NCERT Solutions For Class 8 Maths

Chapter 11 - Exploring Some Geometric Themes

Ganita Prakash Part 2 - Textbook Chapter 4

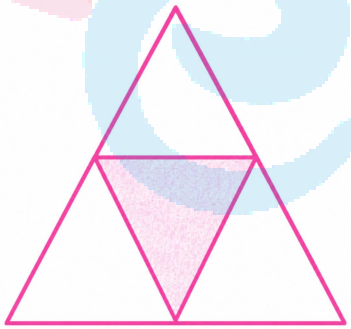
4.1 Fractals

Question 1: Draw the initial few steps (at least till Step 2) of the shape sequence that leads to the Sierpinski Triangle.

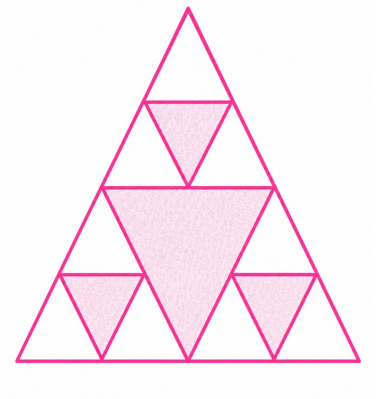
Solution:

Step 0: Draw or take a cut-out of an equilateral triangle.

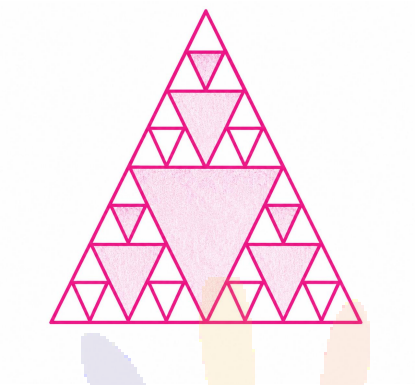
Step 1: Mark the midpoint of each side and join these midpoints. The original triangle is divided into four smaller congruent equilateral triangles. Remove the central inverted triangle.



Step 2: Repeat the same process for each of the three remaining equilateral triangles. Divide each one into four smaller triangles and remove its central triangle.



Step 3: Continue repeating the same process on every remaining triangle to obtain the Sierpinski Triangle.



Question 2: Find the number of holes and the triangles that remain at each step of the shape sequence that leads to the Sierpinski Triangle.

Solution:

At each step, every remaining triangle produces three smaller triangles.

Step 0:

Number of holes = 0

Number of triangles remaining = 1

Step 1:

Number of holes = 1

Number of triangles remaining = 3

Step 2:

Number of holes = 1 + 3 = 4

Number of triangles remaining = $3^2 = 9$

Step 3:

Number of holes = 1 + 3 + 9 = 13

Number of triangles remaining = $3^3 = 27$

Step 4:

Number of holes = 1 + 3 + 9 + 27 = 40

Number of triangles remaining = $3^4 = 81$

Therefore, at Step n:

Number of triangles remaining = 3^n

Number of holes:

= $1 + 3 + 3^2 + \ldots + 3^{n-1}$

= $(3^n - 1)/2$

Question 3: Find the area of the region remaining at the nth step in each of the shape sequences that lead to the Sierpinski fractals. Take the area of the starting square/triangle to the 1 sq. unit.

Solution:

(a) Sierpinski Carpet

Let the area of the starting square be 1 square unit.

At every step, the square is divided into 9 equal parts, and the central part is removed. Therefore, 8/9 of the previous area remains.

Step 0:

Area = 1 square unit

Step 1:

Area = 8/9 square unit

Step 2:

Area = 8/9 × 8/9

= $(8/9)^2$

= 64/81 square unit

Step 3:

Area = $(8/9)^3$

= 512/729 square unit

Therefore, the area remaining after Step n is:

$(8/9)^n$ square units

(b) Sierpinski Triangle

Let the area of the starting triangle be 1 square unit.

At every step, one-fourth of each triangle is removed. Therefore, three-fourths of the previous area remains.

Step 0:

Area = 1 square unit

Step 1:

Area = 3/4 square unit

Step 2:

Area = 3/4 × 3/4

= $(3/4)^2$

= 9/16 square unit

Step 3:

Area = $(3/4)^3$

= 27/64 square unit

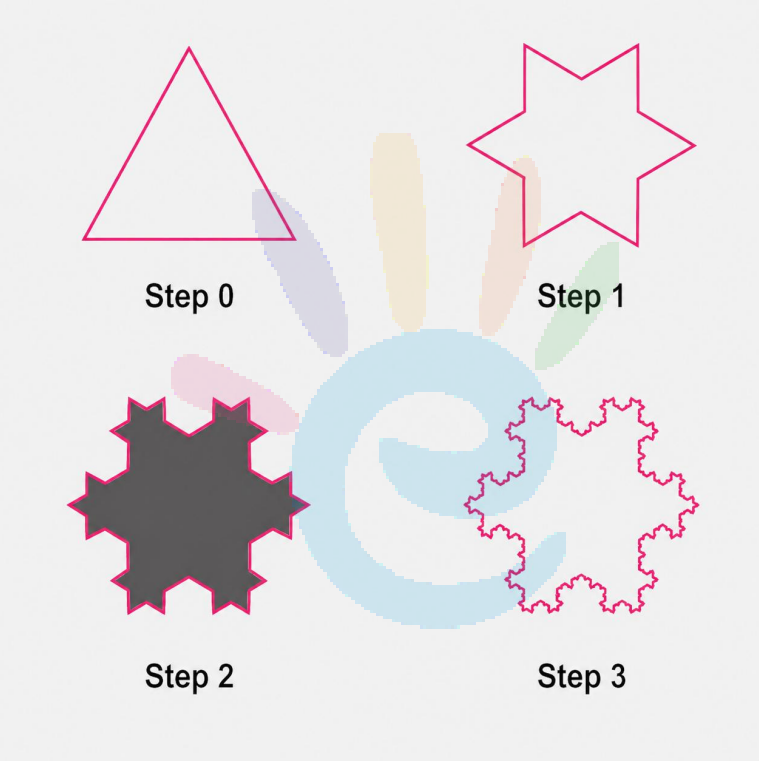
Therefore, the area remaining after Step n is:

$(3/4)^n$ square units

Figure It Out

Question 1: Draw the initial few steps (at least till Step 2) of the shape sequence that leads to the Koch Snowflake.

Solution:



Step 0: Draw an equilateral triangle.

Step 1: Divide each side of the triangle into three equal parts. Replace the middle third of every side with two line segments forming an outward equilateral triangular bump.

Step 2: Repeat the same process on every line segment obtained in Step 1. Divide each segment into three equal parts and replace its middle third with two outward segments.

Repeating this construction continuously produces the Koch Snowflake.

Question 2: Find the number of sides in the nth step of the shape sequence that leads to the Koch Snowflake.

Solution:

Initially, the equilateral triangle has 3 sides.

At every step, each side is replaced by 4 smaller sides.

Step 0:

Number of sides = 3

Step 1:

Number of sides = $3 \times 4 = 12$

Step 2:

Number of sides = $3 \times 4^2 = 48$

Step 3:

Number of sides = $3 \times 4^3 = 192$

Therefore, at Step n:

Number of sides = 3×4^n

Question 3: Find the perimeter of the shape at the nth step of the sequence. Take the equilateral triangle to have a side length of 1 unit.

Solution:

At Step 0, the equilateral triangle has a side length of 1 unit.

Therefore:

Perimeter at Step 0 = 3 units

At every step, each line segment is replaced by four segments, each having one-third of the previous length. Hence, the perimeter is multiplied by 4/3 at every step.

Step 1:

Perimeter = $3 \times 4/3$

Step 2:

Perimeter = $3 \times (4/3)^2$

Step 3:

Perimeter = $3 \times (4/3)^3$

Step 4:

Perimeter = $3 \times (4/3)^4$

Therefore, the perimeter at Step n is:

$3 \times (4/3)^n$ units

4.2 Visualising Solids

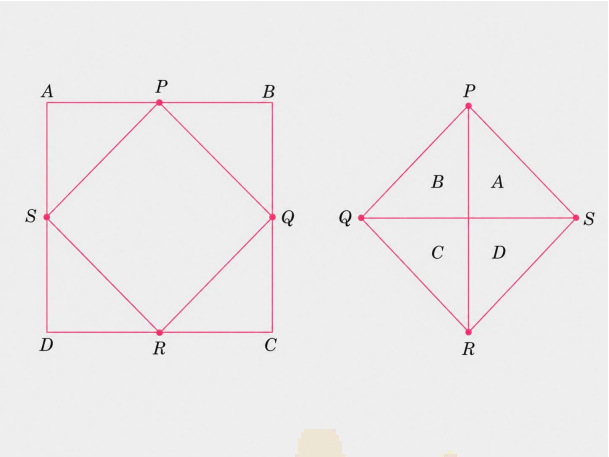
Question 1: Picture your name, then read off the letters backwards. Make sure to do this by sight, not by sound — really see your name! Now try with your friend's name. (Page 75)

Solution:

- This is an activity-based question.
- Visualise your name as it is written and then read the letters from right to left. Repeat the activity using your friend's name. The answer will depend on the names chosen by the student.

Question 2: Cut off the four corners of an imaginary square, with each cut going between midpoints of adjacent edges. What shape is left over? How can you reassemble the four corners to make another square? (Page 76)

Solution:

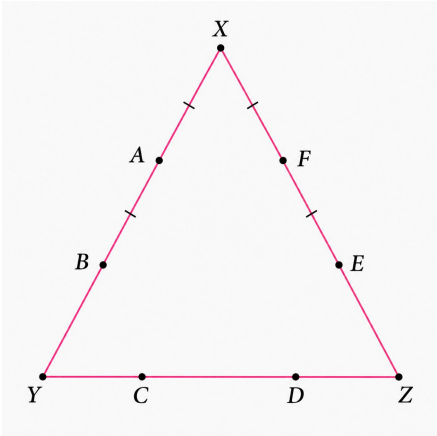


- Let ABCD be a square, and let P, Q, R, and S be the midpoints of AB, BC, CD, and DA, respectively.
- Cut the square along PQ, QR, RS, and SP. The shape left in the centre is another square, PQRS.
- The four corner triangles removed from the original square can be rearranged by joining their equal sides. When placed together appropriately, they form another square having the same area as square PQRS.

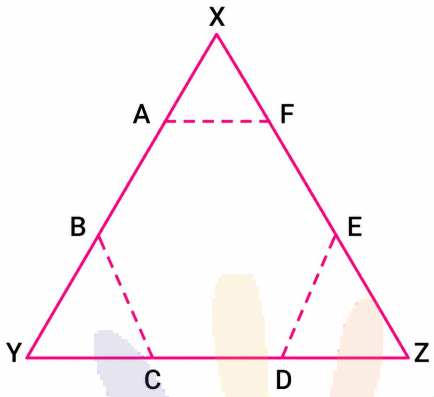
Question 3: Mark the sides of an equilateral triangle into thirds. Cut off each corner of the triangle, as far as the marks. What shape do you get? (Page 76)

Solution:

- Let XYZ be an equilateral triangle.
- Divide each side into three equal parts and cut off the three corners along the marked points.



- After removing the three corner triangles, the remaining shape has six sides.

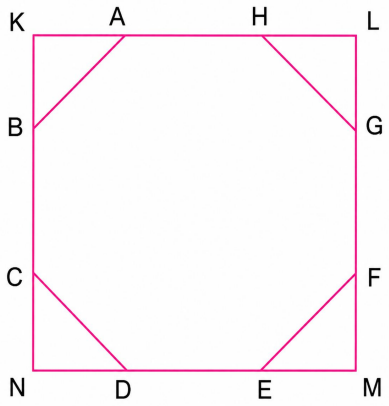


- All six sides are equal, and all its interior angles are equal.
- Therefore, the shape obtained is a regular hexagon.

Question 4: Mark the sides of a square into thirds and cut off each of its corners as far as the marks. What shape is left? (Page 76)

Solution:

- Let KLMN be a square.
- Divide each side of the square into three equal parts. Cut off each corner by joining the nearest division points on the two adjacent sides.
- After removing the four corner triangles, the remaining shape has eight sides.



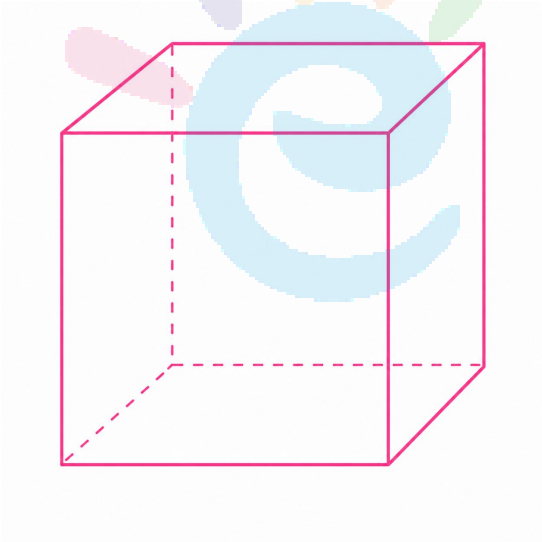
- Therefore, the shape left is an octagon.

Can you describe a solid and a viewpoint that would result in each of the following cases? If it helps, you can imagine the solid passing through a wall like Tom did, and leaving a hole of the appropriate shape. (Page 77)

Question 5: A solid whose profile has a square outline.

Solution:

- A cube can have a square profile.
- When a cube is viewed directly from the front, top, or side, its outline appears as a square.



Question 6: A solid whose profile has a circular outline.

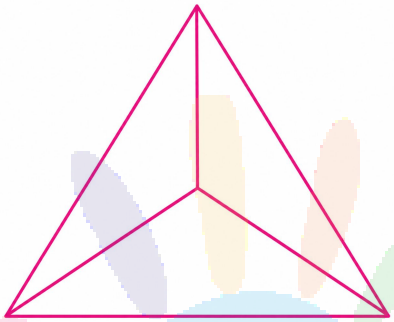
Solution:

- A sphere has a circular profile from every viewpoint.
- A cylinder also has a circular profile when viewed directly from the top or bottom.

Question 7: A solid whose profile has a triangular outline.

Solution:

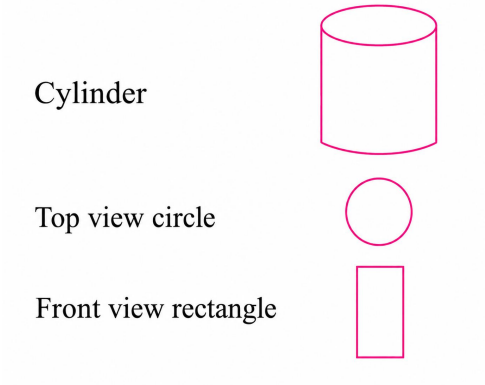
- A triangular pyramid can have a triangular profile when viewed from a suitable direction.
- A cone also appears triangular when viewed directly from the side.



- As we saw with the elephant, a given solid might have very different profiles from different viewpoints. Can you visualise solids that have the following contrasting profiles?
- Spend some time on this, and if you are finding it difficult to visualise, you may look around and use objects that are around you, or that you will make in the next section. Feel free to consider viewpoints from any direction, including directly above the object. (Page 77)

Question 8: A solid with a rectangular profile from one viewpoint and a circular profile from another viewpoint.

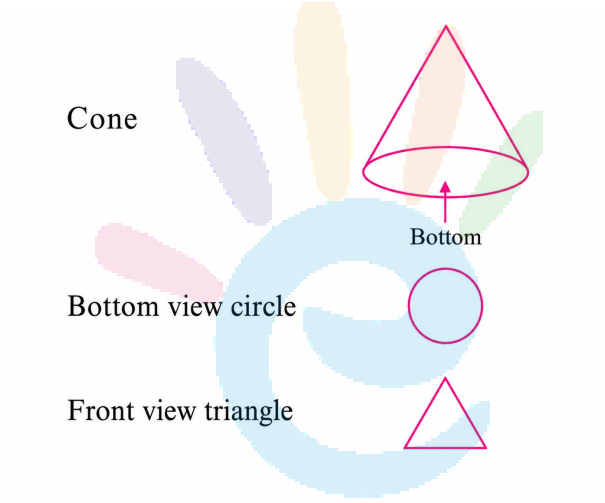
Solution:



- A cylinder satisfies this condition.
- When viewed from the side, the cylinder has a rectangular profile. When viewed directly from the top or bottom, it has a circular profile.

Question 9: A solid with a circular profile from one viewpoint and a triangular one from another viewpoint.

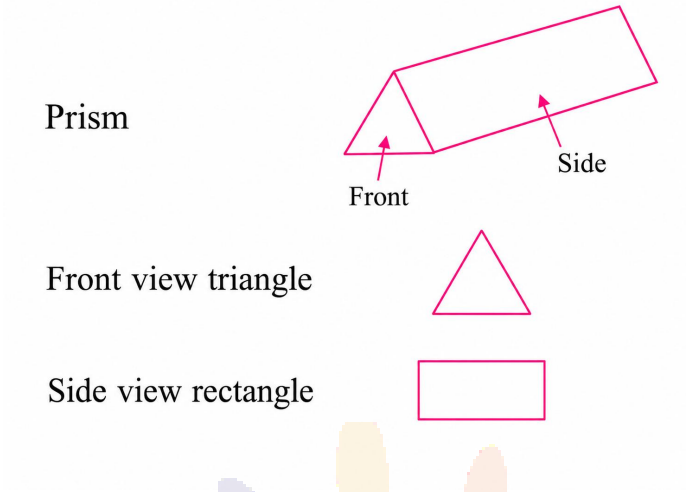
Solution:



- A cone satisfies this condition.
- When viewed from the top, it has a circular profile. When viewed from the side, it has a triangular profile.

Question 10: A solid with a rectangular profile from one viewpoint and a triangular one from another viewpoint.

Solution:

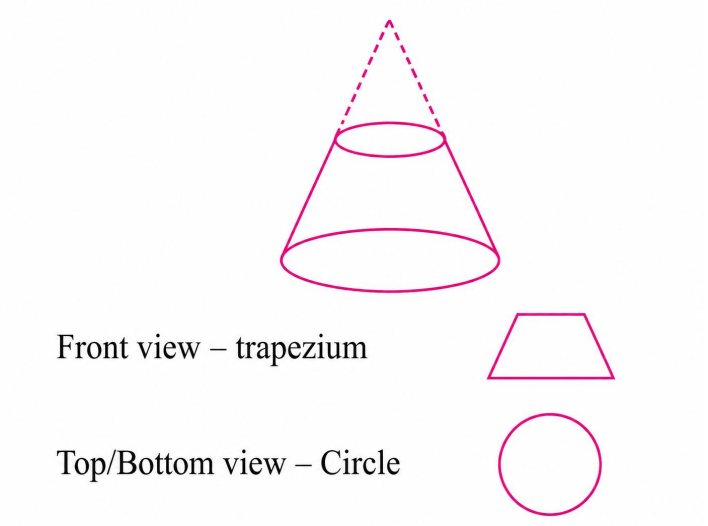


- A triangular prism satisfies this condition.
- When viewed from one end, it has a triangular profile. When viewed from the side, it has a rectangular profile.

Question 11: A solid with a trapezium-shaped profile from one viewpoint and a circular one from another viewpoint.

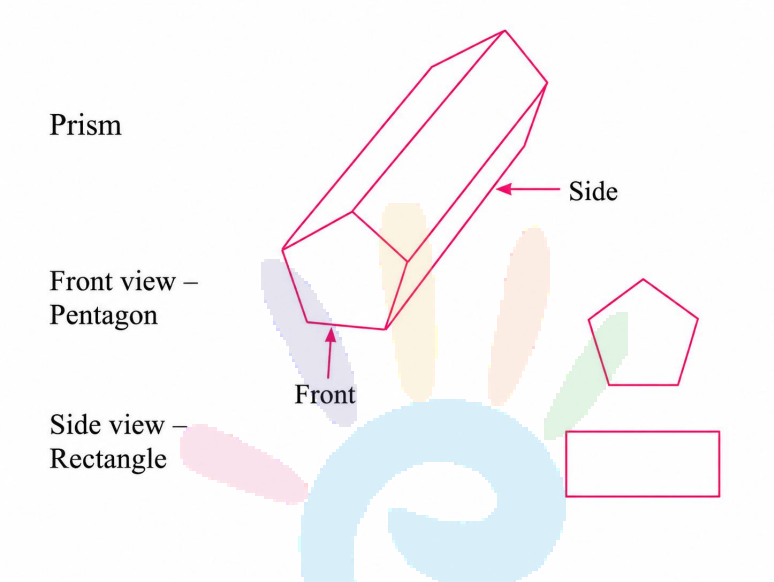
Solution:

- A frustum of a cone satisfies this condition.
- When viewed from the side, it has a trapezium-shaped profile. When viewed from the top or bottom, it has a circular profile.



Question 12: A solid with a pentagonal profile from one viewpoint and a rectangular one from another viewpoint.

Solution:



A pentagonal prism satisfies this condition.

When viewed from either end, it has a pentagonal profile. When viewed from the side, it has a rectangular profile.

4.3 Making Solids, 4.4 Special Solids

Question 1: If the congruent polygons of a prism have 10 sides, how many faces, edges, and vertices does the prism have? What if the polygons have n sides? (Page 79)

Solution:

For a prism with a 10-sided polygon as its base:

Number of sides of each base = 10

Number of faces:

= 10 rectangular side faces + 2 polygonal bases

= 12

Number of edges:

= 10 + 10 + 10

= 30

Number of vertices:

= 2 × 10

= 20

Therefore, the prism has 12 faces, 30 edges, and 20 vertices.

For a prism whose bases are n-sided polygons:

Number of faces = n + 2

Number of edges = 3n

Number of vertices = 2n

Question 2: If the base of a pyramid has 10 sides, how many faces, edges, and vertices does the pyramid have? What if the base is an n-sided polygon? (Page 79)

Solution:

For a pyramid with a 10-sided base:

Number of triangular side faces = 10

Number of bases = 1

Therefore:

Number of faces = 10 + 1 = 11

Number of edges:

= 10 base edges + 10 slant edges

= 20

Number of vertices:

= 10 base vertices + 1 apex

= 11

Therefore, the pyramid has 11 faces, 20 edges, and 11 vertices.

For a pyramid with an n-sided base:

Number of faces = n + 1

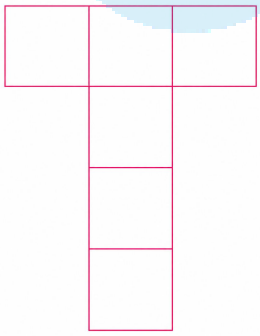
Number of edges = 2n

Number of vertices = n + 1

Question 3: What is a net of a cube? (Page 80)

Solution:

A net of a cube is a two-dimensional arrangement of six congruent squares that can be folded along their common edges to form a cube.

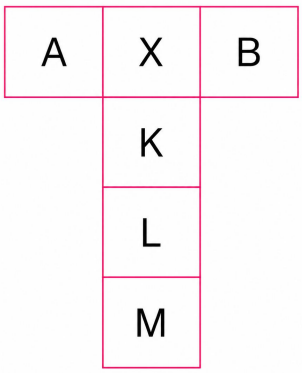


A cube has 11 distinct nets when nets obtained by rotation or reflection are considered the same.

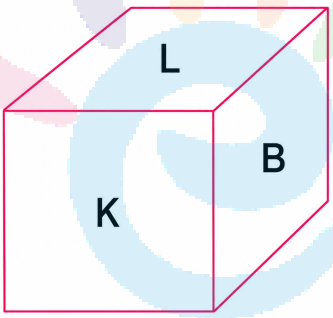
Question 4: Visualise how it can be folded to form a cube. (Page 80)

Solution:

To form a cube, take one square of the net as the base.



Fold the four squares attached to the base upward to form the front, back, left, and right faces.

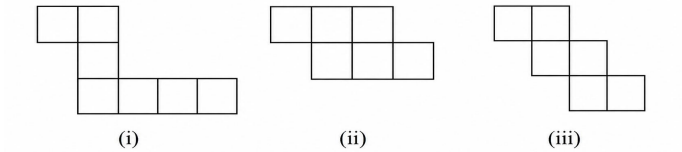


Finally, fold the remaining square over the open top to complete the cube.

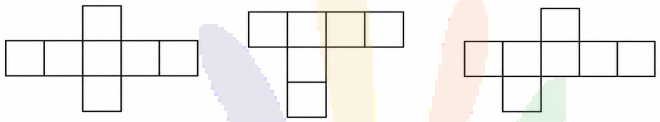
All six squares become the six faces of the cube.

Figure It Out

Question 1: Which of the following are the nets of a cube? First, try to answer by visualisation. Then, you may use cutouts and try.



(i) (ii) (iii)



(iv) (v) (vi)

Solution:

After visualising and folding the given arrangements:

(i) No

(ii) Yes

(iii) Yes

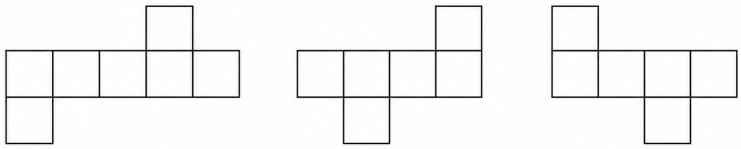
(iv) Yes

(v) No

(vi) Yes

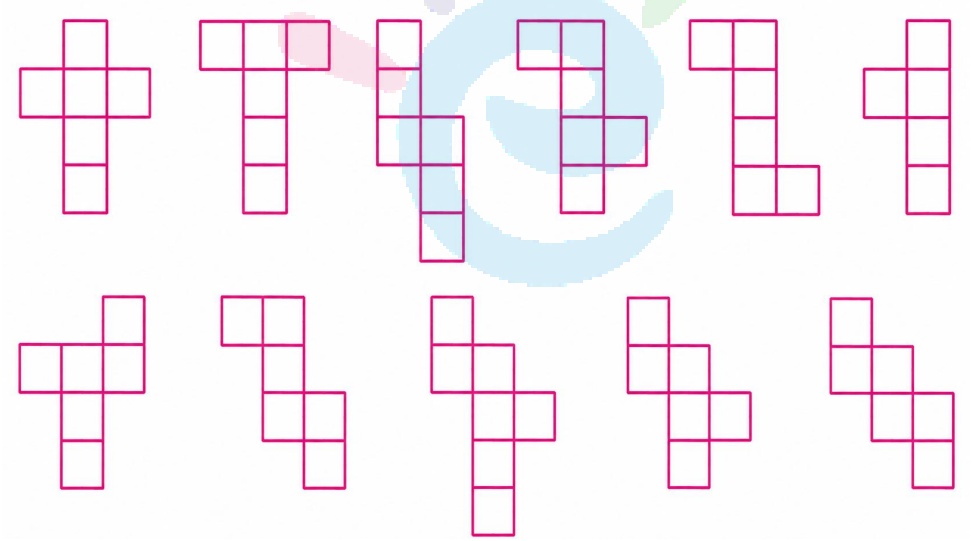
Therefore, arrangements (ii), (iii), (iv), and (vi) are nets of a cube.

Question 2: A cube has 11 possible net structures in total. In this count, two nets are considered the same if one can be obtained from the other by a rotation or a flip. For example, the following nets are all considered the same—



Find all the 11 nets of a cube.

Solution:



A cube has 11 distinct nets.

To find them, arrange six congruent squares so that:

Each square is joined to at least one other square along a complete side.

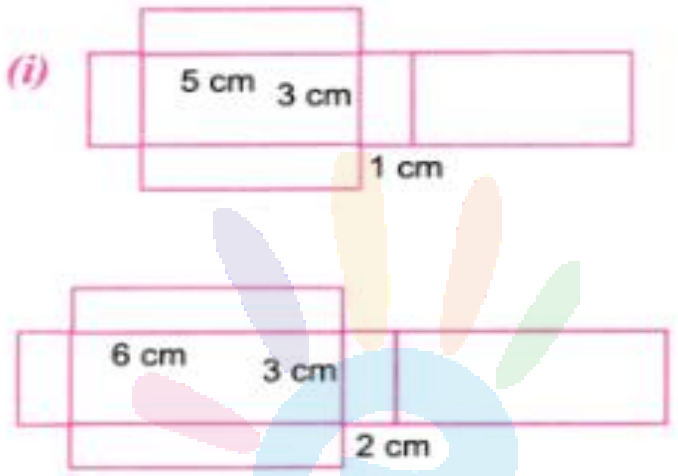
The arrangement can be folded without any two squares overlapping.

All six squares form the six different faces of the cube.

Draw the 11 accepted arrangements shown in the textbook and verify each one by cutting and folding it into a cube.

Question 3: Draw a net of a cuboid having sidelengths:
(i) 5 cm, 3 cm, and 1 cm
(ii) 6 cm, 3 cm, and 2 cm

Solution:



(i) For a cuboid of dimensions 5 cm, 3 cm, and 1 cm, draw:

- Two rectangles of dimensions 5 cm × 3 cm
- Two rectangles of dimensions 5 cm × 1 cm
- Two rectangles of dimensions 3 cm × 1 cm

Arrange these rectangles so that corresponding edges are joined, and the net can be folded into a cuboid.

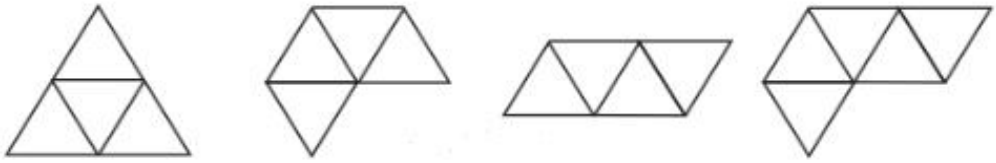
(ii) For a cuboid of dimensions 6 cm, 3 cm, and 2 cm, draw:

- Two rectangles of dimensions 6 cm × 3 cm
- Two rectangles of dimensions 6 cm × 2 cm
- Two rectangles of dimensions 3 cm × 2 cm

Arrange the six rectangles in a valid cuboid net and fold along the common edges.

Intext Questions

Question 1: What is a net of a regular tetrahedron? Which of the following are nets of a regular tetrahedron? (Page 81)



Solution:

- A net of a regular tetrahedron consists of four congruent equilateral triangles joined along their edges.
- After visualising the given arrangements, (ii) and (iv) cannot be folded to form a tetrahedron.
- Therefore, the remaining given arrangements are valid nets of a regular tetrahedron.

Question 2: Draw a net with appropriate measurements that can be folded into a regular tetrahedron. Verify if it works by making an actual cutout. (Page 81)

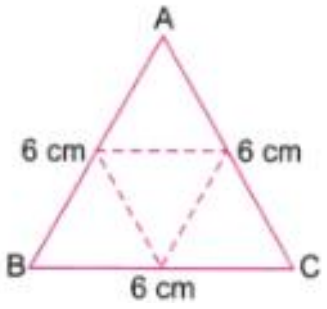
Solution:

Draw an equilateral triangle of side 6 cm.

Mark the midpoint of each side and join the three midpoints. This divides the large triangle into four congruent equilateral triangles.

Cut along the outer boundary and fold the three corner triangles upward along the inner lines.

Join their free edges to form a regular tetrahedron.

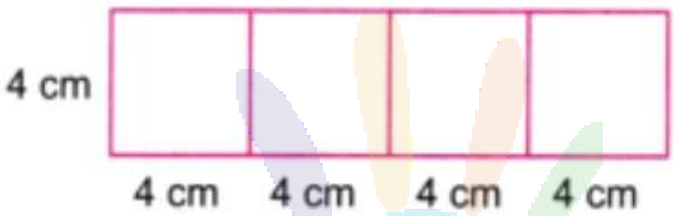


The construction can be verified using an actual paper cutout.

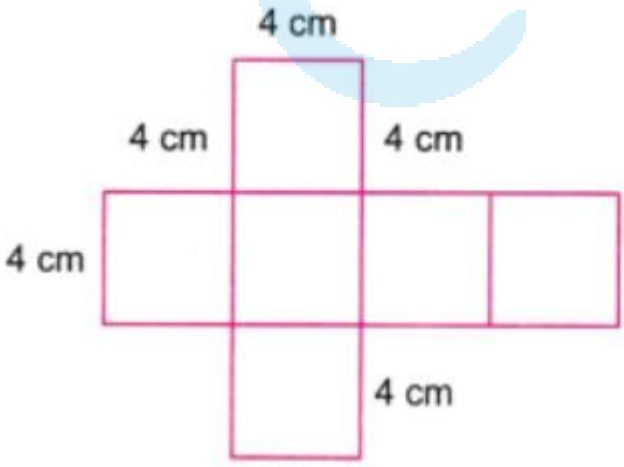
Question 3: Draw a net with appropriate measurements that can be folded into a square pyramid. Verify if it works by making an actual cutout. (Page 81)

Solution:

- Draw a square of side 4 cm to form the base of the pyramid.
- On each of the four sides of the square, draw an identical isosceles triangle whose base is 4 cm.



- Cut along the outer boundary of the complete figure.
- Fold the four triangles upward along the sides of the square and join their slant edges.



- The four triangular faces meet at a common point and form a square pyramid.

Question 4: What is the net of a cylinder? (Page 82)

Solution:

The net of a cylinder consists of:

One rectangle representing the curved surface

Two congruent circles representing the two circular bases

If the cylinder has radius r and height h, the rectangle has:

Length = $2\pi r$

Breadth = h

The two circles each have radius r.

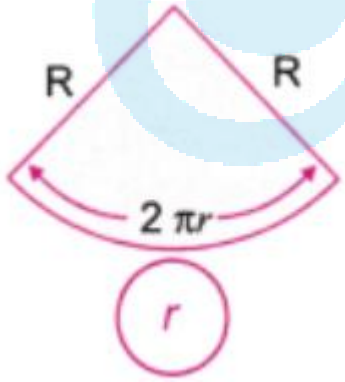
Question 5: How will the net of a cone look? (Page 82)

Solution:

The net of a cone consists of:

A sector of a circle representing the curved surface

A circle representing the base



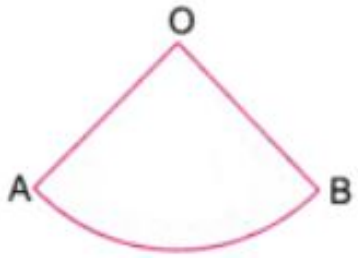
If the cone has slant height R and base radius r, the sector has radius R, and its arc length is equal to the circumference of the base:

Arc length = $2\pi r$

Question 6: What surface do you construct by using the above net, in which O is not the centre of the boundary circle? Make a physical model to help you answer this question! (Page 82)

Solution:

When the two straight edges of the circular sector are brought together and joined, the sector forms the curved surface of a cone.

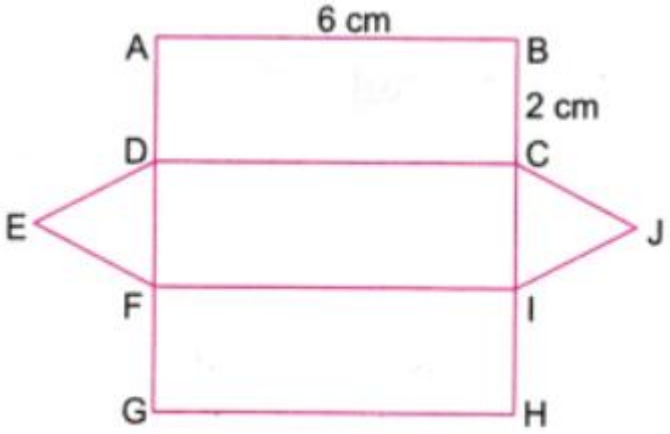


- The point O becomes the vertex of the cone.
- The radius of the sector becomes the slant height of the cone.
- Thus, the surface constructed using the given net is the curved surface of a cone.

Question 7: Draw a net with appropriate measurements that can be folded into a triangular prism. Verify that it works by making an actual cutout. (Page 82)

Solution:

Draw three rectangles of dimensions 6 cm × 2 cm in a row.



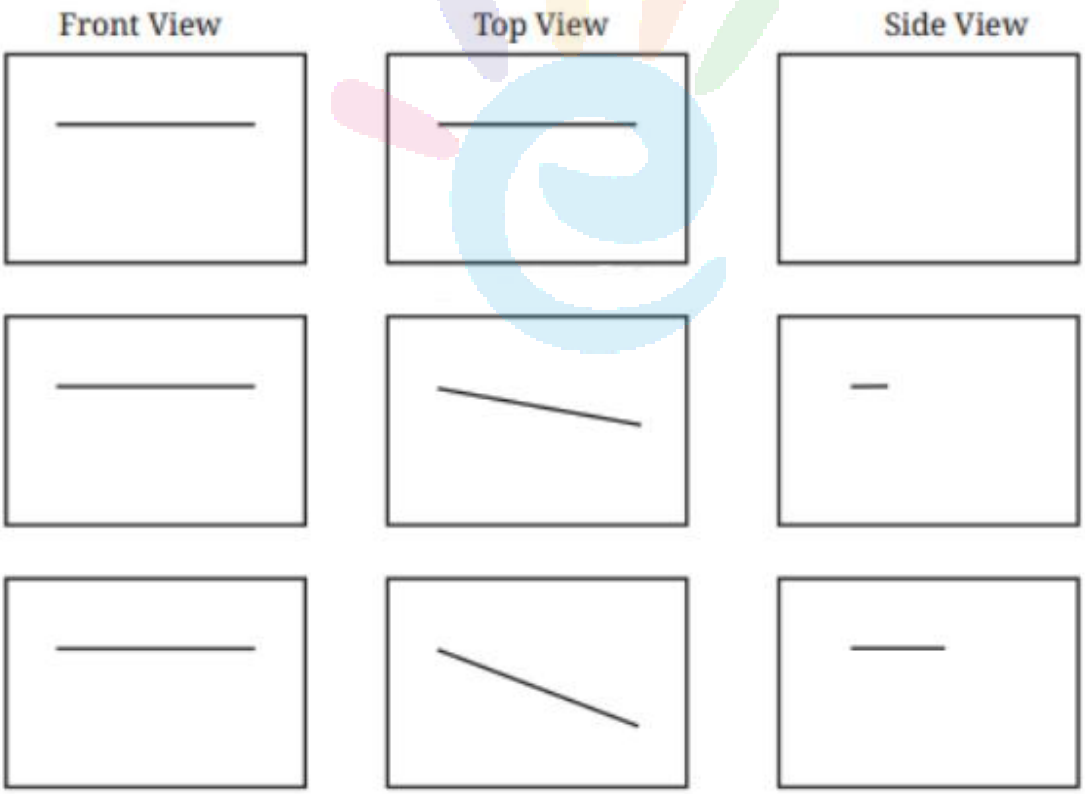
- Attach an equilateral triangle of side 2 cm to one end of the rectangular strip.
- Attach another congruent equilateral triangle to the opposite end.

- Cut along the outer boundary and fold along the common edges.
- The three rectangles form the lateral faces, and the two triangles form the bases of the triangular prism.

Figure It Out

Question 1: Observe the front view, top view, and side view of the different lines in the figure. Is there any relation between their lengths?

Solution:



- The apparent length of a line in a projection depends on its angle with the plane of projection.
- A line parallel to the projection plane appears in its actual length.
- A line tilted with respect to the projection plane appears shorter than its actual length.

- As the inclination of the line increases, the length of its projection changes.
- In the given figure, the top views indicate that line (a) has the shortest projected length, while line (c) has the longest projected length among the three shown.

Question 2: Find the front view, top view, and side view of each of the following solids, fixing its orientation with respect to the vertical, horizontal, and side planes: cube, cuboid, parallelepiped, cylinder, cone, prism, and pyramid. If needed, see the next problem for clues.

Solution:

(a) Cube: Assuming standard orientation with faces parallel to planes

View	Dimensions	Shape
Front view	$l \times l$	Square
Top view	$l \times l$	Square
Side view	$l \times l$	Square

(b) Cuboid: Dimensions

Length = l, Breadth = b, Height = h

View	Dimensions	Shape
Front view	$l \times h$	Rectangle
Top view	$l \times b$	Rectangle
Side view	$b \times h$	Rectangle

(c) Parallelepiped: All faces are parallelograms

View	Shape
Front view	Parallelogram
Top view	Parallelogram
Side view	Parallelogram

(d) Cylinder: Axis vertical

View	Shape
Front view	Rectangle
Top view	Circle
Side view	Rectangle

(e) Cone: Axis vertical, base on horizontal plane

View	Shape
Front view	Isosceles triangle
Top view	Circle
Side view	Isosceles triangle

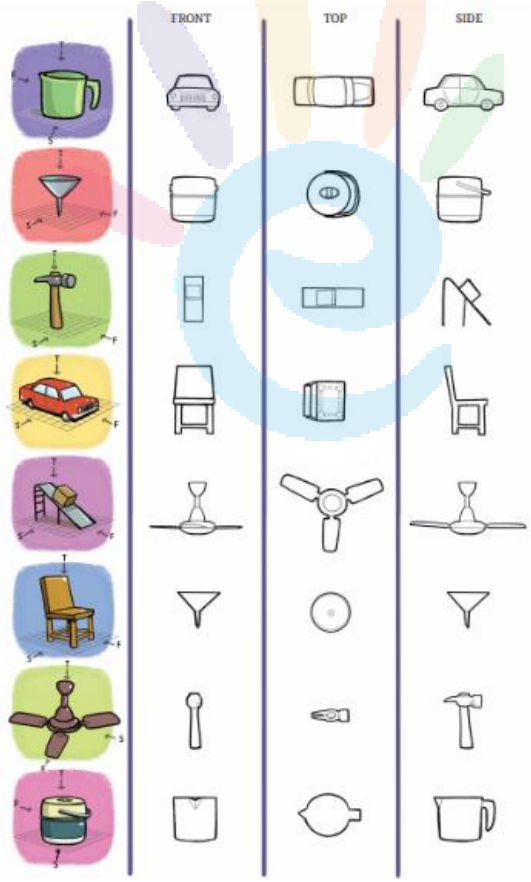
(f) Prism: Regular prism with square base, axis vertical

View	Shape
Front view	Rectangle
Top view	Square
Side view	Rectangle

(g) Pyramid: Square pyramid, axis vertical

View	Shape
Front view	Isosceles triangle
Top view	Square
Side view	Isosceles triangle

Question 3: Match each of the following objects with its projections.



Solution:

The correct matches are:

(a) – (viii)

(b) – (vi)

(c) – (vii)

(d) – (i)

(e) – (iii)

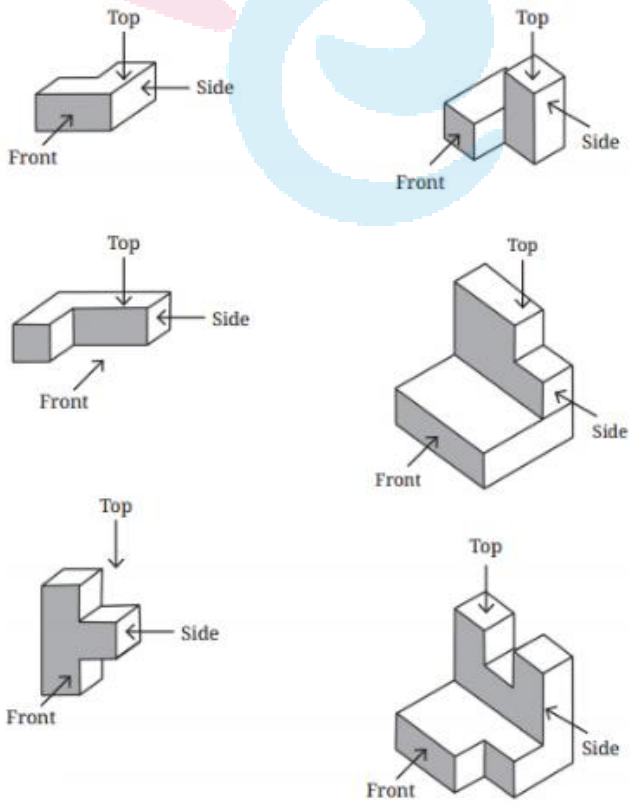
(f) – (iv)

(g) – (v)

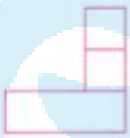
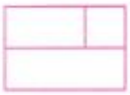
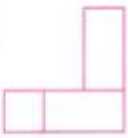
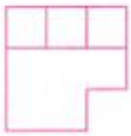
(h) – (ii)

Figure It Out

Question 1: Draw the top view, front view, and side view of each of the following combinations of identical cubes.



Solution:

	Front View	Side View	Top View
(a)			
(b)			
(c)			
(d)			
(e)			
(f)			

Observe each arrangement from the front, top, and side.

- Front view: Draw the visible arrangement of squares when the solid is viewed from the front.
- Top view: Draw the positions occupied by cubes when viewed directly from above.
- Side view: Draw the visible arrangement of squares when viewed from the specified side.

The views should be drawn on a square grid exactly according to the height and position of the cubes shown in each given figure.

Question 2:

Imagine eight identical cubes, glued together along faces to form the letter 'L'.

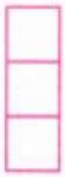
(i) This looks like a 'L' from the front. What does it look like from the side? From the top?

(ii) Glue additional cubes to make a shape that looks like 'L' from the front and 'H' from the top.

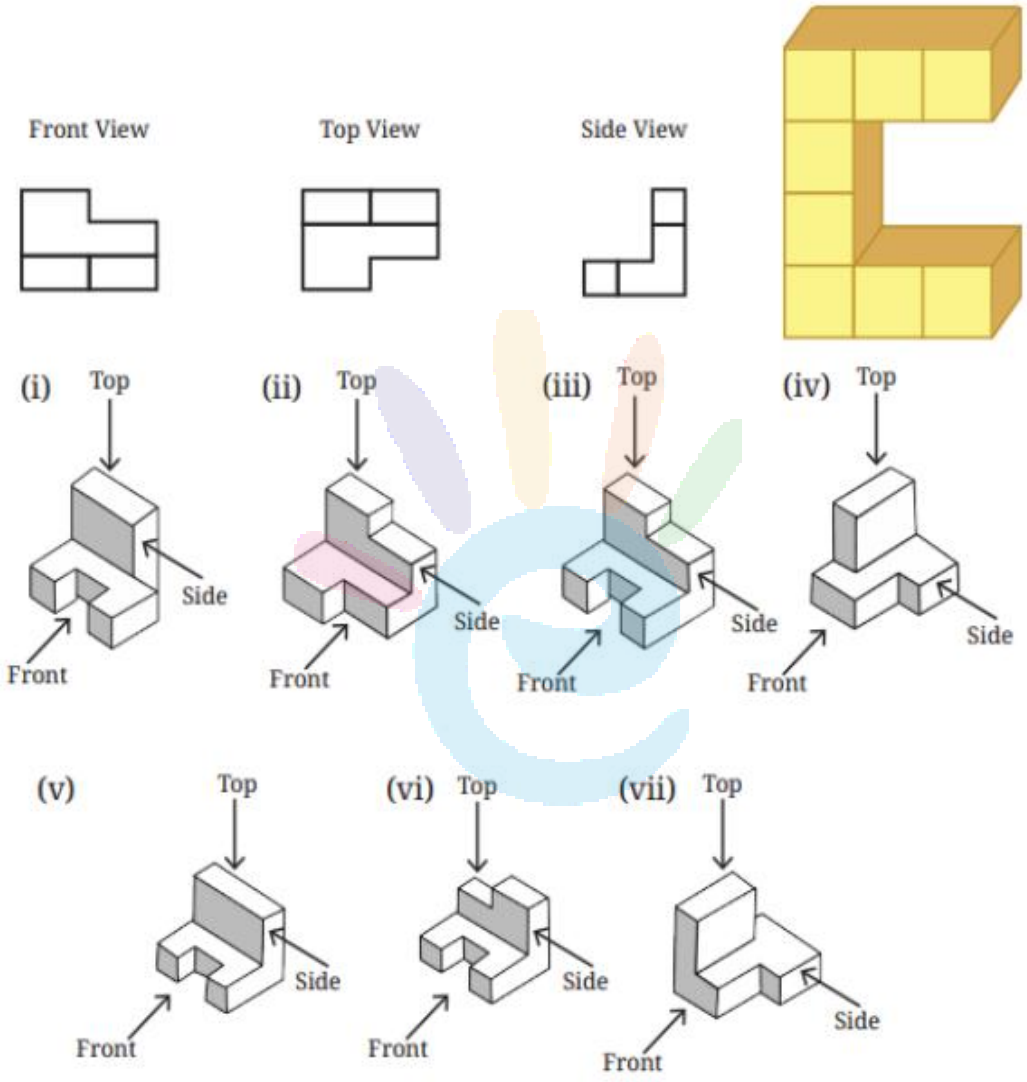
(iii) Now, can you glue even more cubes to make it look like 'L' from the front, 'H' from the top, and 'F' from the side?

(iv) Can you think of other letter combinations to make with a single combination of cubes in this manner?

Solution:

	Side	Top
(i)		
(ii)		
(iii)	Not possible	
(iv)		

Question 3: Which solid corresponds to the given top view, front view, and side view?

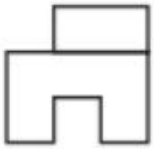


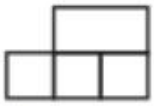
Solution:

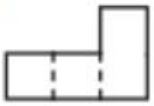
By comparing the number and arrangement of visible squares in the top, front, and side views, the matching solid is:

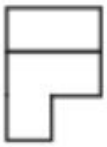
Solid (ii)

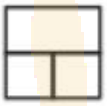
Question 4: Using identical cubes, make a solid that gives the following projections.

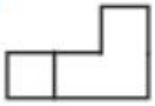
(i) 
Top View

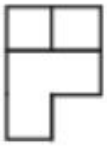
(ii) 
Front View

(iii) 
Side View

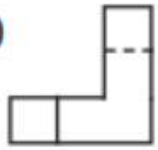
(iv) 
Top View

(v) 
Front View

(vi) 
Side View

(vii) 
Top View

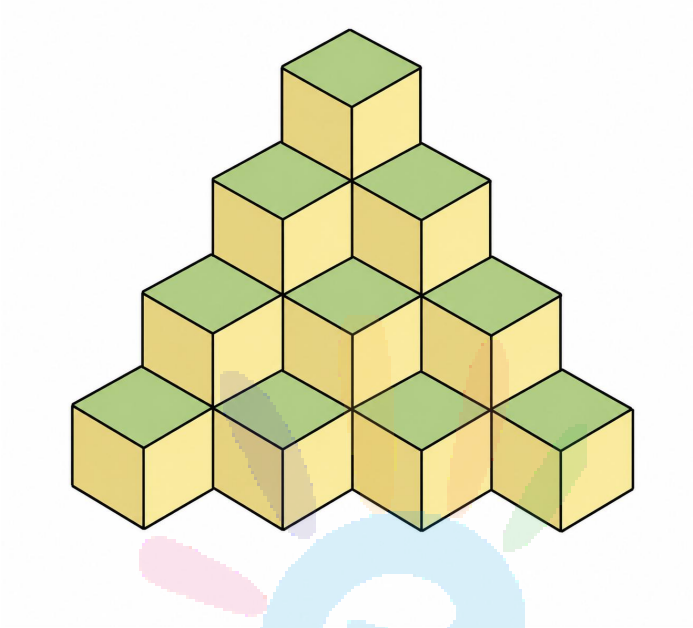
(viii) 
Front View

(ix) 
Side View

Solution:



Question 5: Find the number of cubes in this stack of identical cubes.



Solution:

Count the cubes layer by layer from the top to the bottom.

Top layer = 1 cube

Second layer = 3 cubes

Third layer = 6 cubes

Bottom layer = 10 cubes

Total number of cubes:

= 1 + 3 + 6 + 10

= 20

Hence, the stack contains 20 cubes.

Question 6: What are the different shapes the projection of a cube can make under different orientations?

Solution:

The projection of a cube can produce different shapes depending on its orientation with respect to the projection plane.

Some possible shapes are:

(a) Square

This is obtained when one face of the cube is parallel to the projection plane.

(b) Rectangle

This may appear when the cube is tilted in one direction, and the projection combines visible parts of adjoining faces.

(c) Parallelogram

This can be obtained when the cube is tilted so that a face projects obliquely.

(d) Rhombus

A rhombus-shaped projection may appear in a special tilted orientation.

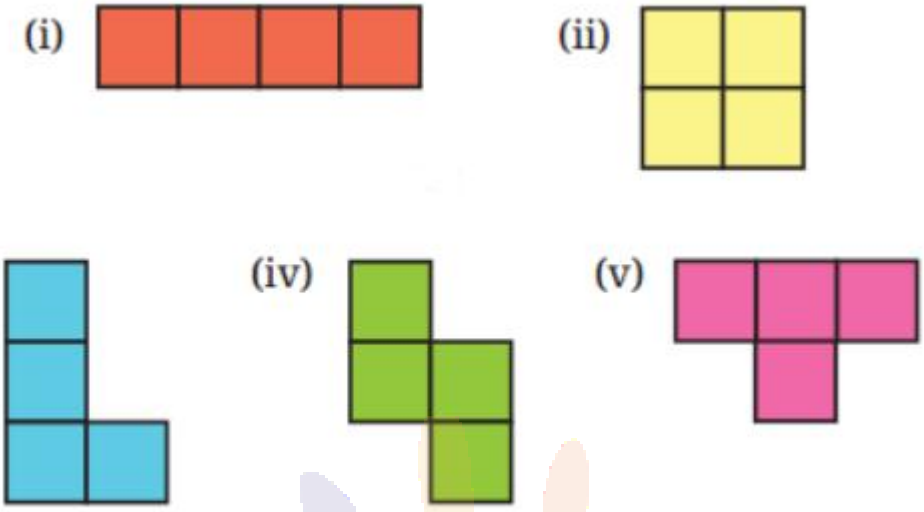
(e) Hexagon

A hexagonal projection can be obtained when the cube is viewed along a body diagonal so that three faces are visible symmetrically.

Thus, a cube can have square, rectangular, parallelogram-shaped, rhombus-shaped, or hexagonal projections under different orientations.

Figure It Out

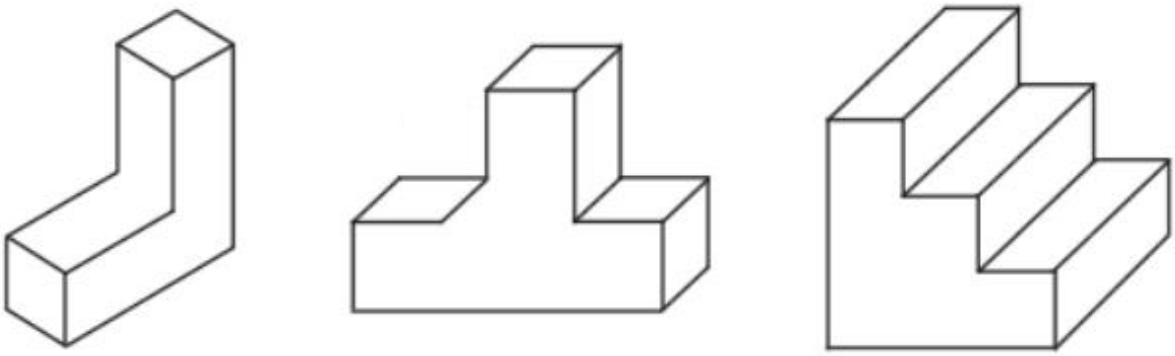
Question 1: In addition to the 5 ways shown in Fig. 4.8, are there any additional ways of gluing four cubes together along faces? Can you visualise and draw these as well? (NCERT Textbook, Page 98)



Solution:

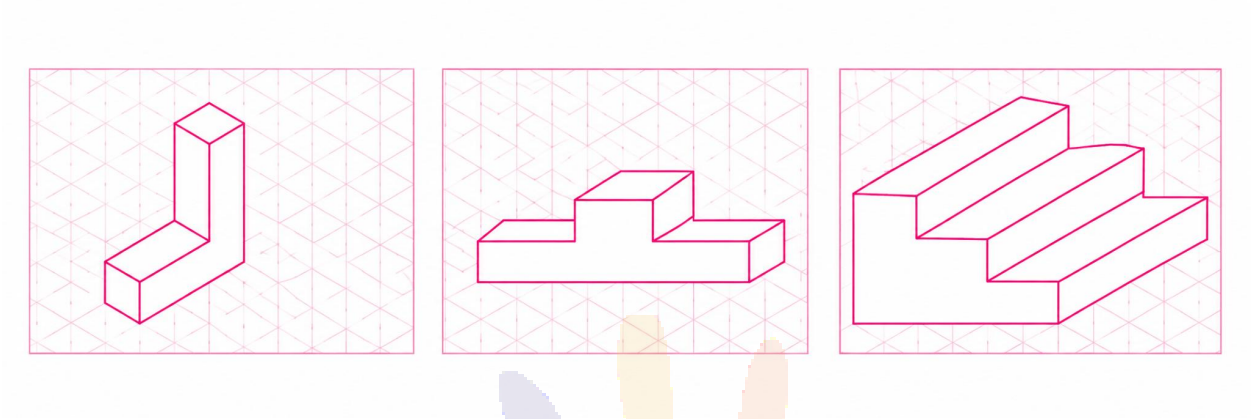


Question 2: Draw the following figures on the isometric grid.

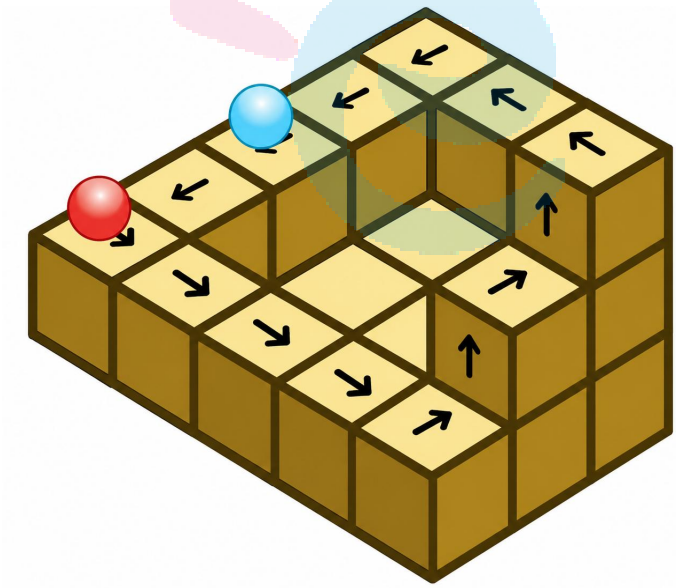


[Hint: It may be useful to determine whether the edge to be currently drawn — say, along the height — goes from down to up or up to down. Accordingly, draw the line segment on the grid either in the direction of the height axis or opposite to it.]

Solution:



Question 3: Is there anything strange about the path of this ball? Recreate it on the isometric grid.



[Hint: Consider a portion of this figure that is physically realisable and identify the 3 primary directions.]

Solution:

The figure represents a Penrose staircase, which is an optical illusion.

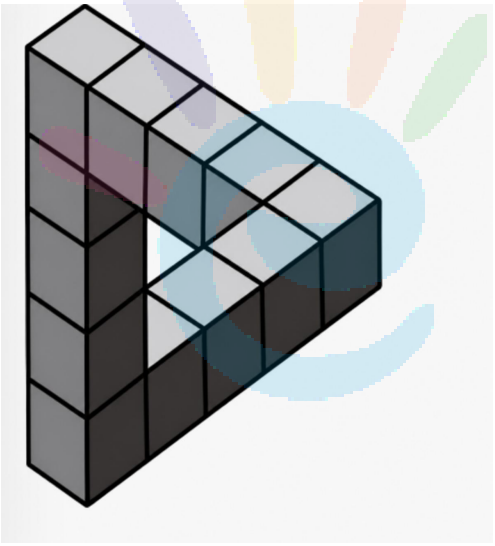
The staircase appears to rise or descend continuously while returning to its starting point. This is impossible in a real three-dimensional structure.

Each small portion of the staircase looks correct when viewed separately. However, when the complete staircase is examined, its heights and directions are inconsistent.

A ball placed on the imaginary staircase appears to roll continuously downward and return to its starting point. In reality, this cannot happen because a ball cannot keep moving downward and return to the same height without gaining energy.

To recreate it on an isometric grid, draw each visible section using the three principal isometric directions. Arrange the sections so that they appear connected from the chosen viewpoint, even though they cannot form a physically possible structure.

Question 4: Observe this triangle.

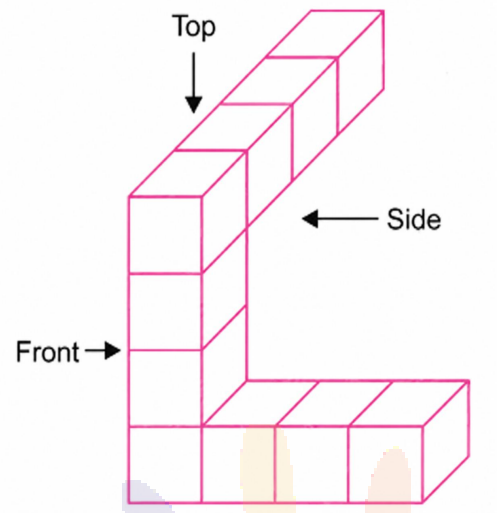


(i) Would it be possible to build a model out of actual cubes? What are the front, top, and side profiles of this impossible triangle?
(ii) Recreate this on an isometric grid.
(iii) Why does the illusion work?

Solution:

(i)

The impossible triangle cannot be constructed as a completely connected and consistent three-dimensional solid using actual cubes.



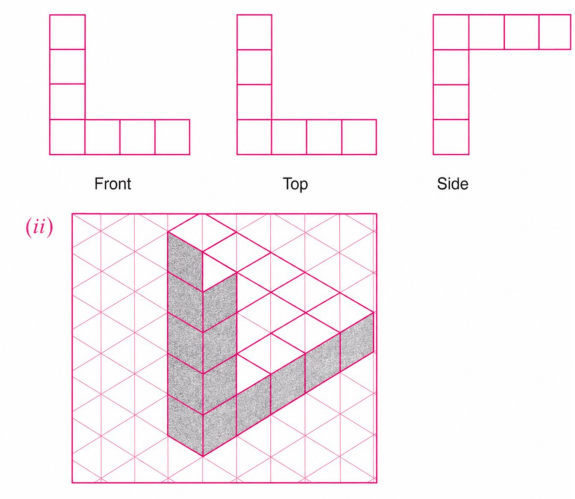
However, separate rows of cubes can be positioned at different distances so that they appear to form a closed triangle when viewed from one carefully selected angle.

From other directions, the gaps and misalignment become visible.

The front, top, and side profiles depend on the exact arrangement used. They reveal that the three arms do not actually meet to form a closed triangular loop.

(ii)

To recreate the figure on an isometric grid:



Draw one horizontal arm using equal cube units.

At one end, draw a vertical or slanting arm.

Draw the third arm in the depth direction so that its endpoint appears to meet the first arm.

Use the three principal directions of the isometric grid to maintain the illusion.

(iii)

The illusion works because the drawing gives conflicting depth information.

Our brain interprets each corner separately as a valid three-dimensional connection.

It assumes that lines that appear continuous are part of the same solid.

Perspective hides the actual separation between the arms.

The visual system prefers smooth and continuous shapes, so it mentally joins the three disconnected arms into one impossible triangle.

Thus, the triangle appears possible in a two-dimensional drawing even though it cannot exist as a complete solid in three-dimensional space.