NCERT Solutions For Class 8 Maths

Chapter 13 - Algebra Play

Ganita Prakash Part 2 - Textbook Chapter 6

6.1 Algebra Play & 6.2 Thinking about 'Think of a Number' Tricks

NCERT Intext Questions (Page 136)

Question 1.
How would you change this game to make the final answer 3? What about 5?

Solution:
(a) To make the final answer 3
Let the number thought of be x.
Step 1: Think of a number
= x
Step 2: Triple the number
= 3x
Step 3: Add 9
= 3x + 9
Step 4: Divide the result by 3
= (3x + 9)/3
= x + 3
Step 5: Subtract the original number
= (x + 3) − x
= 3
Therefore, this set of operations always gives 3 as the final answer.
Example:
Let the number be 23.
Triple it:
3 × 23 = 69
Add 9:
69 + 9 = 78
Divide by 3:
78 ÷ 3 = 26
Subtract the original number:

26 − 23 = 3
Hence, the final answer is 3.
(b) To make the final answer 5
Let the number thought of be x.
Step 1: Think of a number
= x
Step 2: Double the number
= 2x
Step 3: Add 10
= 2x + 10
Step 4: Divide by 2
= (2x + 10)/2
= x + 5
Step 5: Subtract the original number
= (x + 5) − x
= 5
Therefore, this set of operations always gives 5 as the final answer.

Question 2.
Can you come up with more complicated steps that always lead to the same final value?

Solution:
Yes. We can create many such number tricks using algebra.
Consider the following example:
Step 1: Think of a number
= x
Step 2: Multiply it by 5
= 5x
Step 3: Add 25
= 5x + 25
Step 4: Divide the result by 5
= (5x + 25)/5
= x + 5
Step 5: Subtract the original number

= (x + 5) − x
= 5
Thus, the final value will always be 5, regardless of the number chosen.

Example:
Let the number be 18.
Multiply it by 5:
5 × 18 = 90
Add 25:
90 + 25 = 115
Divide by 5:
115 ÷ 5 = 23
Subtract the original number:
23 − 18 = 5
Hence, the final answer is always 5.

Math Talk (Page 137)

Question 1.
Find the dates if the final answers are the following:
(i) 1269
(ii) 394
(iii) 296

Solution:
The rule used in the number trick gives:
Final answer = 100M + 165 + D

Here:
M represents the month number.
D represents the date.

Therefore:
100M + D = Final answer − 165
(i) Final answer = 1269
100M + D = 1269 − 165

100M + D = 1104
1104 = 1100 + 4
Therefore:
M = 11
D = 4
Month 11 is November.
Hence, the date is 4 November, written as 04/11.
(ii) Final answer = 394
100M + D = 394 − 165
100M + D = 229
229 = 200 + 29
Therefore:
M = 2
D = 29
Month 2 is February.
Hence, the date is 29 February, written as 29/02.
Note: This date is valid only in a leap year.
(iii) Final answer = 296
100M + D = 296 − 165
100M + D = 131
131 = 100 + 31
Therefore:
M = 1
D = 31
Month 1 is January.
Hence, the date is 31 January, written as 31/01.

6.3 Number Pyramids

NCERT Intext Questions (Pages 138-139)

Question 1.
Use the same rule to fill these pyramids:

Solution:
The rule is:

Each number in a row is obtained by adding the two numbers directly below it.
(i)
The number at the top is:
6 + 2 = 8
Therefore, the missing number is 8.
(ii)
The numbers in the second row are:
3 + 4 = 7
4 + 3 = 7
The top number is:
7 + 7 = 14
Therefore, the completed numbers are 7, 7 and 14.
(iii)
Starting from the bottom row:
5 + 4 = 9
4 + 5 = 9
5 + 0 = 5
In the next row:
9 + 9 = 18
9 + 5 = 14
The top number is:
18 + 14 = 32
Therefore, the missing numbers are 9, 9, 5, 18, 14 and 32.

Question 2.
Fill the following pyramids:

Solution:
(i)
Let a, b, c, d, e and f represent the missing numbers.
From the upper rows:
a + 22 = 50
a = 50 − 22
a = 28
Also:
b + c = 28 ...(1)

c + d = 22 ...(2)
From the bottom row:
4 + e = b ...(3)
e + 6 = c ...(4)
6 + f = d ...(5)
Adding equations (3) and (4):
b + c = 4 + e + e + 6
b + c = 10 + 2e
Using equation (1):
28 = 10 + 2e
2e = 18
e = 9
Now:
b = 4 + 9
b = 13
c = 9 + 6
c = 15
Using equation (2):
15 + d = 22
d = 7
Using equation (5):
6 + f = 7
f = 1
Therefore:
a = 28
b = 13
c = 15
d = 7
e = 9
f = 1
(ii)
Let a, b, c, d, e and f represent the missing numbers.
From the pyramid:
40 + b = a ...(1)
c + d = 40 ...(2)
d + 9 = b ...(3)

$5 + e = c$ ...(4)
$e + 7 = d$ ...(5)
$7 + f = 9$
Therefore:
$f = 9 - 7$
$f = 2$
Adding equations (4) and (5):
$c + d = (5 + e) + (e + 7)$
$c + d = 12 + 2e$
Using equation (2):
$40 = 12 + 2e$
$2e = 28$
$e = 14$
Now:
$c = 5 + 14$
$c = 19$
$d = 14 + 7$
$d = 21$
$b = 21 + 9$
$b = 30$
$a = 40 + 30$
$a = 70$
Therefore:
$a = 70$
$b = 30$
$c = 19$
$d = 21$
$e = 14$
$f = 2$
(iii)
Let a, b, c, d, e and f represent the missing numbers.
From the pyramid:
$a + b = 35$ ...(1)
$c + d = a$ ...(2)
$d + 7 = b$ ...(3)

From the bottom row:
3 + 5 = c
c = 8
Also:
5 + e = d ...(4)
5 + f = 7 ...(5)
Using the structure of the pyramid:
a = c + d
b = d + 7
Therefore:
a + b = c + d + d + 7
Using a + b = 35 and c = 8:
8 + 2d + 7 = 35
2d + 15 = 35
2d = 20
d = 10
Now:
a = 8 + 10
a = 18
b = 10 + 7
b = 17
Using equation (4):
5 + e = 10
e = 5
Using equation (5):
5 + f = 7
f = 2
Therefore:
a = 18
b = 17
c = 8
d = 10
e = 5
f = 2

Figure It Out (Page 140)

Question 1.
Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.

Solution:
For a three-row number pyramid with bottom-row numbers a, b and c, the top number is:
a + 2b + c
This is because:
Second row numbers are:
a + b and b + c
Therefore, the top number is:
(a + b) + (b + c)
= a + 2b + c
(a)
Bottom row numbers are 4, 13 and 8.
Top number:
= 4 + 2(13) + 8
= 4 + 26 + 8
= 38
Therefore, the topmost number is 38.
(b)
Bottom row numbers are 7, 11 and 3.
Top number:
= 7 + 2(11) + 3
= 7 + 22 + 3
= 32
Therefore, the topmost number is 32.
(c)
Bottom row numbers are 10, 14 and 25.
Top number:
= 10 + 2(14) + 25
= 10 + 28 + 25
= 63
Therefore, the topmost number is 63.

Question 2.
Write an expression for the topmost row of a pyramid with 4 rows in terms of the values in the bottom row.

Solution:
Let the four numbers in the bottom row be:
a, b, c and d
The numbers in the row above are:
e = a + b
f = b + c
g = c + d
The next row contains:
h = e + f
= (a + b) + (b + c)
= a + 2b + c
And:
i = f + g
= (b + c) + (c + d)
= b + 2c + d
The topmost number j is:
j = h + i
= (a + 2b + c) + (b + 2c + d)
= a + 3b + 3c + d
Therefore, the expression for the topmost number is:
a + 3b + 3c + d

Question 3.
Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.

Recall the Virahanka-Fibonacci number sequence 1, 2, 3, 5, … where each number is the sum of the two numbers before it.

Solution:
For a four-row number pyramid whose bottom row is a, b, c and d, the top number is:
a + 3b + 3c + d

(a)
Given:
a = 8
b = 19
c = 21
d = 13
Top number:
= a + 3b + 3c + d
= 8 + 3(19) + 3(21) + 13
= 8 + 57 + 63 + 13
= 141
Therefore, the topmost number is 141.
(b)
Given:
a = 7
b = 18
c = 19
d = 6
Top number:
= a + 3b + 3c + d
= 7 + 3(18) + 3(19) + 6
= 7 + 54 + 57 + 6
= 124
Therefore, the topmost number is 124.
(c)
From the given bottom row:
a = 9
b = 7
c = 5
d = 11
Top number:
= a + 3b + 3c + d
= 9 + 3(7) + 3(5) + 11
= 9 + 21 + 15 + 11
= 56
Therefore, the topmost number is 56.

Question 4.
If the first three Virahanka-Fibonacci numbers are written in the bottom row of a number pyramid with three rows, fill in the rest of the pyramid. What numbers appear in the grid? What is the number at the top? Are they all Virahanka-Fibonacci numbers?

Solution:
The first three Virahanka-Fibonacci numbers are:
1, 2 and 3
Write these in the bottom row.
The left number in the second row is:
1 + 2 = 3
The right number in the second row is:
2 + 3 = 5
The top number is:
3 + 5 = 8
Therefore, the completed pyramid is:
Top row: 8
Second row: 3, 5
Bottom row: 1, 2, 3
The numbers appearing in the pyramid are:
1, 2, 3, 3, 5 and 8
The number at the top is 8.
Yes, every number appearing in the pyramid belongs to the Virahanka-Fibonacci sequence:
1, 2, 3, 5, 8, ...
The number 3 appears twice in the pyramid, but it is still a term of the sequence.

Question 5.
What can you say about the numbers in the pyramid and the number at the top in the following cases?

(i) The first four Virahanka-Fibonacci numbers are written in the bottom row of a four-row pyramid.
(ii) The first 29 Virahanka-Fibonacci numbers are written in the bottom row of a 29-row pyramid.

Solution:

(i)

The first four Virahanka-Fibonacci numbers are:

1, 2, 3, 5

Place them in the bottom row of the four-row pyramid. According to the rule, each number is obtained by adding the two numbers directly below it.

The next row is:

$1 + 2 = 3$

$2 + 3 = 5$

$3 + 5 = 8$

So, the second row is:

3, 5, 8

The next row is:

$3 + 5 = 8$

$5 + 8 = 13$

So, the third row is:

8, 13

The top number is:

$8 + 13 = 21$

Thus, the complete pyramid contains the numbers:

1, 2, 3, 5, 3, 5, 8, 8, 13 and 21

All these numbers belong to the Virahanka-Fibonacci sequence. Some numbers appear more than once.

Therefore, the number at the top of the pyramid is **21**.

(ii)

From the pattern observed above, if the bottom row contains the first n Virahanka-Fibonacci numbers, the number at the top is the $(2n - 1)$th Virahanka-Fibonacci number.

Here:

$n = 29$

Therefore:

$2n - 1 = 2 \times 29 - 1$

$= 58 - 1$

$= 57$

Hence, the number at the top is the **57th Virahanka-Fibonacci number**.

Question 6.

If the bottom row of an n-row pyramid contains the first n Virahanka-Fibonacci numbers, what can we say about the numbers in the pyramid? What can we say about the number at the top?

Solution:
Let the first n Virahanka-Fibonacci numbers be written in the bottom row:
$F_1, F_2, F_3, \ldots, F_n$
In the Virahanka-Fibonacci sequence, every term is obtained by adding the previous two terms.
Therefore, the next row contains:
$F_1 + F_2 = F_3$
$F_2 + F_3 = F_4$
$F_3 + F_4 = F_5$
and so on.

Thus, every row of the pyramid also contains Virahanka-Fibonacci numbers.
As we move upward through the pyramid, the terms continue in the same sequence. For an n-row pyramid, the number at the top is the:
(2n − 1)th Virahanka-Fibonacci number
Therefore:

- All the numbers appearing in the pyramid belong to the Virahanka-Fibonacci sequence.
- The top number is the **(2n − 1)th Virahanka-Fibonacci number**.

6.4 Fun with Grids

Math Talk (Page 142)

Question 1.
Create your own calendar trick. For instance, choose a grid of a different size and shape.

Solution:
We can create calendar tricks by using the fact that:

- The number to the right of any date is 1 more.
- The number directly below any date is 7 more.

(i) A cross-shaped grid of five numbers

Consider the following five numbers:

1, 7, 8, 9 and 15

Their sum is:

1 + 7 + 8 + 9 + 15 = 40

Let the topmost number be a.

Then the other numbers are:

a + 6, a + 7, a + 8 and a + 14

Therefore, their sum is:

a + (a + 6) + (a + 7) + (a + 8) + (a + 14)

= 5a + 35

= 5(a + 7)

Here, a + 7 is the middle number.

Therefore:

Sum of the five numbers = 5 × Middle number

In the given example, the middle number is 8.

So:

5 × 8 = 40

A 3 × 3 calendar grid

Consider the following numbers:

10, 11, 12

17, 18, 19

24, 25, 26

Their sum is:

(10 + 11 + 12) + (17 + 18 + 19) + (24 + 25 + 26)

= 33 + 54 + 75

= 162

Let the top-left number be a.

Then the numbers in the grid are:

a, a + 1, a + 2

a + 7, a + 8, a + 9

a + 14, a + 15, a + 16

Their total is:

a + (a + 1) + (a + 2)

- (a + 7) + (a + 8) + (a + 9)
- (a + 14) + (a + 15) + (a + 16)

= 9a + 72
= 9(a + 8)
Here, a + 8 is the centre number.
Therefore:
Sum of all nine numbers = 9 × Centre number
In this example:
9 × 18 = 162
A 1 × 3 calendar grid
Consider:
28, 29, 30
Their sum is:
28 + 29 + 30 = 87
Let the leftmost number be a.
Then the three numbers are:
a, a + 1 and a + 2
Their sum is:
a + (a + 1) + (a + 2)
= 3a + 3
= 3(a + 1)
Here, a + 1 is the middle number.
Therefore:
Sum of three consecutive dates = 3 × Middle date
In the given example:
3 × 29 = 87
(ii)
Similar tricks can be created using other symmetrical calendar shapes.
The general observation is:
When the selected calendar dates are arranged symmetrically around a central number, their total equals the number of selected cells multiplied by the central number.
Students can select any suitable grid or shape and verify the result by representing the first number as a.

Question 2.
In the following grids, find the values of the shapes and fill in the empty squares:

Solution:

6.5 The Largest Product

Figure It Out (Page 144)

Question 1.
Fill in the digits 1, 3, and 7 Algebra Play Class 8 Solutions Maths Ganita Prakash Part 2 Chapter 6 Page 144 Q1 to make the largest product possible.

Solution:
The digits 1, 3 and 7 must be arranged to form a two-digit number and a one-digit multiplier.

Question 2.
Fill in the digits 3, 5, and 9 to make the largest product possible.

Solution:
The digits 3, 5 and 9 can be arranged in six different ways:
$35 \times 9 = 315$
$39 \times 5 = 195$
$53 \times 9 = 477$
$59 \times 3 = 177$
$93 \times 5 = 465$
$95 \times 3 = 285$
Among these products, the greatest is:
$53 \times 9 = 477$
Therefore, the digits should be arranged as **53 × 9**.
Hence, the largest possible product is **477**.

6.6 Decoding Divisibility Tricks

Figure It Out (Pages 145-147)

Question 1.
In the trick given above, what is the quotient when you divide by 9? Is there a relationship between the two numbers and the quotient?

Solution:
Let the original two-digit number be ab, where b > a.
The original number is:
10a + b
The number obtained by reversing its digits is:
10b + a
Since b > a, the reversed number is larger.
Their difference is:
(10b + a) − (10a + b)
= 10b + a − 10a − b
= 9b − 9a
= 9(b − a)
Therefore, the difference is always divisible by 9.
When it is divided by 9, the quotient is:
9(b − a) ÷ 9
= b − a
Hence, the quotient is equal to the difference between the two digits.

Question 2.
In the trick given above, instead of finding the difference between the two 2-digit numbers, find their sum. What will happen? For example:

We start with 31. After reversing, we get 13. Adding 31 and 13, we get 44.
We start with 28. After reversing, we get 82. Adding 28 and 82, we get 110.
We start with 12. After reversing, we get 21. Adding 12 and 21, we get 33.

Observe that all these numbers are divisible by 11. Is this always true? Can we justify this claim using algebra?

Solution:
Yes, the sum is always divisible by 11.
Let the original two-digit number be:

10a + b
Its reversed number is:
10b + a
Their sum is:
(10a + b) + (10b + a)
= 10a + b + 10b + a
= 11a + 11b
= 11(a + b)
Since a + b is an integer, the sum is always a multiple of 11.
For example:
31 + 13 = 44 = 11 × 4
28 + 82 = 110 = 11 × 10
12 + 21 = 33 = 11 × 3
Therefore, the sum of a two-digit number and its reversed number is always divisible by **11**.

Question 3.
Consider any 3-digit number, say abc (100a + 10b + c). Make two other 3-digit numbers from these digits by cycling these digits around, yielding bca and cab. Now add the three numbers. Using algebra, justify that the sum is always divisible by 37. Will it also always be divisible by 3? [Hint: Look at some multiples of 37.]

Solution:
The three numbers are:
abc = 100a + 10b + c
bca = 100b + 10c + a
cab = 100c + 10a + b
Adding them:
abc + bca + cab
= (100a + 10b + c)
- (100b + 10c + a)
- (100c + 10a + b)

Collecting like terms:
= 111a + 111b + 111c
= 111(a + b + c)

Since:
$111 = 37 \times 3$
we get:
$111(a + b + c) = 37 \times 3(a + b + c)$
Therefore, the sum is always divisible by **37**.
It is also always divisible by **3**, because 111 itself is divisible by 3.
For example, take 153.
The two cyclic numbers are:
531 and 315
Their sum is:
$153 + 531 + 315 = 999$
Now:
$999 = 37 \times 27$
Therefore, 999 is divisible by 37.
Also:
$999 \div 3 = 333$
Hence, the sum is always divisible by both **37 and 3**.

Question 4.
Consider any 3-digit number, say abc. Make it a 6-digit number by repeating the digits, that is, abcabc. Divide this number by 7, then by 11, and finally by 13. What do you get? Try this with other numbers. Figure out why it works. [Hint: Multiply 7, 11, and 13.]

Solution:
Let the original three-digit number be:
$N = 100a + 10b + c$
The repeated six-digit number abcabc can be written as:
$1000N + N$
$= 1001N$
Also:
$1001 = 7 \times 11 \times 13$
Therefore:
$abcabc = 7 \times 11 \times 13 \times N$
When abcabc is divided successively by 7, 11 and 13, the result is:

N
Thus, we get the original three-digit number.
For example, take 836.
Repeating the digits gives:
836836
Since:
$836836 = 1001 \times 836$
and:
$1001 = 7 \times 11 \times 13$
therefore:
$836836 \div 7 \div 11 \div 13 = 836$
Hence, dividing the repeated six-digit number by 7, 11 and 13 gives the **original three-digit number**.

Question 5.
There are 3 shrines, each with a magical pond in the front. If anyone dips flowers into these magical ponds, the number of flowers doubles. A person has some flowers. He dips them all in the first pond and then places some flowers in shrine 1. Next, he dips the remaining flowers in the second pond and places some flowers in shrine 2. Finally, he dips the remaining flowers in the third pond and then places them all in shrine 3. If he placed an equal number of flowers in each shrine, how many flowers did he start with? How many flowers did he place in each shrine?

Solution:
Let the initial number of flowers be x.
Let the equal number of flowers placed in each shrine be k.
After dipping the flowers in the first pond, the number doubles:
$2x$
After placing k flowers in shrine 1, the remaining flowers are:
$2x - k$
After dipping these flowers in the second pond, the number becomes:
$2(2x - k)$
$= 4x - 2k$
After placing k flowers in shrine 2, the remaining flowers are:
$4x - 3k$

After dipping these flowers in the third pond, the number becomes:
$2(4x - 3k)$
$= 8x - 6k$
All these flowers are placed in shrine 3. Since the number placed in shrine 3 is also k:
$8x - 6k = k$
$8x = 7k$
Therefore:
$x = 7k/8$
For x to be a whole number, the smallest possible value of k is 8.
Thus:
$x = 7 \times 8/8$
$x = 7$
Verification:
Initially, the person has 7 flowers.
First pond:
$7 \times 2 = 14$
Flowers placed in shrine 1 = 8
Remaining flowers:
$14 - 8 = 6$
Second pond:
$6 \times 2 = 12$
Flowers placed in shrine 2 = 8
Remaining flowers:
$12 - 8 = 4$
Third pond:
$4 \times 2 = 8$
Flowers placed in shrine 3 = 8
Therefore, the person started with **7 flowers** and placed **8 flowers in each shrine**.

Question 6.
A farm has some horses and hens. The total number of heads of these animals is 55, and the total number of legs is 150. How many horses and how many hens are on the farm? Can you solve this without letter-numbers?
[Hint: If all the 55 animals were hens, then how many legs would there be? Using the difference between this number and 150, can you find the number of horses?]

Solution:

Method 1: Using Algebra

Let the number of horses be x and the number of hens be y.

Since the total number of animals is 55:

$x + y = 55$ …(1)

A horse has 4 legs and a hen has 2 legs.

Therefore:

$4x + 2y = 150$

Dividing by 2:

$2x + y = 75$ …(2)

Subtracting equation (1) from equation (2):

$(2x + y) - (x + y) = 75 - 55$

$x = 20$

Substituting $x = 20$ in equation (1):

$20 + y = 55$

$y = 35$

Therefore:

Number of horses = 20

Number of hens = 35

Method 2: Without Letter-Numbers

Suppose all 55 animals were hens.

The total number of legs would be:

$55 \times 2 = 110$

However, the actual number of legs is 150.

The number of extra legs is:

$150 - 110 = 40$

Replacing one hen with one horse increases the number of legs by:

$4 - 2 = 2$

Therefore, the number of horses is:

$40 \div 2 = 20$

The number of hens is:

$55 - 20 = 35$

Hence, there are **20 horses and 35 hens**.

Question 7.
A mother is 5 times her daughter's age. In 6 years, the mother will be 3 times her daughter's age. How old is the daughter now?

Solution:
Let the daughter's present age be x years.
The mother's present age is:
5x years
After 6 years:
Daughter's age = x + 6
Mother's age = 5x + 6
According to the question:
$5x + 6 = 3(x + 6)$
$5x + 6 = 3x + 18$
$5x - 3x = 18 - 6$
$2x = 12$
$x = 6$
Therefore, the daughter is currently **6 years old**.
Verification:
Mother's present age:
$5 \times 6 = 30$ years
After 6 years:
Daughter's age = 12 years
Mother's age = 36 years
Since:
$36 = 3 \times 12$
the answer is correct.

Question 8.
Two friends, Gauri and Naina, are cowherds. One day, they pass each other on the road with their cows. Gauri says to Naina, "You have twice as many cows as I do". Naina says, "That's true, but if I gave you three of my cows, we would each have the same number of cows". How many cows do Gauri and Naina have?

Solution:
Let the number of cows Gauri has be x.
Since Naina has twice as many cows:
Naina's cows = 2x
If Naina gives 3 cows to Gauri:
Gauri will have:
x + 3
Naina will have:
2x − 3
According to the question, both will then have the same number of cows.
Therefore:
x + 3 = 2x − 3
2x − x = 3 + 3
x = 6
Thus, Gauri has:
6 cows
Naina has:
2 × 6 = 12 cows
Verification:
After Naina gives 3 cows to Gauri:
Gauri has:
6 + 3 = 9 cows
Naina has:
12 − 3 = 9 cows
Therefore, Gauri has **6 cows** and Naina has **12 cows**.

Question 9.
I run a small dosa cart, and my expenses are as follows:
Rent for the dosa cart is ₹ 5000 per day.

The cost of making one dosa (including all the ingredients andfuel) is ₹ 10.
(i) If I can sell 100 dosas a day, what should be the selling price of my dosa to make a profit of ₹ 2000?
(ii) If my customers are willing to pay only ₹ 50 for a dosa, how many dosas should I aim to sell in a day to make a profit of ₹ 2000?

Solution:
Daily rent of the dosa cart = ₹5000
Cost of making one dosa = ₹10
(i)
Number of dosas sold = 100
Cost of making 100 dosas:
100 × ₹10 = ₹1000
Total daily cost:
₹5000 + ₹1000 = ₹6000
Required profit = ₹2000
Therefore, the total revenue required is:
₹6000 + ₹2000 = ₹8000
Selling price of one dosa:
₹8000 ÷ 100 = ₹80
Therefore, each dosa should be sold for **₹80**.
(ii)
Let the number of dosas sold be n.
Selling price of one dosa = ₹50
Total revenue:
₹50n
The total cost includes the fixed rent and the cost of making n dosas:
Total cost = ₹5000 + ₹10n
Required profit = ₹2000
Therefore:
Revenue = Total cost + Profit
50n = 5000 + 10n + 2000
50n − 10n = 7000
40n = 7000
n = 7000/40
n = 175
Therefore, **175 dosas** must be sold to make a profit of ₹2000.

Question 10.
Evaluate the following sequence of fractions:
$1/3,(1+3)/(5+7),(1+3+5)/(7+9+11)$

What do you observe? Can you explain why this happens?

[Hint: Recall what you know about the sum of the first n odd numbers.]

Solution:
The fractions are:
$1/3$
$(1 + 3)/(5 + 7)$
$(1 + 3 + 5)/(7 + 9 + 11)$
Now evaluate them.
First fraction:
$1/3$
Second fraction:
$(1 + 3)/(5 + 7)$
$= 4/12$
$= 1/3$
Third fraction:
$(1 + 3 + 5)/(7 + 9 + 11)$
$= 9/27$
$= 1/3$
Thus, all three fractions are equal to 1/3.
We know that the sum of the first n odd numbers is:
n^2
Therefore, the numerator of the nth fraction is:
n^2
The denominator consists of the next n odd numbers.
The sum of those numbers is:
$3n^2$
Therefore, the value of the fraction is:
$n^2/3n^2$
$= 1/3$
Hence, every fraction in the sequence is equal to **1/3**.

Question 11.

Karim and the Genie

Karim was taking a nap under a tree. He had a dream about a magical lamp and a genie. He heard a voice saying, "I have come to serve you, Oh master". He woke up and to his surprise, it was a genie! "Do you want to make money?" asked the genie. Karim nodded dumbly in bewilderment. The genie continued, "Do you see the banyan tree over there? All you have to do is go around it once. The money in your pocket will double ".

Karim immediately started towards the tree, only to be stopped by the genie. "One moment!" said the genie. "Since I am bringing you great riches, you should share some of your gains with me. You must give me 8 coins each time you go around the tree."

Thinking that was a trifling amount, Karim readily agreed.

He went around the tree once. Just as the genie had said, the number of coins in his pocket doubled! He gave 8 coins to the genie. He made another round. Again, the number of coins doubled. He gave 8 more coins to the genie. He went around the tree for the third time. The number of coins doubled again, but to his horror, he was left with only 8 coins, exactly the number of coins he owed the genie!

As Karim began to wonder how the genie tricked him, the genie let out a loud laugh and disappeared.

(i) How many coins did Karim initially have?
(ii) For what cost per round should Karim agree to the deal, if he wants to increase the number of coins he has?
(iii) Through its magical powers, the genie knows the number of coins that Karim has. How should the genie set the cost per round so that it gets all of Karim's coins?

Solution:
(i)
Let Karim initially have n coins.

After the first round, the number of coins doubles:
$2n$
After giving 8 coins to the genie, Karim has:
$2n - 8$
After the second round, the number doubles again:
$2(2n - 8)$
After paying another 8 coins:
$2(2n - 8) - 8$
$= 4n - 24$
After the third round, the number doubles:
$2(4n - 24)$
$= 8n - 48$
Karim then has exactly 8 coins, which he gives to the genie. Therefore, nothing remains:
$8n - 48 - 8 = 0$
$8n - 56 = 0$
$8n = 56$
$n = 7$
Therefore, Karim initially had **7 coins**.
Verification:
Initially = 7 coins
After the first round and payment:
$2 \times 7 - 8 = 6$
After the second round and payment:
$2 \times 6 - 8 = 4$
After the third round:
$2 \times 4 = 8$
He gives these 8 coins to the genie and is left with 0 coins.
(ii)
Let the cost charged by the genie per round be c coins.
Karim starts with 7 coins.
After the first round and payment:
$2 \times 7 - c$
$= 14 - c$
After the second round and payment:
$2(14 - c) - c$

= 28 − 3c
After the third round and payment:
2(28 − 3c) − c
= 56 − 7c
For Karim to finish with more coins than the 7 coins he started with:
56 − 7c > 7
49 > 7c
c < 7
Therefore, the cost per round must be **less than 7 coins**.
If the charge must be a whole number, the maximum suitable charge is 6 coins per round.
For c = 6:
Final coins:
56 − 7 × 6
= 56 − 42
= 14
Thus, Karim's 7 coins become 14 coins.
(iii)
Let Karim initially have n coins and let the genie charge c coins after each round.
After three rounds, Karim will have:
8n − 7c
For the genie to take all of Karim's coins:
8n − 7c = 0
7c = 8n
c = 8n/7
Therefore, the genie should charge:
8n/7 coins per round
where n is the number of coins Karim initially has.
For this charge to be a whole number, n must be a multiple of 7.