NCERT Solutions For Class 8 Maths

Chapter 14 - Area

Ganita Prakash Part 2 - Textbook Chapter 7

7.1 Rectangle and Squares

Figure It Out (Pages 150-152)

Question 1.
Identify the missing sidelengths.

Solution:
(i)
After naming the figure, consider rectangle ABCD.
Area of a rectangle = Length × Breadth
7 × BC = 21
BC = 21/7
BC = 3 in
Now:
AF = AD + DF
= 3 in + 4 in
= 7 in
In rectangle EFAG:
Area = EF × AF
28 = EF × 7
EF = 28/7
EF = 4 in
Also:
HA = HG + GA
= 3 in + 4 in
= 7 in
In rectangle HIJA:
Area = HA × AJ
35 = 7 × AJ
AJ = 35/7
AJ = 5 in

Now:
AK = AJ + JK
= 5 in + 2 in
= 7 in
In rectangle KLMA:
Area = KL × LM
14 = x × 7
x = 14/7
x = 2 in
Therefore, the missing sidelength is **2 in**.
(ii)
In rectangle ABGH:
Area = AB × AH
29 = AB × 4
AB = 29/4 m
AB = 7.25 m
Area of rectangle HGDC:
= Area of rectangle ABDC − Area of rectangle ABGH
= 50 − 29
= 21 m^2
In rectangle HGDC:
Area = CD × GD
21 = 29/4 × GD
GD = 21 × 4/29
GD = 84/29 m
GD ≈ 2.90 m
In rectangle BEFG:
Area = BG × BE
11 = 4 × BE
BE = 11/4 m
BE = 2.75 m
Therefore:
AB = 29/4 m or 7.25 m
BE = 11/4 m or 2.75 m
GD = 84/29 m or approximately 2.90 m

Question 2.
The figure shows a path (the shaded portion) laid around a rectangular park EFGH.

(i) What measurements do you need to find the area of the path? Once you identify the lengths to be measured, assign possible values of your choice to these measurements and find the area of the path. Give a formula for the area.
An example of a formula — Area of a rectangle = length × width.
[Hint: There is a relation between the areas of EFGH, the path, and ABCD.]

(ii) If the width of the path along each side is given, can you find its area? If not, what other measurements do you need? Assign values of your choice to these measurements and find the area of the path. Give a formula for the area using these measurements.
[Hint: Break the path into rectangles.]

(iii) Does the area of the path change when the outer rectangle is moved while keeping the inner rectangular park EFGH inside it, as shown?

Solution:
(i)
To find the area of the path, we need the following measurements:

- Length of the outer rectangle ABCD = A
- Breadth of the outer rectangle ABCD = B
- Length of the inner rectangular park EFGH = a
- Breadth of the inner rectangular park EFGH = b

Let:
A = 10 m
B = 8 m
a = 6 m
b = 4 m
Area of outer rectangle ABCD:
= A × B
= 10 × 8
= 80 m^2
Area of inner rectangle EFGH:

= a × b
= 6 × 4
= 24 m²
Area of the path:
= Area of outer rectangle − Area of inner rectangle
= 80 − 24
= 56 m²
Therefore, the area of the path is **56 m²**.
Formula:
Area of path = (A × B) − (a × b)
(ii)
The width of the path alone is not sufficient. We must also know the length and breadth of either the inner park or the outer rectangle.
Let:
Width of the path = d = 2 m
Length of the inner park = l = 16 m
Breadth of the inner park = w = 11 m
Since the path has a uniform width on all sides:
Length of outer rectangle:
= l + 2d
= 16 + 2(2)
= 20 m
Breadth of outer rectangle:
= w + 2d
= 11 + 2(2)
= 15 m
Area of outer rectangle:
= 20 × 15
= 300 m²
Area of inner rectangle:
= 16 × 11
= 176 m²
Area of path:
= 300 − 176
= 124 m²
Therefore, the area of the path is **124 m²**.

The same result can be obtained by dividing the path into four rectangles:
Area of left part:
$= w \times d$
$= 11 \times 2$
$= 22\ m^2$
Area of right part:
$= 11 \times 2$
$= 22\ m^2$
Area of top part:
$= (l + 2d) \times d$
$= 20 \times 2$
$= 40\ m^2$
Area of bottom part:
$= 20 \times 2$
$= 40\ m^2$
Total area:
$= 22 + 22 + 40 + 40$
$= 124\ m^2$
Formula:
Area of path $= (l + 2d)(w + 2d) - lw$
On simplifying:
Area of path $= 2d(l + w) + 4d^2$
(iii)
No, the area of the path does not change when the inner rectangular park is moved inside the outer rectangle.
This is because:
Area of path = Area of outer rectangle − Area of inner rectangle
As long as the dimensions of both rectangles remain unchanged, their areas also remain unchanged. Therefore, the area of the path remains the same.

Question 3.
The figure shows a plot with sides 14m and 12m, and with a crosspath. What other measurements do you need to find the area of the crosspath? Once you identify the lengths to be measured, assign some possible values of your choice and find the area of the path. Give a formula for the area based on the measurements you choose.

Solution:

Given:

Length of the plot = 14 m

Breadth of the plot = 12 m

We also need:

- Width of the horizontal path = w_1
- Width of the vertical path = w_2

Let:

$w_1 = 2$ m

$w_2 = 2$ m

Area of the horizontal path:

= Length of plot × Width of horizontal path

= 14 × 2

= 28 m²

Area of the vertical path:

= Breadth of plot × Width of vertical path

= 12 × 2

= 24 m²

The central overlapping region has been counted twice. Its area is:

= $w_1 \times w_2$

= 2 × 2

= 4 m²

Therefore:

Area of crosspath:

= 28 + 24 − 4

= 48 m²

Hence, the area of the crosspath is **48 m²**.

Formula:

Area of crosspath = $(L \times w_1) + (W \times w_2) - (w_1 \times w_2)$

Here:

L = Length of the plot

W = Breadth of the plot

w_1 = Width of the horizontal path

w_2 = Width of the vertical path

Question 4.
Find the area of the spiral tube shown in the figure. The tube has the same width throughout.

[Hint: There are different ways of finding the area. Here is one method.]
What should be the length of the straight tube if it is to have the same area as the bent tube on the left?

Solution:
The width of the spiral tube is 1 unit throughout.
Divide the spiral tube into separate non-overlapping rectangles.
Area of the spiral tube:
= Area of ABEC + Area of DEGF + Area of GHIJ
- Area of JKML + Area of NOPL + Area of PQRS
- Area of STUV + Area of VWYX + Area of XZA_1B_1

= 20 × 1 + 18 × 1 + 20 × 1 + 13 × 1
+ 15 × 1 + 8 × 1 + 10 × 1 + 3 × 1 + 5 × 1
= 20 + 18 + 20 + 13 + 15 + 8 + 10 + 3 + 5
= 112 square units
Therefore, the area of the spiral tube is **112 square units**.
For the bent tube shown on the left:
Area of first rectangular portion:
= 5 × 1
= 5 square units
Area of second rectangular portion:
= 4 × 1
= 4 square units
Total area of the bent tube:
= 5 + 4
= 9 square units
Let the length of the straight tube be x units.
Since its width is 1 unit:
Area of straight tube:
= x × 1
= x square units
The straight and bent tubes must have equal areas.

Therefore:
x = 9
Hence, the straight tube should be **9 units long**.

Question 5.
In this figure, if the sidelength of the square is doubled, what is the increase in the areas of the regions 1, 2, and 3? Give reasons.

Solution:
Let 'a' be the side of the square.
Area of triangle, DCB (region 3)
= 1/2 × base × height = 1/2 × DC × BC
= 1/2 × a × a = $a^2/2$ sq. units
In right angled DCB,
DB = $\sqrt{(DC^2 + BC^2)} = \sqrt{(a^2 + a^2)} = \sqrt{2}a$ units
∴ AE = DE = EB = DB/2
= $\sqrt{2}a/2$
= $(\sqrt{2} \times \sqrt{2})/(2 \times \sqrt{2})$ a
= $2/(2\sqrt{2})$ a
= $a/\sqrt{2}$ units
Area of triangle, AED (region 1)
= 1/2 × DE × AE
= 1/2 × $a/\sqrt{2}$ × $a/\sqrt{2}$
= $a^2/4$ sq. units
Area of triangle, AEB (region 2)
= 1/2 × EB × AE
= 1/2 × $a/\sqrt{2}$ × $a/\sqrt{2}$
= $a^2/2$ × 1/2
= $a^2/4$ sq. units
If the side of the square is doubled = 2a
Area of triangle, BCD (region 3)
= 1/2 × DC × BC
= 1/2 × 2a × 2a = $2a^2$ sq. units
In right angled triangle, BCD.
By Pythagoras theorem,

$BD = \sqrt{(DC^2 + BC^2)}$
$= \sqrt{((2a)^2 + (2a)^2)}$
$= \sqrt{(4a^2 + 4a^2)} = 2\sqrt{2}a$ units
$\therefore AE = ED = EB = BD/2$
$= 2\sqrt{2}a/2$
$= \sqrt{2}a$ units
Area of triangle, AED (region 1)
$= 1/2 \times DE \times AE$
$= 1/2 \times \sqrt{2}a \times \sqrt{2}a$
$= 1/2 \times 2a^2 = a^2$ sq. units
Area of triangle, AEB (region 2)
$= 1/2 \times EB \times AE$
$= 1/2 \times \sqrt{2}a \times \sqrt{2}a$
$= a^2$ sq. units
The increase in area of region 1
$= a^2/(a^2/4) = 4$ times
The increase in area of region 2
$= a^2/(a^2/4) = 4$ times
The increase in area of region 3
$= 2a^2/(a^2/2)$
$= 2a^2/a^2 \times 2 = 4$ times
Thus, the increase in the areas of regions 1, 2, and 3 is 4 times.
Reason: If the sidelength of the square is doubled, then the area becomes 4 times.

Question 6.
Divide a square into 4 parts by drawing two perpendicular lines inside the square as shown in the figure.

Rearrange the pieces to get a larger square, with a hole inside.

You can try this activity by constructing the square using cardboard, thick chart paper, or similar materials.

Solution:
This is an activity-based question. It can be completed through the following steps:

1. Take a square piece of cardboard or thick chart paper, for example, an 8 cm × 8 cm square.
2. Draw two perpendicular lines inside the square as shown in the textbook. The lines should not pass through the centre.
3. Cut the square carefully along the two lines. This will produce four rectangular pieces.
4. Rearrange the four pieces by placing them near the four corners of a larger square.
5. Adjust the pieces so that their outer boundaries form a larger square.
6. A smaller square-shaped hole will appear at the centre.

The total area of the four pieces remains unchanged because no material is added or removed. Only their positions are changed.

Triangles

Figure It Out (Pages 157-159)

Question 1.
Find the areas of the following triangles:

Solution:
(i)
For triangle ABC:
Base BC = 4 cm
Height AE = 3 cm
Area of a triangle:
= 1/2 × Base × Height
Area of triangle ABC:
= 1/2 × 4 × 3
= 6 cm²
Therefore, the area of triangle ABC is **6 cm²**.
(ii)
For triangle DEF:
Base EF = 5 cm
Perpendicular height DN = 3.2 cm

Area of triangle DEF:
= 1/2 × 5 × 3.2
= 2.5 × 3.2
= 8 cm²
Therefore, the area of triangle DEF is **8 cm²**.
(iii)
For triangle NAT:
Base AT = 3 cm
Height NA = 4 cm
Area of triangle NAT:
= 1/2 × 3 × 4
= 6 cm²
Therefore, the area of triangle NAT is **6 cm²**.

Question 2.
Find the length of the altitude BY.

Solution:
Area of triangle AXC:
= 1/2 × XC × AX
Since:
XC = XB + 6
and:
AX = 4
Therefore:
Area of triangle AXC:
= 1/2 × (XB + 6) × 4
= 2(XB + 6)
= 2XB + 12 square units
Area of triangle AXB:
= 1/2 × XB × AX
= 1/2 × XB × 4
= 2XB square units
Area of triangle ABC:
= Area of triangle AXC − Area of triangle AXB
= (2XB + 12) − 2XB

= 12 square units
Using AC = 8 units as the base and BY as the altitude:
Area of triangle ABC:
= 1/2 × AC × BY
12 = 1/2 × 8 × BY
12 = 4BY
BY = 12/4
BY = 3 units
Therefore, the length of altitude BY is **3 units**.

Question 3.
Find the area of ΔSUB, given that it is isosceles, SE is perpendicular to UB, and the area of ΔSEB is 24 sq. units.

Solution:
Given:
Area of triangle SEB = 24 square units
Triangle SUB is an isosceles triangle, with:
SU = SB
SE is perpendicular to the base UB.
In an isosceles triangle, the perpendicular drawn from the vertex to the base also bisects the base.
Therefore:
UE = EB
Triangles SUE and SEB have:

- Equal bases UE and EB
- The same perpendicular height SE

Therefore, their areas are equal.
Area of triangle SUE:
= Area of triangle SEB
= 24 square units
Area of triangle SUB:
= Area of triangle SUE + Area of triangle SEB
= 24 + 24
= 48 square units
Therefore, the area of triangle SUB is **48 square units**.

In the Sulba-Sutras, ancient Indian geometric texts that deal with the construction of altars, we find many interesting problems on area. When altars are built, they must have the exact prescribed shape and area. This gives rise to problems of the kind in which one has to transform a given shape into another with the same area. The Sulba Sutras provide solutions to many such problems. Such problems are also posed and solved in Euclid's Elements. Here are two problems of this kind.

Question 4.
[Sulba-Sutras] Give a method to transform a rectangle into a triangle of equal area.

Solution:
Let ABCD be a rectangle with:
Length = a
Breadth = b
Area of rectangle ABCD:
= a × b
To construct a triangle with the same area:

1. Mark E as the midpoint of side CD.
2. Draw a line perpendicular to CD through E.
3. Mark a point M on this perpendicular such that:

ME = b
Since the distance between AB and CD is also b, the perpendicular distance from M to AB becomes:
b + b = 2b

4. Join M to A and B to form triangle MAB.

Base of triangle MAB:
= AB
= a
Height of triangle MAB:
= 2b
Area of triangle MAB:
= 1/2 × a × 2b
= ab
The area of rectangle ABCD is also ab.
Therefore, triangle MAB and rectangle ABCD have equal areas.

Question 5.
[Sulba-Sutras] Give a method to transform a triangle into a rectangle of equal area.

Solution:
Let triangle ABC have:
Base = b
Height = h
Area of triangle ABC:
= 1/2 × b × h
To construct a rectangle of equal area:

1. Draw the altitude of the triangle.
2. Mark the midpoint M of the altitude.
3. Through M, draw a line parallel to the base of the triangle.
4. Construct a rectangle with:

Length = b
Breadth = h/2
Area of the rectangle:
= b × h/2
= 1/2 × bh
This equals the area of the original triangle.
Therefore, a rectangle having length b and breadth h/2 has the same area as the triangle.

Question 6.
ABCD, BCEF, and BFGH are identical squares.

(i) If the area of the red region is 49 sq. units, then what is the area of the black region?

(ii) In another version of this figure, if the total area enclosed by the black and red regions is 180 sq. units, then what is the area of each square?

Solution:
Let the side length of each identical square be a units.
Since the three squares are identical:
AB = BC = BF = a

The total vertical distance HC is:
HC = HB + BC
= a + a
= 2a
(i)
The red region forms triangle HDC.
Its base is:
DC = a
Its perpendicular height is:
HC = 2a
Area of red triangle HDC:
= 1/2 × DC × HC
= 1/2 × a × 2a
= a^2
Given:
Area of red region = 49 square units
Therefore:
$a^2 = 49$
a = 7 units
From the figure:
AI = a/2
Therefore:
AI = 7/2 units
The black region is triangle AID.
Its base is:
AD = 7 units
Its height is:
AI = 7/2 units
Area of black region:
= 1/2 × AD × AI
= 1/2 × 7 × 7/2
= 49/4
= 12.25 square units
Therefore, the area of the black region is **12.25 square units**.

(ii)
Area of red region:
$= a^2$
Area of black region:
$= a^2/4$
Total area of red and black regions:
$= a^2 + a^2/4$
$= 5a^2/4$
Given:
$5a^2/4 = 180$
$5a^2 = 720$
$a^2 = 720/5$
$a^2 = 144$
The area of each square is a^2.
Therefore, the area of each square is **144 square units**.

Question 7.
If M and N are the midpoints of XY and XZ, what fraction of the area of ΔXYZ is the area of ΔXMN? [Hint: Join XY]

Solution:
Since M and N are the midpoints of XY and XZ:
XM = 1/2 XY
XN = 1/2 XZ
By the midpoint theorem:
MN ‖ YZ
and
MN = 1/2 YZ
Therefore, ΔXMN and ΔXYZ are similar triangles. The ratio of their corresponding sides is:
XM/XY = 1/2
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Area of ΔXMN / Area of ΔXYZ = $(1/2)^2$
= 1/4

Therefore:
Area of ΔXMN = 1/4 × Area of ΔXYZ
Hence, the required fraction is **1/4**.

Question 8.
Gopal needs to carry water from the river to his water tank. He starts from his house. What is the shortest path he can take from his house to the river and then to the water tank? Roughly recreate the map in your notebook and trace the shortest path.

Solution:
Let Gopal's house be represented by H and the water tank by T.
To determine the shortest route:

1. Reflect the water tank T across the straight riverbank and mark its image as T′.
2. Join H and T′ using a straight line.
3. Let this line meet the riverbank at P.
4. Join P to the actual water tank T.

Since T and T′ are at equal perpendicular distances from the riverbank:
PT = PT′
Therefore:
HP + PT = HP + PT′
Since a straight line gives the shortest distance between two points, the shortest route is:

House H → Point P on the riverbank → Water tank T

Area of a Polygon

Figure It Out (Page 160)

Question 1.
Find the area of the quadrilateral ABCD given that AC = 22 cm, BM = 3 cm, DN = 3 cm, BM is perpendicular to AC, and DN is perpendicular to AC.

Solution:
The diagonal AC divides quadrilateral ABCD into two triangles, ΔABC and ΔADC.
Area of quadrilateral ABCD:

= Area of ΔABC + Area of ΔADC
For ΔABC:
Base AC = 22 cm
Height BM = 3 cm
Area of ΔABC:
= 1/2 × AC × BM
= 1/2 × 22 × 3
= 33 cm²
For ΔADC:
Base AC = 22 cm
Height DN = 3 cm
Area of ΔADC:
= 1/2 × AC × DN
= 1/2 × 22 × 3
= 33 cm²
Therefore:
Area of quadrilateral ABCD:
= 33 + 33
= **66 cm²**

Question 2.
Find the area of the shaded region given that ABCD is a rectangle.

Solution:
The shaded area is obtained by subtracting the areas of triangles AEF and EBC from the area of rectangle ABCD.
Given:
AE = 10 cm
EB = 8 cm
AF = 6 cm
FD = 4 cm
Therefore:
AB = AE + EB
= 10 + 8
= 18 cm
AD = AF + FD

= 6 + 4
= 10 cm
Area of rectangle ABCD:
= AB × AD
= 18 × 10
= 180 cm²
Area of ΔAEF:
= 1/2 × AE × AF
= 1/2 × 10 × 6
= 30 cm²
Since ABCD is a rectangle:
BC = AD = 10 cm
Area of ΔEBC:
= 1/2 × EB × BC
= 1/2 × 8 × 10
= 40 cm²
Area of the shaded region:
= 180 − (30 + 40)
= 180 − 70
= **110 cm²**

Question 3.
What measurements would you need to find the area of a regular hexagon?

Solution:
To calculate the area of a regular hexagon, we need to know its side length.
Let the side length be a.
A regular hexagon can be divided into six congruent equilateral triangles, each having side a.
Area of one equilateral triangle:
= √3/4 × a²
Therefore:
Area of regular hexagon:
= 6 × √3/4 × a²
= 3√3/2 × a²
Hence, the required measurement is the **side length of the regular hexagon**.

Formula:
Area of a regular hexagon = $3\sqrt{3}/2 \times a^2$

Question 4.
What fraction of the total area of the rectangle is the area of the red region?

Solution:
Let the length and breadth of rectangle ABCD be l and b, respectively.
Area of rectangle ABCD:
$= l \times b$
Let the perpendicular distances from O to AB and DC be x and y, respectively.
Since AB and DC are opposite sides of the rectangle:
$x + y = b$
Area of ΔAOB:
$= 1/2 \times AB \times x$
$= 1/2 \times l \times x$
Area of ΔDOC:
$= 1/2 \times DC \times y$
$= 1/2 \times l \times y$
Area of the red region:
= Area of ΔAOB + Area of ΔDOC
$= 1/2\ lx + 1/2\ ly$
$= 1/2\ l(x + y)$
Since $x + y = b$:
Area of the red region:
$= 1/2\ lb$
Therefore:
Area of red region / Area of rectangle
$= (1/2\ lb)/(lb)$
$= 1/2$
Hence, the red region occupies **1/2 of the rectangle's total area**.

Question 5.
Give a method to obtain a quadrilateral whose area is half that of a given quadrilateral.

Solution:
Let ABCD be the given quadrilateral.

1. Mark P, Q, R and S as the midpoints of sides AB, BC, CD and DA respectively.
2. Join P to Q, Q to R, R to S and S to P.
3. The quadrilateral PQRS formed by joining these midpoints is the required quadrilateral.

According to the midpoint property, the four triangles formed at the corners together occupy half the area of ABCD.
Therefore, the remaining midpoint quadrilateral occupies the other half.
Thus:
Area of PQRS = 1/2 × Area of ABCD

Parallelogram

Figure It Out (Pages 162-164)

Question 1.

Observe the parallelograms in the figure below.
(i) What can we say about the areas of all these parallelograms?
(ii) What can we say about their perimeters? Which figure appears to have the maximum perimeter, and which has the minimum perimeter?

Solution:
(i)
The area of a parallelogram is:
Area = Base × Perpendicular height
Each parallelogram has:
Base = 5 units
Height = 3 units
Therefore:
Area of each parallelogram:

= 5 × 3
= 15 square units
Hence, all the parallelograms have equal areas of 15 square units.
(ii)
The perimeters differ because the sloping sides of the parallelograms have different lengths.
Perimeter of a parallelogram:
= 2 × (Base + Side)
As a parallelogram becomes more slanted, its side length generally increases, and so does its perimeter.
From the given figure:

- Figure (d) appears to have the minimum perimeter.
- Figure (g) appears to have the maximum perimeter.

Thus, figures can have the same area but different perimeters.

Question 2.
Find the areas of the following parallelograms:

Solution:
Area of a parallelogram:
= Base × Perpendicular height
(i)
Base = 7 cm
Height = 4 cm
Area:
= 7 × 4
= **28 cm²**
(ii)
Base = 5 cm
Height = 3 cm
Area:
= 5 × 3
= **15 cm²**
(iii)
Base = 5 cm
Height = 4.8 cm

Area:
= 5 × 4.8
= **24 cm²**
(iv)
Base = 2 cm
Height = 4.4 cm
Area:
= 2 × 4.4
= **8.8 cm²**

Question 3.
Find QN.

Solution:
In right-angled triangle PNQ:
∠PNQ = 90°
Given:
PN = 7.6 cm
PQ = 12 cm
PQ is the hypotenuse.
Using the Baudhayana-Pythagoras Theorem:
$PQ^2 = PN^2 + QN^2$
$12^2 = 7.6^2 + QN^2$
$144 = 57.76 + QN^2$
$QN^2 = 144 - 57.76$
$QN^2 = 86.24$
$QN = \sqrt{86.24}$
$QN \approx 9.29$ cm
Therefore, the length of QN is approximately **9.29 cm**.

Question 4.
Consider a rectangle and a parallelogram of the same sidelengths: 5 cm and 4 cm. Which has the greater area?

[Hint: Imagine constructing them on the same base.]

Solution:
For the rectangle:
Length = 5 cm
Breadth = 4 cm
Area of rectangle:
= 5 × 4
= 20 cm²
For the parallelogram:
Base = 5 cm
Slanting side = 4 cm
Its perpendicular height is less than 4 cm unless the parallelogram is itself a rectangle.
Therefore:
Area of parallelogram:
= Base × Height
< 5 × 4
< 20 cm²
Hence, the **rectangle has the greater area**.
If the parallelogram has a right angle, it becomes a rectangle, and both areas will be equal.

Question 5.
Give a method to obtain a rectangle whose area is twice that of a given triangle. What are the different methods that you can think of?

Solution:
Let the triangle have base b and height h.
Area of the triangle:
= 1/2 × b × h
Twice the area of the triangle:
= 2 × 1/2 bh
= bh
Method 1: Using the same base and height
Construct a rectangle with:
Length = b
Breadth = h
Area of rectangle:

= b × h
= 2 × Area of the triangle

Method 2: Using two copies of the triangle

1. Make an identical copy of the given triangle.
2. Rotate one copy and place it beside the original triangle.
3. Arrange the two triangles to form a parallelogram.
4. Rearrange the parallelogram, if required, to obtain a rectangle with the same base and height.

Since the rectangle is formed from two identical triangles, its area is twice the area of one triangle.

Question 6.
[Sulba-Sutras] Give a method to obtain a rectangle of the same area as a given triangle.

Solution:
Let the given triangle have:
Base = b
Height = h
Area of the triangle:
= 1/2 × b × h
Construct a rectangle with:
Length = b
Breadth = h/2
Its area is:
= b × h/2
= 1/2 bh
Therefore, the rectangle has the same area as the triangle.
Alternatively, a rectangle with length b/2 and breadth h will also have the same area.

Question 7.
[Sulba-Sutras] An isosceles triangle can be converted into a rectangle by dissection in a simpler way. Can you find out how to do it?

[Hint: Show that triangles ΔADB and ΔADC can be made into halves of a rectangle. Figure out how they should be assembled to get a rectangle. Use cut-outs if necessary.]

Solution:
Let ABC be an isosceles triangle with:
AB = AC
Draw altitude AD perpendicular to BC.
In an isosceles triangle, the altitude from the vertex also bisects the base.
Therefore:
BD = DC
Triangles ADB and ADC are congruent right-angled triangles.
To form a rectangle:

1. Cut the triangle along altitude AD.
2. This gives two congruent right triangles, ΔADB and ΔADC.
3. Rotate one triangle and arrange the two triangles along their hypotenuses.
4. The two pieces form a rectangle.

The dimensions of the rectangle are:
Length = AD
Breadth = BC/2
Area of rectangle:
= AD × BC/2
= 1/2 × BC × AD
This equals the area of the original isosceles triangle.

Question 8.
[Sulba-Sutras] Give a method to convert a rectangle into an isosceles triangle by dissection.

Solution:
This is the reverse of the method used in the previous question.

1. Take a rectangle PQRS.
2. Draw one diagonal of the rectangle.
3. Cut along the diagonal to obtain two congruent right-angled triangles.
4. Place the two triangles on opposite sides of one equal leg.
5. Join them so that their other equal legs form one straight line.

The two hypotenuses form the equal sides of the required isosceles triangle.
Since the same two pieces are only rearranged, the area remains unchanged.
Therefore, the resulting isosceles triangle has the same area as the original rectangle.

Question 9.
Which has a greater area — an equilateral triangle or a square of the same side length as the triangle? Which has a greater area: two identical equilateral triangles together or a square of the same side length as the triangle? Give reasons.

Solution:
Let the common side length be a.
Area of an equilateral triangle:
$= \sqrt{3}/4 \times a^2$
Area of a square:
$= a^2$
Since:
$\sqrt{3}/4 \approx 0.433$
Therefore:
$\sqrt{3}/4\ a^2 < a^2$
Hence, a square has a greater area than one equilateral triangle of the same side length.
Now, the area of two identical equilateral triangles is:
$2 \times \sqrt{3}/4\ a^2$
$= \sqrt{3}/2\ a^2$
Since:
$\sqrt{3}/2 \approx 0.866$
Therefore:
$\sqrt{3}/2\ a^2 < a^2$
Hence, the square also has a greater area than two identical equilateral triangles together.
Thus:
Area of square > Area of two equilateral triangles > Area of one equilateral triangle

Rhombus & Trapezium

Figure It Out (Pages 169-170)

Question 1.
Find the area of a rhombus whose diagonals are 20 cm and 15 cm.

Solution:
Area of a rhombus:
= 1/2 × Product of diagonals
Given:
First diagonal = 20 cm
Second diagonal = 15 cm
Area:
= 1/2 × 20 × 15
= 10 × 15
= **150 cm²**
Therefore, the area of the rhombus is **150 cm²**.

Question 2.
Give a method to convert a rectangle into a rhombus of equal area using dissection.

Solution:
Let the rectangle have length l and breadth w. If w is greater than l, rotate the rectangle first so that $l \geq w$.
To convert it into a rhombus:

1. Calculate a length x such that:

$x^2 + w^2 = l^2$
Therefore:
$x = \sqrt{(l^2 - w^2)}$

2. Mark a segment of length x on the upper side of the rectangle.
3. Cut the right-angled triangular portion having base x and height w.
4. Move this triangular piece to the opposite side of the rectangle.
5. The rearranged shape becomes a parallelogram.

Its slanting side is:
$\sqrt{(x^2 + w^2)}$

$= \sqrt{l^2}$
$= l$
Since its base is also l, the parallelogram is a rhombus.
No area is added or removed, so the rhombus has the same area as the rectangle.

Question 3.
Find the areas of the following figures:

Solution:
Area of a trapezium:
= 1/2 × Sum of parallel sides × Perpendicular height
(i)
Parallel sides = 10 ft and 7 ft
Height = 16 ft
Area:
= 1/2 × (10 + 7) × 16
= 1/2 × 17 × 16
= **136 ft²**
(ii)
Parallel sides = 36 m and 24 m
Height = 14 m
Area:
= 1/2 × (36 + 24) × 14
= 1/2 × 60 × 14
= **420 m²**
(iii)
Parallel sides = 14 in and 6 in
Height = 10 in
Area:
= 1/2 × (14 + 6) × 10
= 1/2 × 20 × 10
= **100 in²**
(iv)
Parallel sides = 18 ft and 12 ft
Height = 8 ft
Area:

= 1/2 × (18 + 12) × 8
= 1/2 × 30 × 8
= **120 ft²**

Question 4.
[Sulba-Sutras] Give a method to convert an isosceles trapezium to a rectangle using dissection.

Solution:
Let ABCD be an isosceles trapezium in which:
AB ∥ CD
and:
AD = BC
Draw perpendiculars from D and C to AB, meeting it at P and Q.
This divides the trapezium into:

- Rectangle PQCD
- Right triangle APD
- Right triangle BQC

Since the trapezium is isosceles:
ΔAPD ≅ ΔBQC
Therefore:
AP = BQ
Now cut triangle APD and place it beside triangle BQC. The pieces together form a rectangle.
The resulting rectangle has:
Height = h
Length:
= CD + AP
Since:
AB = AP + CD + BQ
and AP = BQ:
AP = (AB − CD)/2
Therefore:
Length of rectangle:
= CD + (AB − CD)/2

= (AB + CD)/2
Area of rectangle:
= (AB + CD)/2 × h
This is equal to the area of the original trapezium.

Question 5.
Here is one of the ways to convert trapezium ABCD into a rectangle EFGH of equal area-

Given the trapezium ABCD, how do we find the vertices of the rectangle EFGH? [Hint: If ΔAHI ≅ ΔDGI and ΔBEJ ≅ ΔCFJ, then the trapezium and rectangle have equal areas.]

Solution:
Let the parallel sides of trapezium ABCD be AB and CD, and let its height be h.

1. Mark I and J as the midpoints of the non-parallel sides AD and BC.
2. Join I and J. The segment IJ is parallel to AB and CD.
3. By the midpoint property of a trapezium:

IJ = (AB + CD)/2

4. Construct a rectangle with length IJ and breadth equal to the height h of the trapezium.
5. Locate vertices E, F, G and H so that the triangular portions removed from the trapezium fit exactly into the missing portions of the rectangle.

The congruent triangle pairs are:
ΔAHI ≅ ΔDGI
and
ΔBEJ ≅ ΔCFJ
Therefore, the parts removed and added have equal areas.
Area of rectangle EFGH:
= (AB + CD)/2 × h
This is equal to the area of trapezium ABCD.

Question 6.
Using the idea of converting a trapezium into a rectangle of equal area, and vice versa, construct a trapezium of area 144 cm2.

Solution:
We need to choose two parallel sides and a height satisfying:
Area of trapezium:
= 1/2 × (Sum of parallel sides) × Height
Choose:
Parallel sides = 10 cm and 8 cm
Height = 16 cm
Then:
Area:
= 1/2 × (10 + 8) × 16
= 1/2 × 18 × 16
= 9 × 16
= 144 cm²
Construction:

1. Draw a rectangle of dimensions 16 cm × 9 cm.
2. Its area is:

16 × 9 = 144 cm²

3. Cut a triangular portion from one side of the rectangle.
4. Move it to the opposite side to form a trapezium with parallel sides 10 cm and 8 cm and a height 16 cm.

Thus, the required trapezium has an area of **144 cm²**.

Question 7.
A regular hexagon is divided into a trapezium, an equilateral triangle, and a rhombus, as shown. Find the ratio of their areas.

Solution:
A regular hexagon can be divided into six congruent equilateral triangles.
Let the area of one small equilateral triangle be A.
From the figure:

- The equilateral triangle contains 1 small triangle.
- The rhombus contains 2 small triangles.

- The trapezium contains 3 small triangles.

Therefore:
Area of equilateral triangle = A
Area of rhombus = 2A
Area of trapezium = 3A
Hence:
Equilateral triangle : Rhombus : Trapezium
= A : 2A : 3A
= **1 : 2 : 3**

Question 8.
ZYXW is a trapezium with ZY || WX. A is the midpoint of XY. Show that the area of the trapezium ZYXW is equal to the area of ΔZWB.

Solution:
Since A is the midpoint of XY:
AY = AX
Also:
∠ZAY = ∠BAX
These are vertically opposite angles.
Since ZY ∥ WX and X, W and B are collinear:
∠ZYA = ∠BXA
These are alternate interior angles.
Therefore:
ΔZAY ≅ ΔBAX
by the ASA congruence rule.
Hence:
Area of ΔZAY = Area of ΔBAX
The triangle ZWB is obtained by removing ΔZAY from the trapezium and adding the equal-area triangle ΔBAX.
Therefore, the total area remains unchanged.
Hence:
Area of trapezium ZYXW = Area of ΔZWB

Areas in Real Life (Pages 170-171)

Question 1.
What do you think is the area of an A4 sheet? Its sidelengths are 21 cm and 29.7 cm. Now find its area.

Solution:
An A4 sheet is rectangular.
Length = 29.7 cm
Breadth = 21 cm
Area:
= Length × Breadth
= 29.7 × 21
= **623.7 cm²**
Therefore, the area of an A4 sheet is **623.7 cm²**.

Question 2.
Express the following lengths in centimeters:
(i) 5 in
(ii) 7.4 in

Solution:
We know:
1 in = 2.54 cm
(i)
5 in:
= 5 × 2.54
= **12.7 cm**
(ii)
7.4 in:
= 7.4 × 2.54
= **18.796 cm**
Therefore:
5 in = 12.7 cm
7.4 in = 18.796 cm

Question 3.
Express the following lengths in inches:
(i) 5.08 cm
(ii) 11.43 cm

Solution:
We know:
2.54 cm = 1 in
Therefore:
Length in inches = Length in centimetres ÷ 2.54
(i)
5.08 cm:
= 5.08 ÷ 2.54
= **2 in**
(ii)
11.43 cm:
= 11.43 ÷ 2.54
= **4.5 in**
Therefore:
5.08 cm = 2 in
11.43 cm = 4.5 in

Question 4.
How many in2 is 1 ft2?

Solution:
We know:
1 ft = 12 in
Therefore:
1 ft^2 = 12 in × 12 in
= **144 in^2**
Hence:
1 ft^2 = 144 in^2

Question 5.
How many m2 is a km2?

Solution:
We know:
1 km = 1000 m
Therefore:
1 km^2:
= 1000 m × 1000 m
= 1,000,000 m^2
= 10^6 m^2
Hence:
1 km^2 = 1,000,000 m^2