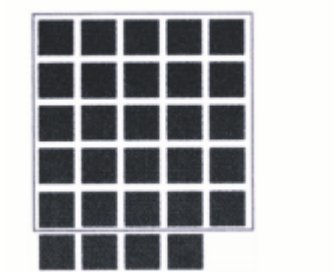
(b) (i)



Number of unit squares in step 1 = 5 = $2^2 + 1$

Number of unit squares in step 2 = 11 = $3^2 + 2$

Number of unit squares in step 3 = 19 = $4^2 + 3$



(ii) Step 1 has $(1 + 1)^2 + 1 = 5$ squares.

Step 2 has $(2 + 1)^2 + 2 = 11$ squares.

Step 3 has $(3 + 1)^2 + 3 = 19$ squares.

Hence, Step 10 has $(10 + 1)^2 + 10 = 131$ squares.

(iii) Step y has $[(y + 1)^2 + y]$ squares.