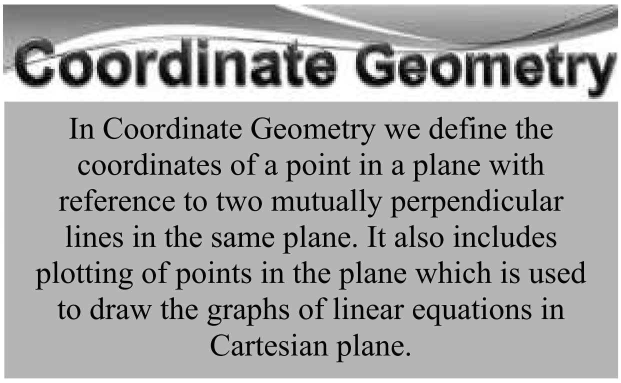
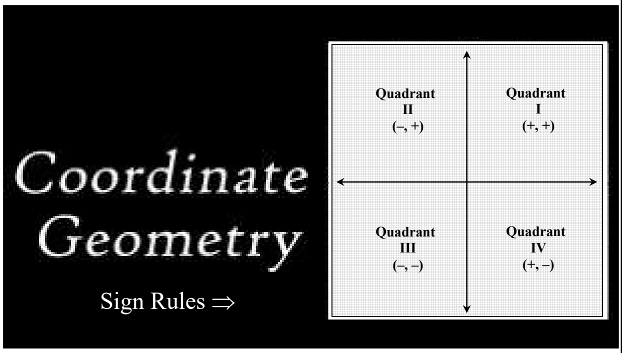
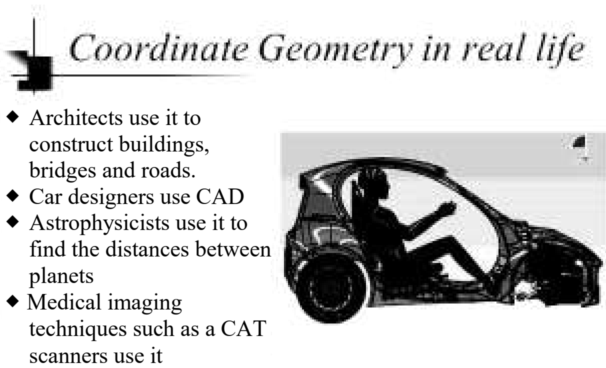
Orienting Yourself : The Use of Coordinates

Chapter Outline

- Introduction of Co-ordinate geometry
- 2-D Cartesian Co-ordinate System
- Plotting the Points
- Mirror Image of Co-ordinates of a point
- Distance Between two points
- Mid Point of Two Co-odinates







GEOMETRY BEFORE CO-ORDINATE SYSTEM

- In ancient times, geometry was studied by Greek mathematicians like Euclid
- Geometry was based on:
  - Shapes
  - Constructions
  - Theorems

Contribution of René Descartes

- Coordinate Geometry was developed by René Descartes (1596–1650)
- He introduced a revolutionary idea:
- **Link Algebra with Geometry**

The Story Behind the Idea

- It is said that Descartes observed a fly moving on a ceiling
- He thought:
- "Can I describe its position using numbers?"
- This led to the concept of:
- Using numbers to represent positions

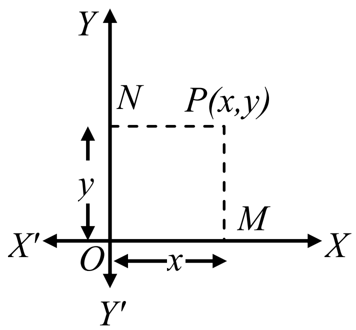
CO-ORDINATE GEOMETRY

A system of geometry where the position of points on the plane is described using an ordered pair of numbers w.r.t. two mutually perpendicular lines.

If we were to place a point on the plane, co-ordinate geometry gives us a way to describe exactly where it is by using two numbers

CARTESIAN CO-ORDINATES (RECTANGULAR CO-ORDINATES)

In Cartesian co-ordinates the position of a point P is determined by knowing the distances from two perpendicular lines passing through the fixed point. Let O be the fixed point called the origin and XOX′ and YOY′, the two perpendicular lines through O, called Cartesian or Rectangular co-ordinates axes.



Draw PM and PN perpendiculars on OX and OY respectively. OM (or NP) and ON (or MP) are called the x-co-ordinate (or abscissa) and y-co-ordinate (or ordinate) of the point P respectively.

- **Axes of Co-ordinates**

  In the figure OX and OY are called as x-axis and y-axis respectively and both together are known as axes of co-ordinates.

- **Origin**

  It is point O of intersection of the axes of co-ordinates.

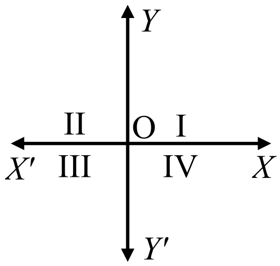
- **Co-ordinates of the Origin**

  It has zero distance from both the axes so that its abscissa and ordinate are both zero. Therefore, the co-ordinates of origin are (0, 0).

❖ Quadrant

The axes divide the plane into four parts. These four parts are called quadrants. So, the plane consists of axes and quadrants. The plane is called the Cartesian plane or the co-ordinate plane or the xy-plane. These axes are called the co-ordinate axes.

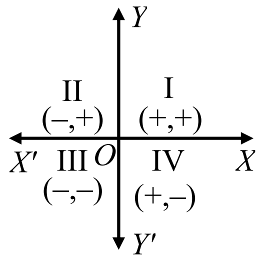
A quadrant is $\frac{1}{4}$ part of a plane divided by co-ordinate axes.



(i) XOY is called the first quadrant
(ii) YOX′ the second.
(iii) X′OY′ the third.
(iv) Y′OX the fourth as marked in the figure.

RULES OF SIGNS OF CO-ORDINATES

(i) In the first quadrant, both co-ordinates i.e., abscissa and ordinate of a point are positive.
(ii) In the second quadrant, for a point, abscissa is negative and ordinate is positive.
(iii) In the third quadrant, for a point, both abscissa and ordinate are negative.
(iv) In the fourth quadrant, for a point, the abscissa is positive and the ordinate is negative.



Quadrant	x-co-ordinate	y-cordinate	Point
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

Note :
(i) If the point lies on x-axis its co-ordinate is of the form (a, 0).
(ii) If the point lies on y-axis its co-ordinate is of the form (0, b).

PLOTTING OF A POINT

In order to plot the points in a plane, we may use the following algorithm.

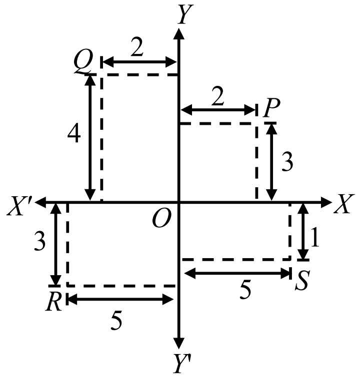
Step I : Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

Step II : Mark their intersection point as **O** (origin).

Step III : Choose a suitable scale on **X-axis** and **Y-axis** and mark the points on both the axis.

Step IV : Obtain the co-ordinates of the point which is to be plotted. Let the point be **P(a,b)**. To plot this point start from the origin and move $|a|$ units along **OX**, **OX'** accordingly as **'a'** is positive or negative respectively. Suppose we arrive at point **M**. From point **M** move vertically upward or downward through $|b|$ units accordingly as **'b'** is positive or negative. The point where we arrive finally is the required point **P (a, b)**.

Ex.1 Write down the (i) abscissa (ii) ordinate (iii) Co-ordinates of P, Q, R and S as given in the figure.



Sol. **Point P**

Abscissa of P = 2; Ordinate of P = 3

Co-ordinates of P = (2, 3)

Point Q

Abscissa of Q = – 2; Ordinate of Q = 4

Co-ordinate of Q = (–2, 4)

Point R

Abscissa of R = – 5; Ordinate of R = – 3

Co-ordinates of R = (–5, –3)

Point S

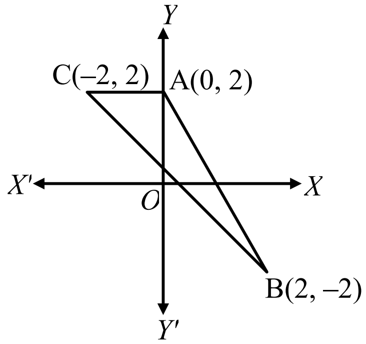
Abscissa of S = 5; Ordinate of S = – 1

Co-ordinates of S = (5, – 1)

Ex.2 Draw a triangle ABC where vertices A, B and C are (0, 2), (2, – 2), and (–2, 2) respectively.

Sol. Plot the point A by taking its abscissa 0 and ordinate = 2.

Similarly, plot points B and C taking abscissa 2 and –2 and ordinates – 2 and 2 respectively. Join A, B and C. This is the required triangle.

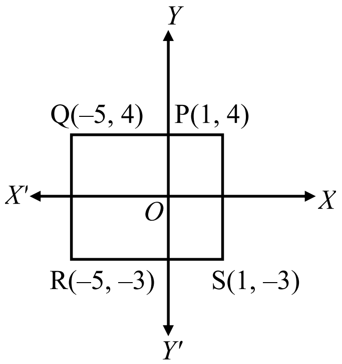


Ex.3 Draw a rectangle PQRS in which vertices P, Q, R and S are (1, 4), (–5, 4), (–5, –3) and (1, – 3) respectively.

Sol. Plot the point P by taking its abscissa 1 and ordinate – 4.

Similarly, plot the points Q, R and S by taking abscissa as –5, –5 and 1 and ordinates as 4, – 3 and –3 respectively.

Join the points PQR and S. PQRS is the required rectangle.



Ex.4 Write the quadrants for the following points.

(i) A (3, 4) (ii) B (– 2, 3) (iii) C (– 5,– 2) (iv) D (4, – 3) (v) E (– 5,– 5)

Sol. (i) Here both co-ordinates are positive therefore point A lies in Ist quadrant.

(ii) Here x is negative and y is positive therefore point B lies in IInd quadrant.

(iii) Here both co-ordinates are negative therefore point C lies in IIIrd quadrant.

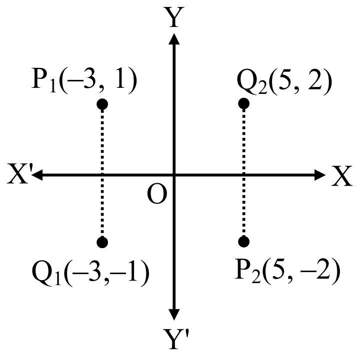
(iv) Here x is positive and y is negative therefore point D lies in IVth quadrant.

(v) Here both co-ordinates are negative therefore point E lies in IIIrd quadrant.

MIRROR IMAGE OF COORDINATES OF A POINT

◈ **With respect to x - axis**

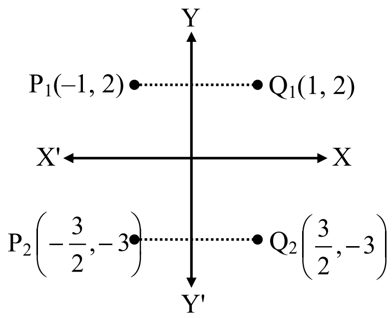
For the image of a point P(x, y) with respect to x axis, the change is only in the sign of ordinate of point so the image is Q(x, –y).



P_1 (–3, 1) & P_2 (5, –2) are two points in II & IV quadrant respectively then their images are Q_1 (–3, –1) & Q_2 (5, 2) in III & I quadrant respectively.

◈ **With respect to y - axis**

In image of a point P(x, y) the change is only in the sign of abscissa of point so the image is Q(–x, y).



$$P_1\ (-1, 2) \xrightarrow{\text{image}} Q_1\ (1, 2)$$

$$P_2\left(-\frac{3}{2},-3\right) \xrightarrow{\text{image}} Q_2\left(\frac{3}{2},-3\right)$$

Ex.5 Find the images of the following points with respect to x axis,

$(1, 2)$, $\left(\frac{3}{8},\frac{4}{3}\right)$, $\left(-\frac{2}{3},3\right)$, (2, 5), (5, 0), (0, 7), (– 3, – 4)

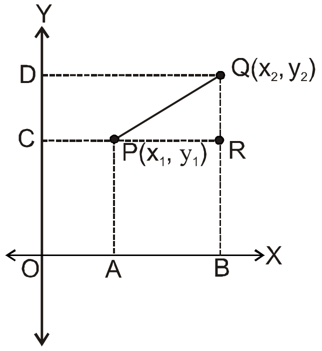
Sol. The images are (1, –2), $\left(\frac{3}{8},-\frac{4}{3}\right)$, $\left(-\frac{2}{5},-3\right)$, (2, –5), (5, 0), (0, –7), (–3, 4) respectively.

Ex.6 Find the images, of points (0, 0), (3, 0), (0, 2), (5, 1), (–2, 3), (–3, –3), (6, –7) with respect to y axis.

Sol. The images are (0, 0), (–3, 0), (0, 2), (–5, 1), (2, 3), (3, –3), (–6, –7) respectively.

DISTANCE BETWEEN TWO POINTS

Let two points be **P (x_1, y_1)** and **Q (x_2, y_2)**. Take two mutually perpendicular lines as the coordinate axis with **O** as origin. Mark the points **P (x_1, y_1)** and **Q (x_2, y_2)**. Draw lines **PA**, **QB** perpendicular to **X-axis**, from the points **P** and **Q**, which meet the **X-axis** in points A and B, respectively.



Draw lines PC and QD perpendicular to **Y-axis**, which meet the **Y-axis** in C and D, respectively. Produce CP to meet BQ in R. Now

OA = abscissa of P = x_1

Similarly, OB = x_2, OC = y_1 and OD = y_2

Therefore, we have

PR = AB = OB – OA = $x_2 - x_1$

Similarly, QR = QB – RB = QB – PA = $y_2 - y_1$

Now, using Pythagoras Theorem, in right angled triangle PRQ, we have

$PQ^2 = PR^2 + RQ^2$

or $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since the distance or length of the line-segment PQ is always non-negative, on taking the positive square root, we get the distance as

$$\mathbf{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This result is known as **distance formula**.

Corollary : The distance of a point **P (x_1, y_1)** from the origin **(0, 0)** is given by

$$\mathbf{OP} = \sqrt{x_1^2 + y_1^2}\,.$$

Ex.7 Find the distance between the points

(i) P(–6, 7) and Q(–1, –5) (ii) R(a + b, a – b) and S(a – b, –a – b)

(iii) $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Sol. (i) Here,

$x_1 = -6$, $y_1 = 7$ and $x_2 = -1$, $y_2 = -5$

$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow PQ = \sqrt{(-1+6)^2 + (-5-7)^2}$

$\Rightarrow PQ = \sqrt{25+144} = \sqrt{169} = 13$

(ii) We have,

$RS = \sqrt{(a-b-a-b)^2 + (-a-b-a+b)^2}$

$\Rightarrow RS = \sqrt{4b^2 + 4a^2} = 2\sqrt{a^2 + b^2}$

(iii) We have,

$AB = \sqrt{(at_2{}^2 - at_1{}^2)^2 + (2at_2 - 2at_1)^2}$

$\Rightarrow AB = \sqrt{a^2(t_2 - t_1)^2(t_2 + t_1)^2 + 4a^2(t_2 - t_1)^2}$

$\Rightarrow AB = a(t_2 - t_1)\sqrt{(t_2 + t_1)^2 + 4}$

Ex.8 Find the value of x, if the distance between the points (x, – 1) and (3, 2) is 5.

Sol. Let P(x, – 1) and Q(3, 2) be the given points, Then,

PQ = 5 (Given)

$\Rightarrow \sqrt{(x-3)^2+(-1-2)^2} = 5$

$\Rightarrow (x-3)^2 + 9 = 5^2$

$\Rightarrow x^2 - 6x + 18 = 25 \quad \Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x-7)(x+1) = 0 \Rightarrow \qquad x = 7 \quad \text{or} \quad x = -1$

Ex.9 Show that the points (1, – 1), (5, 2) and (9, 5) are collinear.

Sol. Let A (1, –1), B (5, 2) and C (9, 5) be the given points. Then, we have

$AB = \sqrt{(5-1)^2+(2+1)^2} = \sqrt{16+9} = 5$

$BC = \sqrt{(5-9)^2+(2-5)^2} = \sqrt{16+9} = 5$

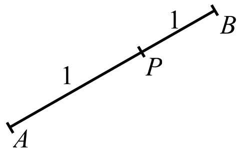
and, $AC = \sqrt{(1-9)^2+(-1-5)^2} = \sqrt{64+36} = 10$

Clearly, AC = AB + BC

Hence, A, B, C are collinear points.

MID POINT BETWEEN TWO POINTS

Let A and B be two points in the plane of the paper as shown in fig. and P be a point on the segment joining A and B such that AP : BP = 1 : 1. Then, the point P divides segment AB internally in the ratio 1 : 1.



$$P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Ex.10 Find the midpoint of the line segment joining P(4, 8) and Q(6, 12).

Sol. Identify coordinates. $x_1 = 4, y_1 = 8$ and $x_2 = 6, y_2 = 12$.

Apply the formula for x: $\frac{4+6}{2} = \frac{10}{2} = 5$.

Apply the formula for y: $\frac{8+12}{2} = \frac{20}{2} = 10$.

The midpoint is (5, 10).

Ex.11 Find the coordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts.

Sol. To divide a line into 4 parts, we find the midpoint of AB (let's call it M_2), then the midpoint of AM_2 (called M_1), and the midpoint of M_2B (called M_3).

(Find M_2): **Midpoint of** $A(-2,2)$ and $B(2,8) = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \mathbf{(0,5)}$.

(Find M_1**):** Midpoint of $A(-2,2)$ and $M_2(0,5) = \left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \mathbf{(-1,3.5)}$.

(Find M_3**):** Midpoint of $M_2(0,5)$ and $B(2,8) = \left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \mathbf{(1,6.5)}$.

The three points are $(-1,3.5), (0,5)$, and $(1,6.5)$.