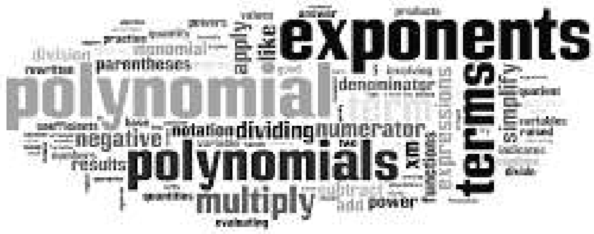
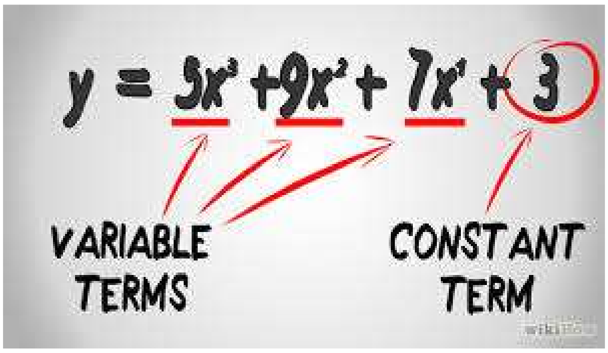
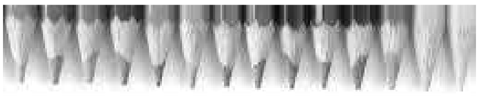
Introduction to Linear Polynomials

Chapter Outline

- Algebraic expressions
- Definition of a polynomial.
  Degree of a polynomial
- Introduction to linear polynomials and applications
- Exploring linear patterns
- Modelling linear growth and linear decay
- Linear relationships
- Visualising linear relationships
- Slope and y-intercept of a line







- Polynomials contain three types of terms :
  (1) monomial : A polynomial with one term
  (2) binomial : A polynomial with two terms.
  (3) Trinomial : A polynomial with three terms.

$3xy^2$	$5x - 1$	$3x + 5y^2 - 3$
Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)

DEFINITION OF POLYNOMIAL

An algebraic expression f(x) of the form $\mathbf{f(x) = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_nx^n}$, where $\mathbf{a_0, a_1, a_2, \dots\dots\dots, a_n}$ are real numbers and all the index of **'x'** are non-negative integers is called a **polynomials** in x.

Identification of Polynomial :

For this, we have following examples :

(i) $\sqrt{3}x^2 + x - 5$ is a polynomial in variable x as all the exponents of x are non negative integers.

(ii) $\sqrt{3}x^2 + \sqrt{x} - 5x$ is not a polynomial as the exponent of second term ($\sqrt{x} = x^{1/2}$) is not a non–negative integer.

(iii) $5x^3 + 2x^2 + 3x - \frac{5}{x} + 6$ is not a polynomial as the exponent of fourth term $\left[-\frac{5}{x} = 5x^{-1}\right]$ is not a non–negative integer.

◈ **Polynomial in one variable**

The algebraic expression like p(x) = 8x, q(x) = 7x +3, r(y) =4y–3, f(z) =6 – z^2, , s(y) =$3y^2$ + 8y + 9, etc. each of which involves only one variable (literal) are called polynomial in one variable.

◈ **Polynomials in two or more variables**

An algebraic expression, whose terms involve two to or more variables (literals) such that the exponent of each variable is a whole number, is called a polynomial in two or more variables.

For examples :

(a) $p(x,y) = 3x^2 - 6xy + 8y^2$ is a polynomial in two variables x and y.

(b) $p(x,y,z) = x + xy^3 - 8x^2yz - 15$ is a polynomial in three variables x, y and z.

Ex.1 Find which of the following algebraic expression is a polynomial.

(i) $3x^2 - 5x$ (ii) $x + \frac{1}{x}$ (iii) $\sqrt{y} - 8$ (iv) $z^5 - \sqrt[3]{z} + 8$

Sol. (i) $3x^2 - 5x = 3x^2 - 5x^1$

It is a polynomial.

(ii) $x + \frac{1}{x} = x^1 + x^{-1}$

It is not a polynomial.

(iii) $\sqrt{y} - 8 = y^{1/2} - 8$

Since, the power of the first term ($\sqrt{y}$) is $\frac{1}{2}$, which is not a whole number.

Therefore it is not a polynomial.

(iv) $z^5 - \sqrt[3]{z} + 8 = z^5 - z^{1/3} + 8$

Since, the exponent of the second term is 1/3, which is not a whole number. Therefore, the given expression is not a polynomial.

DEGREE OF A POLYNOMIAL

Highest index of **x** in algebraic expression is called the **degree of the polynomial**.

So the degree of polynomial

$\mathbf{f(x) = a_0 + a_1x + a_2x^2 + \dots\dots + a_nx^n}$, will be n if $a_n \neq 0$

Ex.2 In the following polynomial find

(a) Degree of each term

(b) Degree of polynomial

Sol.(a) In polynomial $5x^2 - 8x^7 + 3x$:

(i) The degree of term $5x^2 = 2$

(ii) The degree of term $-8x^7 = 7$

(iii) The degree of term $3x = 1$

Since, the greatest power is 7, therefore degree of the polynomial $5x^2 - 8x^7 + 3x$ is 7

(b) The degree of polynomial :

(i) $4y^3 - 3y + 8$ is 3

(ii) $7p + 2$ is 1

(iii) $2m - 7m^8 + m^{13}$ is 13 and so on.

TYPES OF POLYNOMIAL

Generally, we divide the polynomials in the following categories.

(i) Based on degree : There are following types of polynomials based on degree. These are listed below :

(A) Zero degree polynomial : Any non-zero number (constant) is regarded as a polynomial of degree zero or **zero degree polynomial**. i.e. $f(x) = a$, where $a \neq 0$, is a zero degree polynomial, since we can write $f(x) = a$ as $f(x) = ax^0$.

(B) Linear Polynomial : A polynomial of degree one is called a **linear polynomial**. The general form of linear polynomial is $\mathbf{ax + b}$, where **a** and **b** are any real constant and $\mathbf{a \neq 0}$.

(C) Quadratic Polynomial : A polynomial of degree two is called a **quadratic polynomial**. The general form of a quadratic polynomial is $\mathbf{ax^2 + bx + c}$, where $\mathbf{a \neq 0}$.

(D) Cubic Polynomial : A polynomial of degree three is called a **cubic polynomial**. The general form of a cubic polynomial is $\mathbf{ax^3 + bx^2 + cx + d}$, where $\mathbf{a \neq 0}$.

(E) Biquadratic Polynomials : A polynomial of degree four is called a **biquadratic polynomial**. The general form of a biquadratic polynomial is
$\mathbf{ax^4 + bx^3 + cx^2 + dx + e}$, where $\mathbf{a \neq 0}$.

Note : A polynomial of degree five or more than five does not have any particular name. Such polynomials are usually called as polynomial of degree five or six or .... etc.

(ii) Based on number of terms

There are following types of polynomials based on number of terms. These are as follows :

(A) Monomial : A polynomial is said to be a **monomial** if it has only one term. e.g. x, $9x^2$, $5x^3$ all are monomials.

(B) Binomial : A polynomial is said to be a **binomial** if it contains two terms. e.g. $2x^2 + 3x$, $3x + 5x^3$, $-8x^3 + 3$, all are binomials.

(C) Trinomial : A polynomial is said to be a **trinomial** if it contains three terms.

e.g. $3x^3 - 8x - \frac{5}{3}$, $5 - 7x + 8x^9$, $x^{10} + 8x^4 - 3x^2$ are all trinomials.

VALUE OF POLYNOMIAL

The value of a polynomial **f(x)** at $\mathbf{x = \alpha}$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $\mathbf{f(\alpha)}$.

Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$,

If we replace x by -2 everywhere in f(x), we get

$f(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6$

$f(-2) = -8 - 24 - 22 - 6$

$f(-2) = -60$.

So, we can say that value of f(x) at $x = -2$ is -60.

Similarly, the value of polynomial

$f(x) = 3x^2 - 4x + 2$,

(i) at $x = -2$ is $f(-2) = 3(-2)^2 - 4(-2) + 2 = 12 + 8 + 2 = 22$

(ii) at $x = 0$ is $f(0) = 3(0)^2 - 4(0) + 2 \quad = 0 - 0 + 2 = 2$

(iii) at $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2 = \frac{3}{4} - 2 + 2 = \frac{3}{4}$

ZEROES OR ROOTS OF A POLYNOMIAL

- The real number α is a root or zero of a polynomial **f(x)**, if $\mathbf{f(\alpha) = 0}$.

Consider the polynomial $f(x) = 2x^3 + x^2 - 7x - 6$,

If we replace x by 2 everywhere in f(x), we get

$f(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 - 6 = 0$

Hence, $x = 2$ is a root of f(x).

For example :

(i) For polynomial $p(x) = x - 2$; $p(2) = 2 - 2 = 0$

$\therefore$ $x = 2$ or simply 2 is a zero of the polynomial

$p(x) = x - 2$.

(ii) For the polynomial $g(u) = u^2 - 5u + 6$;

$g(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

$\therefore$ 3 is a zero of the polynomial g(u)

Also, $g(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$

$\therefore$ 2 is also a zero of the polynomial g(u)

Note :

(a) Every linear polynomial has one and only one zero.
(b) A given polynomial may have one or more than one zeroes.
(c) If the degree of a polynomial is n; the largest number of zeroes it can have is also n.

For example :
If the degree of a polynomial is 5, the polynomial can have at most 5 zeroes; if the degree of a polynomial is 8; largest number of zeroes it can have is 8.

(d) A zero of a polynomial need not to be 0.
For example : If $f(x) = x^2 - 4$,
then $f(2) = (2)^2 - 4 = 4 - 4 = 0$
Here, zero of the polynomial $f(x) = x^2 - 4$ is 2 which itself is not 0.

(e) 0 may be a zero of a polynomial.
For example : If $f(x) = x^2 - x$,
then $f(0) = 0^2 - 0 = 0$
Here 0 is the zero of polynomial $f(x) = x^2 - x$.

Ex.3 Find the zero of the polynomial in each of the following cases :
(i) $p(x) = x + 5$ (ii) $p(x) = 2x + 5$ (iii) $p(x) = 3x - 2$

Sol. To find the zero of a polynomial p(x) means to solve the polynomial equation p(x) = 0.

(i) For the zero of polynomial $p(x) = x + 5$
$p(x) = 0 \Rightarrow x + 5 = 0 \Rightarrow x = -5$
$\therefore$ $x = -5$ is a zero of the polynomial $p(x) = x + 5$.

(ii) $p(x) = 0 \Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5$ and $x = \frac{-5}{2}$
$\therefore$ $x = -\frac{5}{2}$ is a zero of $p(x) = 2x + 5$.

(iii) $p(x) = 0 \Rightarrow 3x - 2 = 0 \Rightarrow 3x = 2$ and $x = \frac{2}{3}$.
$x = \frac{2}{3}$ is a zero of $p(x) = 3x - 2$

Ex.4 If x = 2 & x = 0 are two roots of the polynomialf(x) = $2x^3 - 5x^2 + ax + b$. Find the values of a and b.

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0 \Rightarrow 16 - 20 + 2a + b = 0$
$\Rightarrow 2a + b = 4$ ...(i)
$f(0) = 2(0)^3 - 5(0)2 + a(0) + b = 0 \Rightarrow b = 0$
Put b = 0 in eq. (i)
$\Rightarrow 2a + 0 = 4$
So, $2a = 4 \Rightarrow a = 2$.

INTRODUCTION TO LINEAR POLYNOMIALS AND APPLICATIONS

A **linear polynomial** is an algebraic expression of the form ax+b where the highest power of the variable is 1. While seemingly simple, these polynomials are fundamental tools for modeling real-world situations where one quantity changes at a **constant rate** in relation to another.

Applications of Linear Polynomials

- **Financial Planning and Budgeting:** Linear polynomials help calculate total costs, savings, and remaining balances.
  Example: If a notebook costs ₹10, the total cost C for x notebooks is modeled by C = 10x.
  If you have a fixed starting amount of money and spend a certain amount daily, your remaining balance can be modeled as y = Initial Amount – (Daily Expense × Days).
- **Distance, Time, and Speed:** When an object moves at a constant speed, its position is described by a linear relationship.
  Example: The formula Distance = Speed × Time (e.g., d = 60t) is a linear polynomial. It is used to predict travel time or how far a vehicle will go over a specific period.
- **Physics and Engineering:** Many physical laws are expressed as linear polynomials when the rate of change is steady.
  Ohm's Law: In electronics, the relationshipV = I R(where R is constant resistance) is a linear polynomial used to find voltage or current.
  Newton's Second Law: The relationship between Force, Mass, and Acceleration (F = ma) is linear if mass is constant.

- **Geometry and Measurement:** Linear polynomials are used to represent dimensions like perimeter.
  Example: The perimeter P of a square with side x is P = 4x.
  Similarly, finding unknown angles in geometric figures often involves solving linear equations derived from polynomials.
- **Medicine and Healthcare:** Nurses and doctors use linear formulas to calculate correct medication dosages based on a patient's body weight.

Ex.5 Cost of Buying Items
Suppose a pen costs ₹5 and you also pay ₹10 delivery charge.
If you buy x pens, total cost becomes:
$5x + 10$
Example
If x = 4 pens:
$5(4) + 10 = 30$
So total cost = ₹30
This is a linear polynomial application.

Ex.6 Salary Calculation
Suppose a worker gets:
₹200 per day
Extra ₹500 bonus
Total salary for x days:
$200x + 500$
Example
If x = 10 days:
$200(10) + 500 = 2500$
Total salary = ₹2500

Ex.7 Temperature Change
Suppose temperature increases 2°C every hour and initial temperature is 25°C.
Temperature after x hours:
$2x + 25$
Example
If x = 3 hours:
$2(3) + 25 = 31$°C

EXPLORING LINEAR PATTERNS

In algebra, a linear pattern is a sequence of numbers or a relationship between two variables where the change is constant. If you graph these patterns, they always form a straight line

1. The Core Concept

A linear relationship is generally expressed in the form:
$y = mx + c$

- y **(Dependent Variable):** The outcome or total value.
- x **(Independent Variable):** The input (e.g., number of items, time, days).
- m **(Slope/Constant Change):** How much y increases or decreases for every 1 unit of x.
- c **(Y-intercept/Initial Value):** The value of y when x = 0.

2. How to Identify a Linear Pattern

- **In a Table:** The difference between consecutive terms (the "common difference") is always the same.
- **In a Graph:** The points can be connected by a single straight line.
- **In an Equation:** The variables x and y have a maximum power of 1 (no x^2 or $\sqrt{x}$).

No.	Scenario	Linear Equation	Constant Change (m)
1	**Taxi Fare:** Base fare of ₹50 and ₹10 per km.	y = 10x + 50	₹10/km
2	**Savings:** Starting with ₹1000, adding ₹200 monthly.	y = 200x + 1000	₹200/month
3	**Phone Battery:** 100% full, drops 5% every hour.	y = –5x + 10	-5%/hour
4	**Water Tank:** Starts with 200L, leaks 2L per hour.	y = –2x + 200	-2L/hour
5	**Notebook Cost:** Each notebook costs ₹25.	y = 25x	₹25/unit
6	**Plant Growth:** 5cm tall, grows 0.5cm every day.	y = 0.5x + 5	0.5cm/day
7	**Fixed Deposit:** ₹5000 initial + ₹50 interest monthly.	y = 50x + 5000	₹50/month
8	**Cricket Score:** Team is at 40 runs, scoring 8 per over.	y = 8x + 40	8 runs/over
9	**Candle Burning:** 12cm tall, melts 1.5cm every hour.	y = -1.5x + 12	-1.5cm/hour
10	**Delivery:** Shop charges ₹100 flat + ₹5 per item.	y = 5x + 100	₹5/item

SLOPE AND Y-INTERCEPT OF A LINE Y = AX + B

In the linear equation y = ax + b, the letters **a** and **b** are constants that define exactly how the line looks and where it sits on a graph. This is often referred to as the **Slope-Intercept Form**.

In simple terms, Slope measures the "steepness" or the "slant" of a line. In the context of linear polynomials, it tells you exactly how much the output (y) changes for every single unit of change in the input (x).

Think of it as the Rate of Change.

Slope in the Polynomial Equation

When we write a linear polynomial in the form y = mx + c:

m is the Slope.

c is the y-intercept (where the line hits the vertical axis).

Example: In the equation y = 3x + 2, the slope is 3.

Four Types of Slopes

You can identify the nature of the slope just by looking at the line:

1. Positive Slope (m > 0): The line goes up from left to right (Linear Growth).
2. Negative Slope (m < 0): The line goes down from left to right (Linear Decay).
3. Zero Slope (m = 0): The line is perfectly horizontal (no change in y).
4. Undefined Slope: The line is perfectly vertical (no change in x).

The Y-Intercept (b)

The constant term (the number standing alone) is the **y-intercept**.

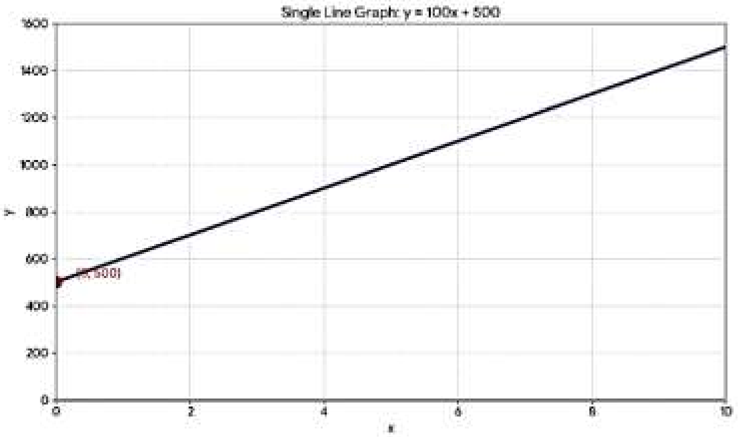
- **What it tells you:** This is the point where the line crosses the **vertical y-axis**.
- **The Coordinate:** At this point, the value of x is always **0**. Therefore, the coordinates of the y-intercept are always (0, b).
- **Starting Value:** In real-life word problems, b usually represents the "starting fee" or "initial amount."

Equation	Slope (a)	Y-Intercept (b)	Direction
y = 3x + 5	3	5	Goes Up
y = -2x + 8	-2	8	Goes Down
y = 0.5x – 4	0.5	-4	Gradual Rise
y = 7	0	7	Horizontal Line

LINEAR GROWTH AND LINEAR DECAY

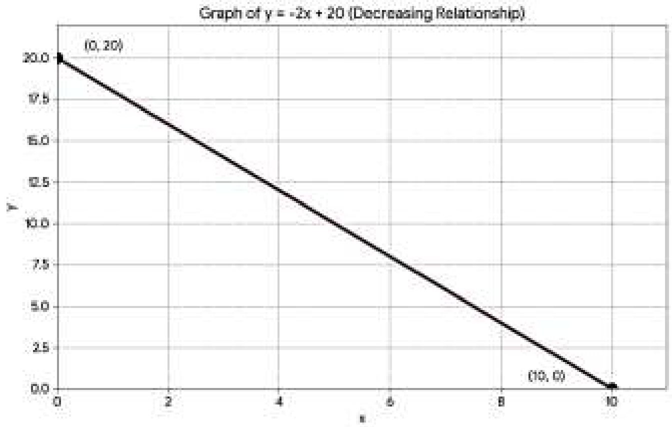
Linear growth happens when a quantity increases by the same amount over equal time intervals.

- Real-Life Example: You start with ₹500 in a piggy bank and add ₹100 every week.
- The Pattern: 500, 600, 700, 800...
- The Equation: y = mx + c
  - c (Starting Value) = 500
  - m (Rate of Growth) = +100
  - Equation: y = 100x + 500
- Graph: The line goes upward from left to right.



Linear decay happens when a quantity decreases by the same amount over equal time intervals.

- Real-Life Example: A candle is 20 cm tall and burns away 2 cm every hour.
- The Pattern: 20, 18, 16, 14...
- The Equation: y = c - mx
    - c (Initial Height) = 20
    - m (Rate of Decay) = 2
    - Equation: y = -2x + 20
- Graph: The line goes downward from left to right.



Feature	Linear Growth	Linear Decay
Change	Constant Addition (+)	Constant Subtraction (-)
Slope (m)	Positive (m > 0)	Negative (m < 0)
Examples	Simple Interest, Population (fixed growth), Filling a tank	Depreciation of a car, Burning fuel, Emptying a tank
Equation Form	y = mx + c	y = -mx + c

DEFINING LINEAR RELATIONSHIPS

A relationship between two variables (x and y) is linear if every time x changes by a certain amount, y changes by a fixed, constant amount.

- In Algebra: It is represented by a first-degree polynomial: y = mx + c.
- In Geometry: It is represented by a Straight Line.

Ex.8 The "Taxi Fare" Story

Imagine a taxi that charges a fixed "booking fee" of ₹50 and then ₹10 for every kilometer.

- 0 km → ₹50
- 1 km → ₹60 (+10)
- 2 km → ₹70 (+10)
- 3 km → ₹80 (+10)

Because the increase is always ₹10, this is a linear relationship.

VISUALIZING LINEAR RELATIONSHIPS

Visualizing means moving from a table of values or a word problem to a Coordinate Plane.

A. The Table of Values

Before drawing, we create a table. Let's use the equation y = 2x + 1:

x (Input)	Calculation (2x+1)	y (Output)	Point (x,y)
0	2(0) + 1	1	(0, 1)
1	2(1) + 1	3	(1, 3)
2	2(2) + 1	5	(2, 5)