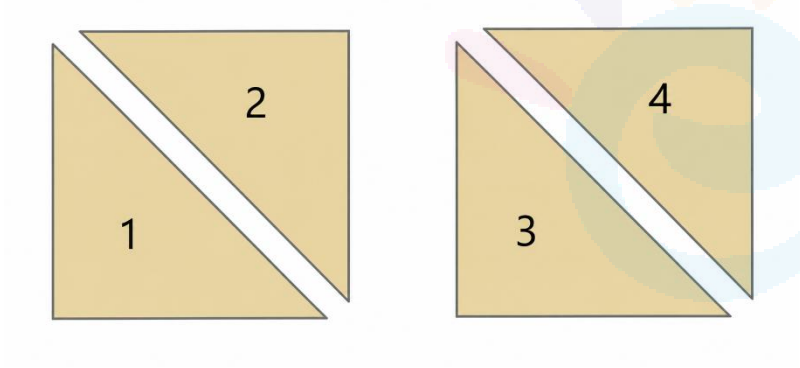
NCERT Solutions For Class 8 Maths

Chapter 9 - The Baudhayana-Pythagoras Theorem (Part 2, Chapter 2)

2.1 Doubling A Square, 2.2 Halving A Square & 2.3 Hypotenuse of an Isosceles Right Triangle

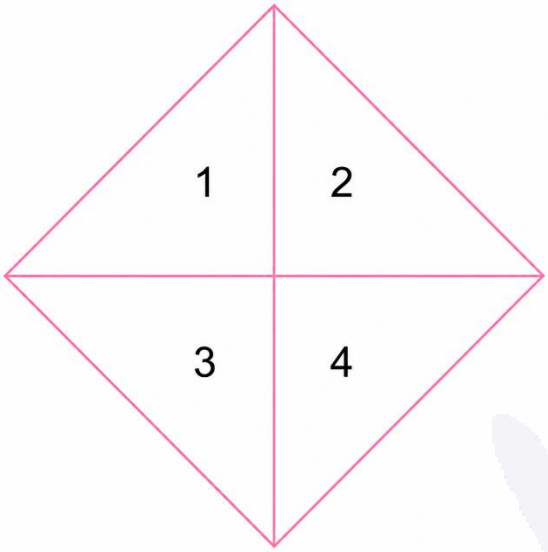
Figure It Out (Pages 39-40)

Question 1.
Earlier, we saw a method to create a square with double the area of a given square piece of paper. There is another method to do this in which two identical square papers are cut in the following way.



Can you arrange these pieces to create a square with double the area of either square?

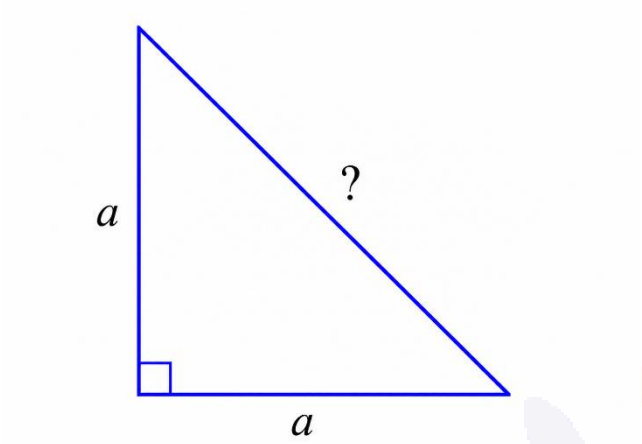
Solution:
Take two identical square sheets, each having side a.



Cut each square along its diagonal. Each square will be divided into two congruent right-angled isosceles triangles.
Thus, the two squares give four identical triangles.
Area of one original square = a^2
Area of two original squares = $2a^2$
Now arrange the four triangles so that their equal sides form the outer boundary of a new square. Since all four triangles together have the same total area as the two original squares, the area of the newly formed square is:
$2a^2$
Therefore, the new square has twice the area of either of the original squares.

Question 2.
The length of the two equal sides of an isosceles right triangle is given. Find the length of the hypotenuse. Find bounds on the length of the hypotenuse such that they have at least one digit after the decimal point.
(i) 3
(ii) 4
(iii) 6
(iv) 8
(v) 9

Solution:

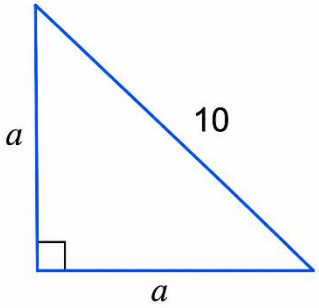


Let the length of each equal side be a.
Using the Baudhayana-Pythagoras Theorem:
Hypotenuse² = a² + a²
Hypotenuse² = 2a²
Hypotenuse = a√2
(i) a = 3
Hypotenuse = 3√2
= √18
Since:
$4.2^2 = 17.64$
$4.3^2 = 18.49$
Therefore:
$4.2 < \sqrt{18} < 4.3$
Hence:
$4.2 < 3\sqrt{2} < 4.3$
The hypotenuse is approximately 4.24 units.
(ii) a = 4
Hypotenuse = 4√2
= √32
Since:
$5.6^2 = 31.36$
$5.7^2 = 32.49$
Therefore:
$5.6 < \sqrt{32} < 5.7$

Hence:
$5.6 < 4\sqrt{2} < 5.7$
The hypotenuse is approximately 5.66 units.
(iii) a = 6
Hypotenuse = $6\sqrt{2}$
$= \sqrt{72}$
Since:
$8.4^2 = 70.56$
$8.5^2 = 72.25$
Therefore:
$8.4 < \sqrt{72} < 8.5$
Hence:
$8.4 < 6\sqrt{2} < 8.5$
The hypotenuse is approximately 8.49 units.
(iv) a = 8
Hypotenuse = $8\sqrt{2}$
$= \sqrt{128}$
Since:
$11.3^2 = 127.69$
$11.4^2 = 129.96$
Therefore:
$11.3 < \sqrt{128} < 11.4$
Hence:
$11.3 < 8\sqrt{2} < 11.4$
The hypotenuse is approximately 11.31 units.
(v) a = 9
Hypotenuse = $9\sqrt{2}$
$= \sqrt{162}$
Since:
$12.7^2 = 161.29$
$12.8^2 = 163.84$
Therefore:
$12.7 < \sqrt{162} < 12.8$
Hence:
$12.7 < 9\sqrt{2} < 12.8$
The hypotenuse is approximately 12.73 units.

Question 3.
The hypotenuse of an isosceles right triangle is 10. What are its other two side lengths?
[Hint: Find the area of the square composed of two such right triangles.]

Solution:



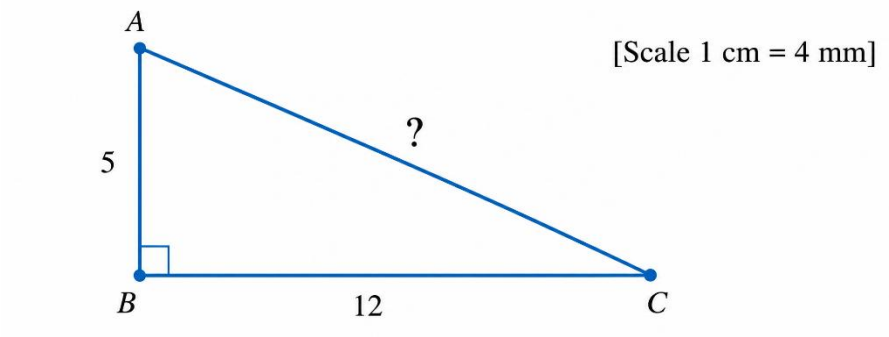
Let each of the two equal sides be a.
Using the Baudhayana-Pythagoras Theorem:
$a^2 + a^2 = 10^2$
$2a^2 = 100$
$a^2 = 50$
$a = \sqrt{50}$
$a = \sqrt{(25 \times 2)}$
$a = 5\sqrt{2}$
Therefore, the other two sides are:
$5\sqrt{2}$ units and $5\sqrt{2}$ units
Each side is approximately 7.07 units long.

2.4 Combining Two Different Squares

Figure It Out (Page 47)

Question 1.
If a right-angled triangle has shorter sides of lengths 5 cm and 12 cm, then what is the length of its hypotenuse? First, draw the right-angled triangle with these sidelengths and measure the hypotenuse, then check your answer using Baudhayana's Theorem.

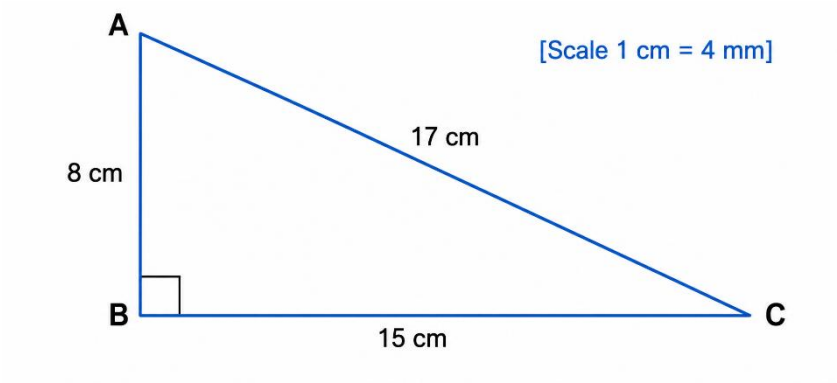
Solution:



The two shorter sides are:
AB = 5 cm
BC = 12 cm
Let AC be the hypotenuse.
Using Baudhayana's Theorem:
$AC^2 = AB^2 + BC^2$
$AC^2 = 5^2 + 12^2$
$AC^2 = 25 + 144$
$AC^2 = 169$
$AC = \sqrt{169}$
AC = 13 cm
Therefore, the hypotenuse of the right-angled triangle is 13 cm.

Question 2.
If a right-angled triangle has a short side of length 8 cm and a hypotenuse of length 17 cm, what is the length of the third side? Again, try drawing the triangle and measuring, and then check your answer using Baudhayana's Theorem.

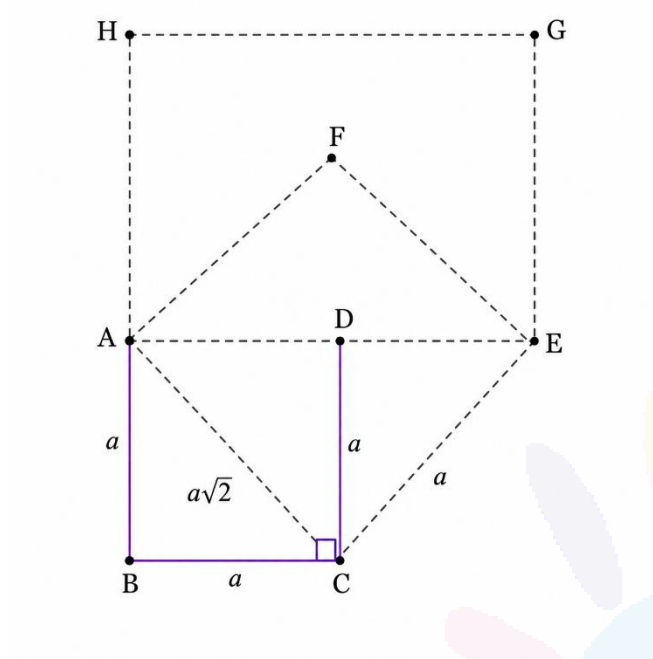
Solution:



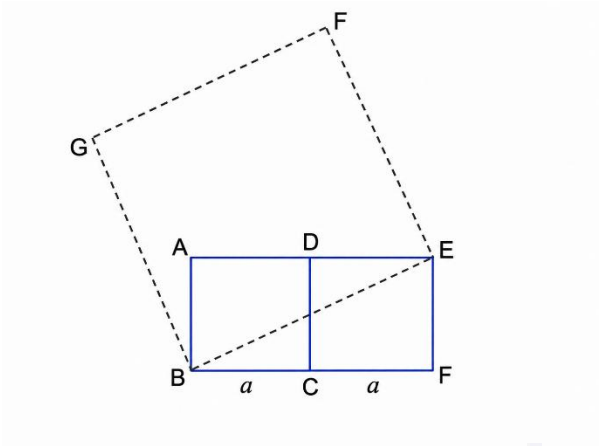
Given:
One shorter side = 8 cm
Hypotenuse = 17 cm
Let the third side be x cm.
Using Baudhayana's Theorem:
$8^2 + x^2 = 17^2$
$64 + x^2 = 289$
$x^2 = 289 - 64$
$x^2 = 225$
$x = \sqrt{225}$
x = 15 cm
Therefore, the third side of the right-angled triangle is 15 cm.
Verification:
$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

Question 3.
Using the constructions you have now seen, how would you construct a square whose area is triple the area of a given square? Five times the area of a given square? (Baudhayana's Sulba-Sutra, Verse 1.10)

Solution:
Let the side of the given square be a.
Its area is:
a^2
(a) Construction of a square with triple the area



First, construct a square ABCD of side a.
Its diagonal is:
AC = √(a² + a²)
AC = a√2
Now construct a right-angled triangle with perpendicular sides of length a√2 and a.
Let its hypotenuse be AE.
Using Baudhayana's Theorem:
AE² = (a√2)² + a²
AE² = 2a² + a²
AE² = 3a²
AE = a√3
Now construct a square with a side of√3.
Area of the new square:
= (a√3)²
= 3a²
Therefore, the new square has three times the area of the original square.
(b) Construction of a square with five times the area



Construct a rectangle whose length is 2a and breadth is a.
Let its diagonal be d.
Using Baudhayana's Theorem:
$d^2 = (2a)^2 + a^2$
$d^2 = 4a^2 + a^2$
$d^2 = 5a^2$
$d = a\sqrt{5}$
Now construct a square with a side $a\sqrt{5}$.
Area of the new square:
$= (a\sqrt{5})^2$
$= 5a^2$
Therefore, the new square has five times the area of the original square.

Question 4.
Let a, b, and c denote the lengths of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in each of the following cases:
(i) a = 5, b = 7
(ii) a = 8, b = 12
(iii) a = 9, c = 15
(iv) a = 7, b = 12
(v) a = 1.5, b = 3.5

Solution:
Using Baudhayana's Theorem:
$c^2 = a^2 + b^2$
(i) a = 5, b = 7

$c^2 = 5^2 + 7^2$
$c^2 = 25 + 49$
$c^2 = 74$
$c = \sqrt{74}$
Therefore, the missing side is $\sqrt{74}$ units, approximately 8.60 units.
(ii) $a = 8, b = 12$
$c^2 = 8^2 + 12^2$
$c^2 = 64 + 144$
$c^2 = 208$
$c = \sqrt{208}$
$c = \sqrt{(16 \times 13)}$
$c = 4\sqrt{13}$
Therefore, the missing side is $4\sqrt{13}$ units, approximately 14.42 units.
(iii) $a = 9, c = 15$
Using:
$c^2 = a^2 + b^2$
$15^2 = 9^2 + b^2$
$225 = 81 + b^2$
$b^2 = 225 - 81$
$b^2 = 144$
$b = \sqrt{144}$
$b = 12$
Therefore, the missing side is 12 units.
(iv) $a = 7, b = 12$
$c^2 = 7^2 + 12^2$
$c^2 = 49 + 144$
$c^2 = 193$
$c = \sqrt{193}$
Therefore, the missing side is $\sqrt{193}$ units, approximately 13.89 units.
(v) $a = 1.5, b = 3.5$
$c^2 = 1.5^2 + 3.5^2$
$c^2 = 2.25 + 12.25$
$c^2 = 14.5$
$c = \sqrt{14.5}$
Therefore, the missing side is $\sqrt{14.5}$ units, approximately 3.81 units.

2.5 Right-Triangles Having Integer Sidelengths

Figure It Out (Page 50)

Question 1.
Find 5 more Baudhayana triples using this idea.

Solution:
A Baudhayana triple consists of three positive integers a, b and c satisfying:
$a^2 + b^2 = c^2$
Using the pattern in which one shorter side is an odd number n, the other shorter side is:
$(n^2 - 1)/2$
and the hypotenuse is:
$(n^2 + 1)/2$
Five Baudhayana triples are:

1. For n = 7

Other side:
$(7^2 - 1)/2$
$= (49 - 1)/2$
$= 24$
Hypotenuse:
$(7^2 + 1)/2$
$= (49 + 1)/2$
$= 25$
Therefore:
$24^2 + 7^2 = 25^2$
Triple: (7, 24, 25)

2. For n = 9

Other side:
$(9^2 - 1)/2$
$= (81 - 1)/2$
$= 40$
Hypotenuse:
$(9^2 + 1)/2$
$= (81 + 1)/2$
$= 41$

Therefore:
$40^2 + 9^2 = 41^2$
Triple: (9, 40, 41)

3. For n = 11

Other side:
$(11^2 - 1)/2$
$= (121 - 1)/2$
$= 60$
Hypotenuse:
$(11^2 + 1)/2$
$= (121 + 1)/2$
$= 61$
Therefore:
$60^2 + 11^2 = 61^2$
Triple: (11, 60, 61)

4. For n = 13

Other side:
$(13^2 - 1)/2$
$= (169 - 1)/2$
$= 84$
Hypotenuse:
$(13^2 + 1)/2$
$= (169 + 1)/2$
$= 85$
Therefore:
$84^2 + 13^2 = 85^2$
Triple: (13, 84, 85)

5. For n = 15

Other side:
$(15^2 - 1)/2$
$= (225 - 1)/2$
$= 112$
Hypotenuse:
$(15^2 + 1)/2$
$= (225 + 1)/2$
$= 113$

Therefore:
$112^2 + 15^2 = 113^2$
Triple: (15, 112, 113)
Hence, five more Baudhayana triples are:
(7, 24, 25)
(9, 40, 41)
(11, 60, 61)
(13, 84, 85)
(15, 112, 113)

Question 2.
Does this method yield non-primitive Baudhayana triples?
[Hint: Observe that among the triples generated, one of the smaller sidelengths is one less than the hypotenuse.]

Solution:
A primitive Baudhayana triple is a triple in which the highest common factor of all three numbers is 1.
Consider the triples generated by this method:
(7, 24, 25)
HCF of 7, 24 and 25 = 1
(9, 40, 41)
HCF of 9, 40 and 41 = 1
(11, 60, 61)
HCF of 11, 60 and 61 = 1
In each triple, the longer leg and the hypotenuse are consecutive integers. Consecutive integers have no common factor other than 1.
Also, the odd side cannot share a common factor with both consecutive numbers.
Therefore, the triples obtained through this method are primitive Baudhayana triples.
Hence, this method does not produce non-primitive triples.

Question 3.
Are there primitive triples that cannot be obtained through this method? If yes, give examples.

Solution:
Yes, there are primitive Baudhayana triples that cannot be obtained through this method.
In the method discussed earlier, an odd number n is selected and the triple is generated as:
$n, (n^2 - 1)/2, (n^2 + 1)/2$
In every triple produced by this method, one shorter side and the hypotenuse are consecutive integers.
For example:
For n = 7:
(7, 24, 25)
Here, 24 and 25 are consecutive integers.
However, consider the primitive triples:
(8, 15, 17)
$8^2 + 15^2 = 64 + 225 = 289 = 17^2$
HCF of 8, 15 and 17 = 1
Therefore, it is a primitive triple. However, neither shorter side is one less than the hypotenuse.
Similarly:
(16, 63, 65)
$16^2 + 63^2 = 256 + 3969 = 4225 = 65^2$
HCF of 16, 63 and 65 = 1
Therefore, it is also a primitive triple, but it cannot be generated using the given method.
Another example is:
(20, 21, 29)
$20^2 + 21^2 = 400 + 441 = 841 = 29^2$
Thus, primitive triples such as (8, 15, 17), (16, 63, 65) and (20, 21, 29) cannot be obtained through this particular method.

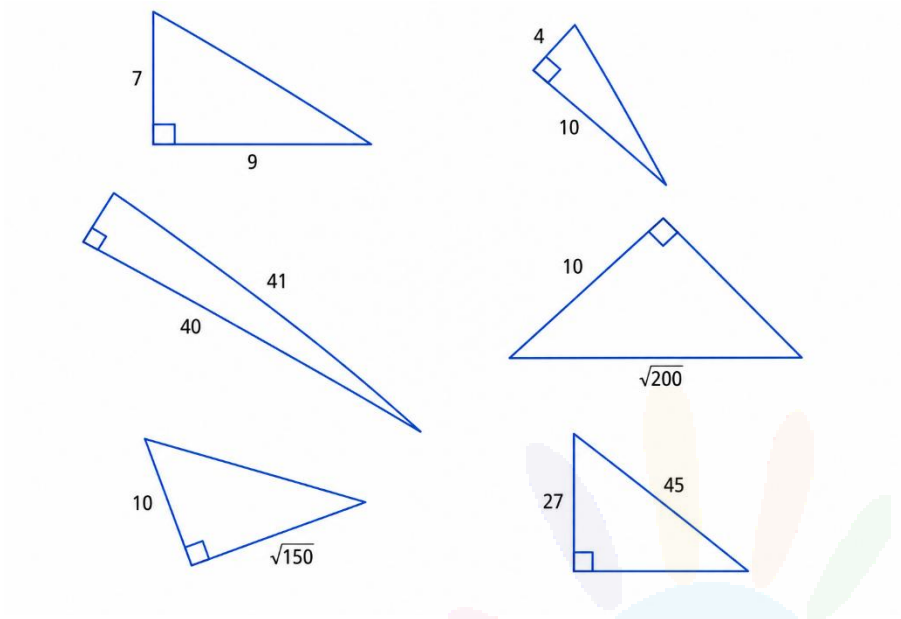
2.6 A Long-Standing Open Problem, 2.7 Further Applications of the Baudhayana-Pythagoras Theorem

Figure It Out (Pages 52-54)

Question 1.
Find the diagonal of a square with sidelength 5 cm.

Solution:
Let ABCD be a square with side 5 cm.
The diagonal BD divides the square into two right-angled triangles.
Using Baudhayana's Theorem:
$BD^2 = 5^2 + 5^2$
$BD^2 = 25 + 25$
$BD^2 = 50$
$BD = \sqrt{50}$
$BD = \sqrt{(25 \times 2)}$
$BD = 5\sqrt{2}$ cm
Using $\sqrt{2} \approx 1.414$:
$BD \approx 5 \times 1.414$
$BD \approx 7.07$ cm
Therefore, the diagonal of the square is $5\sqrt{2}$ cm, or approximately 7.1 cm.

Question 2.
Find the missing sidelengths in the following right triangles:



Solution:

(a)

The two perpendicular sides are 7 units and 9 units.

Using Baudhayana's Theorem:

$a^2 = 7^2 + 9^2$

$a^2 = 49 + 81$

$a^2 = 130$

$a = \sqrt{130}$

Therefore, the missing side is $\sqrt{130}$ units, approximately 11.40 units.

(b)

The two perpendicular sides are 4 units and 10 units.

Using Baudhayana's Theorem:

$b^2 = 4^2 + 10^2$

$b^2 = 16 + 100$

$b^2 = 116$

$b = \sqrt{116}$

$b = \sqrt{(4 \times 29)}$

$b = 2\sqrt{29}$

Therefore, the missing side is $2\sqrt{29}$ units, approximately 10.77 units.

(c)

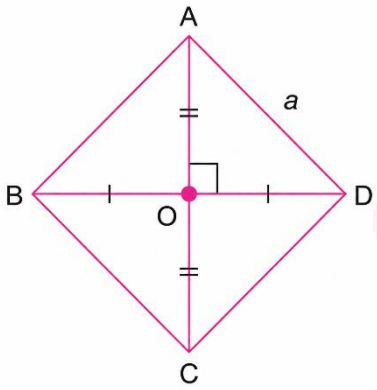
The hypotenuse is 41 units and one shorter side is 40 units.

Using Baudhayana's Theorem:
$40^2 + c^2 = 41^2$
$1600 + c^2 = 1681$
$c^2 = 1681 - 1600$
$c^2 = 81$
$c = \sqrt{81}$
$c = 9$
Therefore, the missing side is 9 units.
(d)
The hypotenuse is $\sqrt{200}$ units and one shorter side is 10 units.
Using Baudhayana's Theorem:
$10^2 + d^2 = (\sqrt{200})^2$
$100 + d^2 = 200$
$d^2 = 200 - 100$
$d^2 = 100$
$d = \sqrt{100}$
$d = 10$
Therefore, the missing side is 10 units.
(e)
The two perpendicular sides are 10 units and $\sqrt{150}$ units.
Using Baudhayana's Theorem:
$e^2 = 10^2 + (\sqrt{150})^2$
$e^2 = 100 + 150$
$e^2 = 250$
$e = \sqrt{250}$
$e = \sqrt{(25 \times 10)}$
$e = 5\sqrt{10}$
Therefore, the missing side is $5\sqrt{10}$ units, approximately 15.81 units.
(f)
The hypotenuse is 45 units and one shorter side is 27 units.
Using Baudhayana's Theorem:
$27^2 + f^2 = 45^2$
$729 + f^2 = 2025$
$f^2 = 2025 - 729$
$f^2 = 1296$

f = √1296
f = 36
Therefore, the missing side is 36 units.

Question 3.
Find the sidelength of a rhombus whose diagonals are of length 24 units and 70 units.

Solution:



The diagonals of a rhombus bisect each other at right angles.
Let the diagonals intersect at O.
Half of the first diagonal:
OA = 24/2 = 12 units
Half of the second diagonal:
OD = 70/2 = 35 units
Triangle AOD is a right-angled triangle.
Let the side of the rhombus be a.
Using Baudhayana's Theorem:
$a^2 = OA^2 + OD^2$
$a^2 = 12^2 + 35^2$
$a^2 = 144 + 1225$
$a^2 = 1369$
$a = \sqrt{1369}$
a = 37
Therefore, the side length of the rhombus is 37 units.

Question 4.
Is the hypotenuse the longest side of a right triangle? Justify your answer.

Solution:
Yes, the hypotenuse is always the longest side of a right-angled triangle.
Let a and b be the two perpendicular sides and c be the hypotenuse.
According to Baudhayana's Theorem:
$c^2 = a^2 + b^2$
Since b^2 is positive:
$c^2 > a^2$
Therefore:
$c > a$
Similarly, since a^2 is positive:
$c^2 > b^2$
Therefore:
$c > b$
Thus, c is greater than both a and b.
Hence, the hypotenuse is the longest side of a right triangle.

Question 5.
True or False — Every Baudhayana triple is either a primitive triple or a scaled version of a primitive triple.

Solution:
True.
Let (a, b, c) be a Baudhayana triple satisfying:
$a^2 + b^2 = c^2$
Let the HCF of a, b and c be d.
Case 1: d = 1
If the HCF of a, b and c is 1, then the triple is a primitive Baudhayana triple.
For example:
(3, 4, 5)
HCF of 3, 4 and 5 = 1
Therefore, (3, 4, 5) is a primitive triple.
Case 2: d > 1
If the HCF is greater than 1, divide all three terms by d.

Suppose:
$a = dx$, $b = dy$ and $c = dz$
Then:
$a^2 + b^2 = c^2$
$d^2x^2 + d^2y^2 = d^2z^2$
Dividing by d^2:
$x^2 + y^2 = z^2$
Thus, (x, y, z) is also a Baudhayana triple.
For example:
(10, 24, 26)
HCF of 10, 24 and 26 = 2
Dividing each term by 2:
(5, 12, 13)
The triple (5, 12, 13) is primitive.
Therefore, (10, 24, 26) is a scaled version of the primitive triple (5, 12, 13).
Hence, every Baudhayana triple is either primitive or a scaled version of a primitive triple.

Question 6.
Give 5 examples of rectangles whose sidelengths and diagonals are all integers.

Solution:
The length, breadth and diagonal of a rectangle form a Baudhayana triple because:
$\text{Diagonal}^2 = \text{Length}^2 + \text{Breadth}^2$
Five examples are:

1. Length = 3 units, breadth = 4 units

$\text{Diagonal}^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
Diagonal = 5 units
Rectangle: 3 × 4, diagonal 5

2. Length = 5 units, breadth = 12 units

$\text{Diagonal}^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
Diagonal = 13 units

Rectangle: 5 × 12, diagonal 13

3. Length = 7 units, breadth = 24 units

Diagonal² = 7² + 24²

= 49 + 576

= 625

Diagonal = 25 units

Rectangle: 7 × 24, diagonal 25

4. Length = 8 units, breadth = 15 units

Diagonal² = 8² + 15²

= 64 + 225

= 289

Diagonal = 17 units

Rectangle: 8 × 15, diagonal 17

5. Length = 9 units, breadth = 40 units

Diagonal² = 9² + 40²

= 81 + 1600

= 1681

Diagonal = 41 units

Rectangle: 9 × 40, diagonal 41

Therefore, five examples are:

• 3 units, 4 units and 5 units

• 5 units, 12 units and 13 units

• 7 units, 24 units and 25 units

• 8 units, 15 units and 17 units

• 9 units, 40 units and 41 units

Question 7.

Construct a square whose area is equal to the difference of the areas of squares of side lengths 5 units and 7 units.

Solution:

Area of the square with side 7 units:

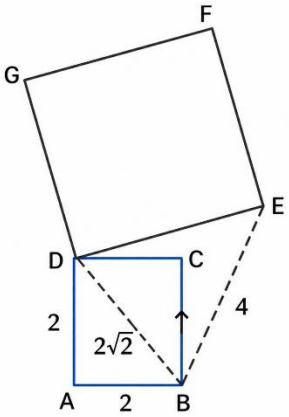
= 7²

= 49 square units

Area of the square with side 5 units:

= 5²

= 25 square units
Required area:
= 49 − 25
= 24 square units
Therefore, the side of the required square must be:
√24 units



Construction:

1. Draw a line segment AB = 5 units.
2. At A, draw a perpendicular line AX.
3. With B as centre and radius 7 units, draw an arc cutting AX at C.

Thus, triangle ABC is right-angled at A, with:
AB = 5 units
BC = 7 units

4. Using Baudhayana's Theorem:

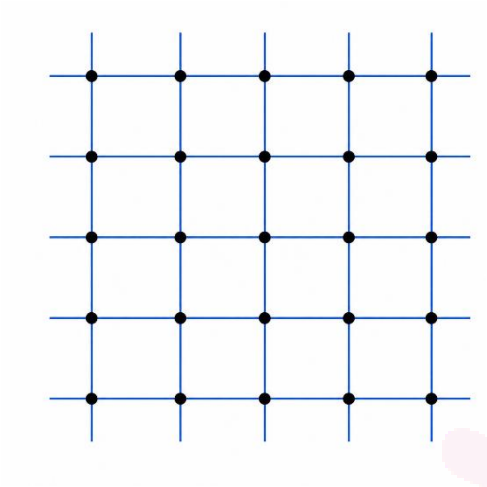
$AC^2 + AB^2 = BC^2$
$AC^2 + 5^2 = 7^2$
$AC^2 + 25 = 49$
$AC^2 = 24$
AC = √24 units

5. Construct a square on AC as its side.

Area of the constructed square:
$= AC^2$
$= (\sqrt{24})^2$
= 24 square units

Therefore, the constructed square has an area equal to the difference between the areas of squares with sides 7 units and 5 units.

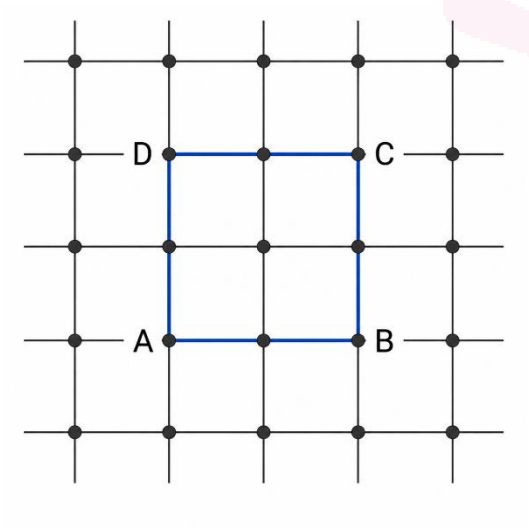
Question 8.



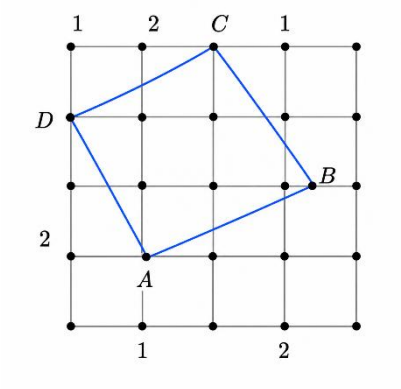
(i) Using the dots of a grid as the vertices, can you create a square that has an area of (a) 2 sq. units, (b) 3 sq. units, (c) 4 sq. units, and (d) 5 sq. units?
(ii) Suppose the grid extends indefinitely. What are the possible integer-valued areas of squares you can create in this manner?

Solution:
For a square drawn on a dot grid, suppose one side moves a units horizontally and b units vertically.
Using Baudhayana's Theorem:
Side^2 = $a^2 + b^2$
Since the area of a square is side^2:
Area = $a^2 + b^2$
Here, a and b are integers.
(i)
(a) Area = 2 square units
$2 = 1^2 + 1^2$
Choose two neighbouring grid points that differ by 1 unit horizontally and 1 unit vertically.
The side length is:
$\sqrt{(1^2 + 1^2)} = \sqrt{2}$ units

Area:
$= (\sqrt{2})^2$
= 2 square units
Therefore, a square of area 2 square units can be created.
(b) Area = 3 square units
For such a square, we would need integers a and b satisfying:
$a^2 + b^2 = 3$
The possible square numbers not exceeding 3 are 0 and 1.
However:
$1^2 + 1^2 = 2$
There are no integers a and b such that:
$a^2 + b^2 = 3$
Therefore, a square of area 3 square units cannot be created using grid points as vertices.
(c) Area = 4 square units



$4 = 2^2 + 0^2$
Draw an ordinary square of side 2 units along the horizontal and vertical grid lines.
Area:
$= 2 \times 2$
= 4 square units
Therefore, a square with an area of 4 square units can be created.
(d) Area = 5 square units



$5 = 2^2 + 1^2$

Choose a side that moves 2 units horizontally and 1 unit vertically.

Its length is:

$\sqrt{(2^2 + 1^2)}$

$= \sqrt{5}$ units

Area of the square:

$= (\sqrt{5})^2$

= 5 square units

Therefore, a square with an area of 5 square units can be created.

(ii)

An integer-valued area x is possible if it can be expressed as:

$x = a^2 + b^2$

where a and b are integers.

Examples:

$1 = 1^2 + 0^2$

$2 = 1^2 + 1^2$

$4 = 2^2 + 0^2$

$5 = 2^2 + 1^2$

$8 = 2^2 + 2^2$

$9 = 3^2 + 0^2$

$10 = 3^2 + 1^2$

$13 = 3^2 + 2^2$

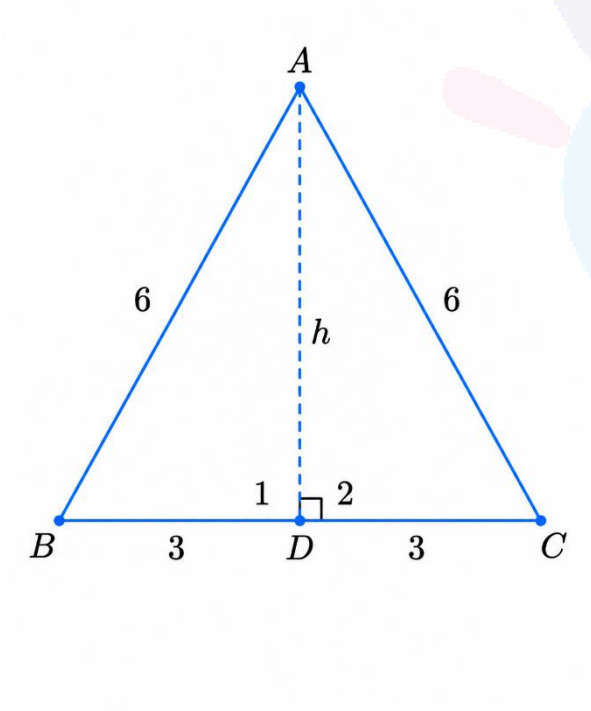
$16 = 4^2 + 0^2$

$17 = 4^2 + 1^2$

However, integers such as 3, 6, 7, 11, and 12 cannot be expressed as the sum of two squares of integers.
Therefore, the possible integer-valued areas are exactly those positive integers that can be written in the form:
$a^2 + b^2$
where a and b are integers.

Question 9.
Find the area of an equilateral triangle with sidelength 6 units.
[Hint: Show that an altitude bisects the opposite side. Use this to find the height.]

Solution:



Let ABC be an equilateral triangle with:
AB = BC = CA = 6 units
Draw AD perpendicular to BC.
We first show that D is the midpoint of BC.
In triangles ADB and ADC:
AB = AC = 6 units
AD = AD, common side

∠ADB = ∠ADC = 90°
Therefore:
ΔADB ≅ ΔADC by the RHS congruence rule.
Hence, by CPCT:
BD = DC
Since:
BC = 6 units
BD = DC = 6/2
BD = DC = 3 units
Now consider the right-angled triangle ADC.
AC = 6 units
DC = 3 units
Let AD = h.
Using Baudhayana's Theorem:
$h^2 + 3^2 = 6^2$
$h^2 + 9 = 36$
$h^2 = 27$
$h = \sqrt{27}$
$h = 3\sqrt{3}$ units
Now:
Area of triangle ABC
= 1/2 × Base × Height
= 1/2 × 6 × $3\sqrt{3}$
= $9\sqrt{3}$ square units
Therefore, the area of the equilateral triangle is $9\sqrt{3}$ square units.
Its approximate value is:
9 × 1.732 = 15.588
Therefore, the area is approximately 15.59 square units.