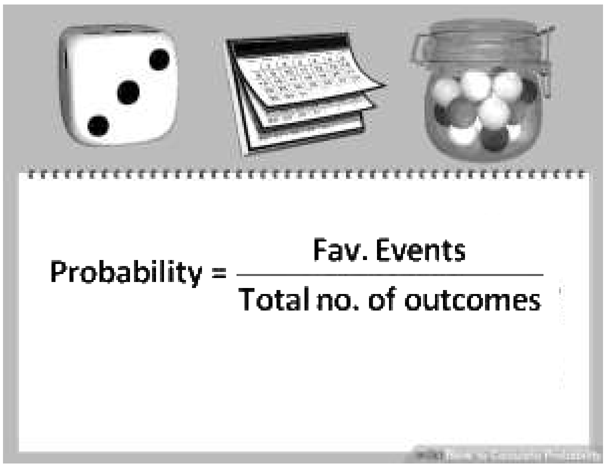
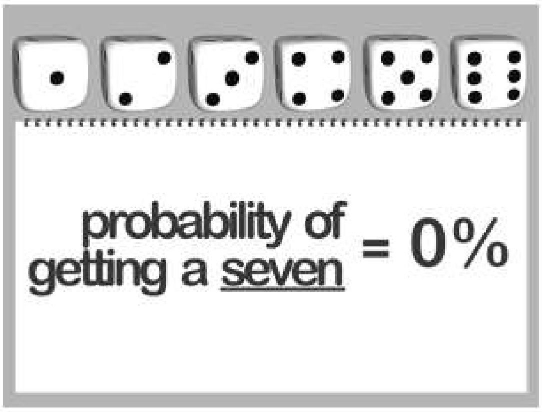
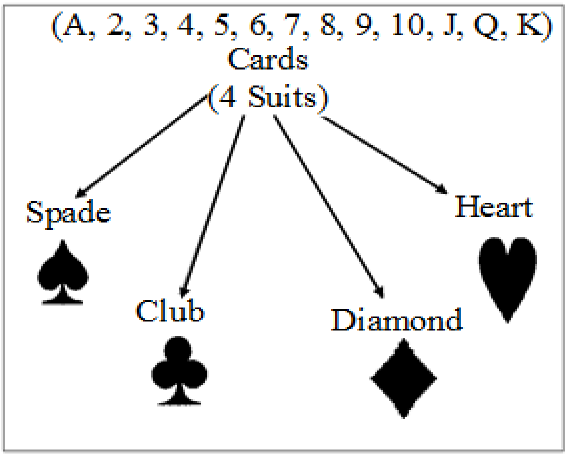
The Mathematics of Maybe: Introduction to Probability

Chapter Out Line

- Introduction
- Mathematical Definition of Probability







DEFINITIONS

Word 'chance', probably shows uncertainty in our statements.
The uncertainty of 'probability' can be measured numerically by means of 'probability'.

TYPES OF EXPERIMENT

1. **Deterministic Experiment :** Deterministic experiments are those experiments which when repeated under identical conditions produce the same result or outcome every time. For example, Physics or Chemistry experiments performed under identical conditions.

2. **Random or probabilistic Experiment :** If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment. For example, in the tossing of a coin one it is not sure if a head (H) or tail (T) will be obtained, so it is a random experiment. Similarly, rolling an unbiased die is an example of a random experiment.

TERMS

❖ **Trial and Event :**

An experiment is called a **trial** if it results in anyone of the possible outcomes and all the possible outcomes are called **events**.

For Example

(i) Participation of player in the game to win a game, is a trial but winning or losing is an event.

(ii) Tossing of a fair coin is a trial and turning up head or tail are events.

(iii) Throwing of a dice is a trial and occurrence of number 1 or 2 or 3 or 4 or 5 or 6 are events.

(iv) Drawing a card from a pack of playing cards is a trial and getting an ace or a queen is an event.

❖ **Favourable Events :**

Those outcomes of a trial which belongs to the event are called **favourable cases** for that event.

For Example –

(i) If a coin is tossed then favourable cases of getting head is 1.

(ii) If a dice is thrown then favourable case for getting prime number (2, 3, 5) are 3.

(iii) If two dice are thrown, then favourable cases of getting a sum of numbers as 9 are four i.e (4,5), (5,4), (3,6), (6,3).

❖ **Sample Space :**

The set of all possible outcomes of a trial is called its **sample space**. It is generally denoted by S.

For example

(i) If a die is thrown once, then its sample space

$S = \{1, 2, 3, 4, 5, 6\}$

(ii) If two coins are tossed together then its sample space

$S = \{HT, TH, HH, TT\}$.

The Probability Scale

Probability is measured on a scale from 0 to 1 (or 0% to 100%). It helps us understand and compare how likely different events are.

MATHEMATICAL DEFINITION OF PROBABILITY

Let there are n cases for an event A and m of those are favourable to it, then probability of happening of the event A is defined by the ratio m/n which is denoted by P(A). Thus

$$P(A) = \frac{m}{n} = \frac{\text{Number of favourable cases to A}}{\text{Totle number of cases to A}}$$

Note : It is obvious that $0 \le m \le n$. If an event A is certain to happen, then m = n thus P (A) = 1.

If A is impossible to happen then m = 0 and so P (A) = 0. Hence we conclude that

$0 \le P(A) \le 1$

if , P(A) = 0 , then A is called impossible event

if , P(A) =1, then, A is called sure event

Further, if $\overline{A}$ denotes negative of A i.e. not happening of event A, then for above case; we have

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$\therefore P(A) + P(\overline{A}) = 1$

Playing Cards :

- Total : 52 (26 red, 26 black)
- Four suits : Heart, Diamond, Spade, Club-13 of cards each
- Face Card : 12 (4 Kings, 4 queens, 4 jacks)

Analysing Statistical Data Using Probability

The way of collecting evidence through statistical data is used extensively in the world of business for marketing, forecasting sales, insurances and in science and social science research.

Ex.1 Suppose you anonymously collect information regarding the favourite fruit of 50 students in your class. Let us assume that the results are: 20 students like mango, 15 students like apples, 10 students like bananas, and 5 students like grapes. Let us play a game! Suppose we randomly pick one student from the class and try to guess their favourite fruit. What's the probability that the student's favourite fruit is mango? Statistical probability is based on collected data. So, a reasonable estimate that the student's favourite fruit is mango is Probability of mango as the favourite fruit

$$\frac{\text{Number of students who like mango}}{\text{Total number of students}} = \frac{20}{50} = 0.4$$

So, there is a 0.4 or 40% chance that a randomly chosen student in the class likes mango

The Gambler's Fallacy

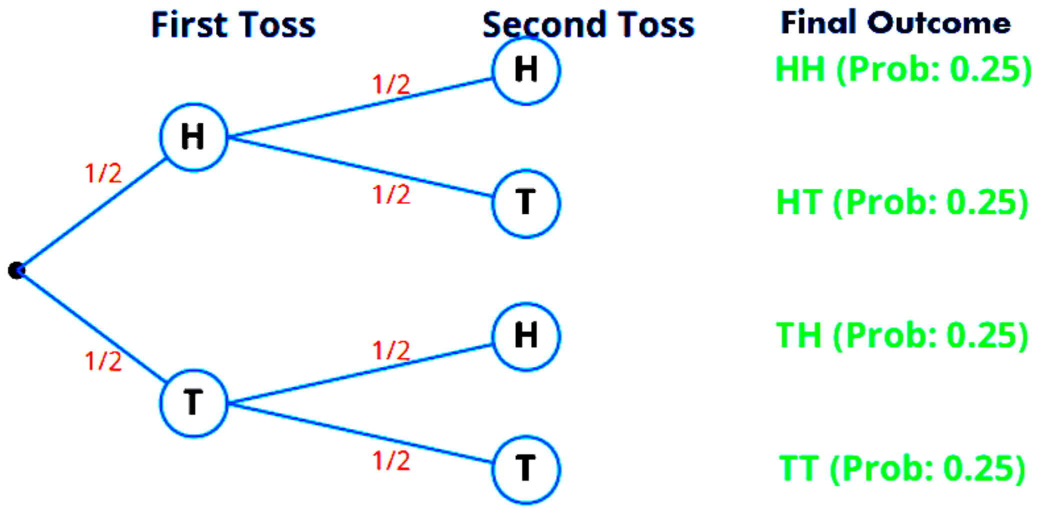
A very common misunderstanding in probability is assuming that past independent random events affect future outcomes. Randomness has no memory!

The Snakes & Ladders Trap

Suppose you are playing the ancient Indian game of *Jñān-Chaupad* (Snakes and Ladders). You roll a fair 6-sided die and get a **'6'** three times in a row. You might think,
"I've rolled three 6s, there is no way I will get a 6 next time!"
This is the **Gambler's Fallacy**. Each roll is an **independent event**. The probability of rolling a **6** on the fourth try remains exactly $\frac{1}{6}$. The die does not remember the previous rolls.

Multi-Step Experiments & Tree Diagrams

A multi-step experiment involves a series of independent trials (like tossing two coins, or picking two fruits from different baskets). The best way to visualize these is by using a Tree Diagram. A Tree Diagram branches out at each step to represent all possible paths, making it easy to list the sample space and calculate combined probabilities.



By multiplying the probability along the branches (1/2 × 1/2), we can easily determine that the theoretical probability of getting any specific combined outcome (like HH) is 1/4 or 25%.

❖ EXAMPLES ❖

Ex.1 Find the probability of getting a number less than 5 in a single throw of a die.

Sol. There are 4 numbers which are less than 5, i.e. 1, 2, 3 and 4.

Number of such favourable outcomes = 4.

$\because$ The number marked on all the faces of a die are 1, 2, 3, 4, 5 or 6

$\therefore$ Total number of possible outcomes = 6

$\therefore$ P(a number less than 5) $= \frac{4}{6} = \frac{2}{3}$

Ex.2 Two coins are tossed simultaneously. Find the probability of getting

(i) two heads (ii) at least one head (iii) no head

Sol. Let H denotes head and T denotes tail.

$\therefore$ On tossing two coins simultaneously, all the possible outcomes are {HH,HT,TH,TT}.

(i) The probability of getting two heads = P(HH)

$$= \frac{\text{Event of occurence of two heads}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

(ii) The probability of getting at least one head

= P(HT or TH or HH)

$$= \frac{\text{Event of occurence of at least one head}}{\text{Total number of possible outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head = P(TT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

Ex.3 Two dice are thrown at a time. Find the probability of the following -

(i) The numbers shown are equal;

(ii) the difference of numbers shown is 1.

Sol. The sample space in a throw of two dice

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2, 2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3, 3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5, 2), (5, 3), (5,4) (5,5) (5, 6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

total no. of cases n (S) = 6 × 6 = 36.

(i) Here E_1 = the event of showing equal number on both dice

= {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) }

$\therefore$ n (E_1) = 6

$\therefore$ P (E_1) = $\frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) Here E_2 = the event of showing numbers whose difference is 1.

= {(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)}

$\therefore$ n (E_2) = 10

$\therefore$ P(E_2) = $\frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

Ex.4 Three coins are tossed together -

(i) Find the probability of getting exactly two heads,

(ii) Find the probability of getting at least two tails.

Sol. The sample space in tossing three coins

S = { HHH, HHT, HTH, THH, TTH, THT, HTT, TTT }

$\therefore$ Total no. of cases n (S) = 2 × 2 × 2 = 8

(i) Here E_1 = the event of getting exactly two heads

= {HHT, HTH, THH}

$\therefore$ n (E_1) = 3 $\therefore$ P (E_1) = $\frac{n(E_1)}{n(S)} = \frac{3}{8}$

(ii) E_2 = {HTT, THT, TTH, TTT}

$\therefore$ n (E_2) = 4,

$\therefore$ P(E_2) = $\frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Ex.5 One card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing:

(i) an ace

(ii) '2' of spades

(iii) '10' of black suit

Sol. (i) There are 4 aces in deck.

$\therefore$ Number of favourable outcomes = 4

$\because$ Total number of cards in deck = 52.

$\therefore$ Total number of possible outcomes = 52.

$\therefore$ P(an ace) = $\frac{4}{52} = \frac{1}{13}$.

(ii) Number of '2' of spades = 1

Number of favourable outcomes = 1

Total number of possible outcomes = 52

$\therefore$ P('2' of spades) = $\frac{1}{52}$

(iii) There are 2 '10' of black suits (i.e. spade and club)

$\therefore$ Number of favourable outcomes = 2

Total number of possible outcomes = 52

$\therefore$ P('10' of a black suit) = $\frac{2}{52} = \frac{1}{26}$

Ex.6 17 Cards numbered 1, 2, 3 ... 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is

(i) Odd

(ii) A prime

(iii) Divisible by 3

(iv) Divisible by 3 and 2 both.

Sol. Out of 17 cards, in the box, one card can be drawn in 17 ways.

$\therefore$ Total number of elementary events = 17.

(i) There 9 odd numbered cards, namely, 1, 3, 5, 7, 9, 11, 13, 15, 17.

$\therefore$ Favourable number of outcomes = 9.

Hence, required probability = $\frac{9}{17}$.

(ii) There are 7 prime numbered cards, namely, 2, 3, 5, 7, 11, 13, 17.

$\therefore$ Favourable number of outcomes = 7.

Hence, P (Getting a prime number) = $\frac{7}{17}$.

(iii) Let A denote the event of getting a card bearing a number divisible by 3. Clearly, event A occurs if we get a card bearing one of the numbers 3, 6, 9, 12, 15.

$\therefore$ Favourable number of outcomes = 5.

Hence, P (Getting a card bearing a number divisible by 3) = $\frac{5}{17}$.

(iv) If a number is divisible by both 3 and 2, then it is a multiple of 6. In cards bearing number 1, 2, 3 ..., 17 there are only 2 cards which bear a number divisible by 3 and 2 both i.e. by 6. These cards bear numbers 6 and 12

$\therefore$ Favourable number of outcomes = 2

Hence, P (Getting a card bearing a number divisible by 3 and 2) = $\frac{2}{17}$.

Ex.7 A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is

(i) Black (ii) Red (iii) Not green.

Sol. Total number of balls in the bag = 5 + 8 + 4 + 7 = 24

$\therefore$ Total number of elementary events = 24

(i) There are 7 black balls in the bag.

$\therefore$ Favourable number of elementary events = 7

Hence, P (Getting a black ball) = $\frac{7}{24}$.

(ii) There are 5 red balls in the bag.

$\therefore$ Favourable number of outcomes = 5

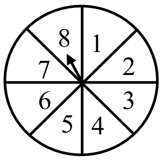
Hence, P (Getting a red ball) = $\frac{5}{24}$

(iii) There are 5 + 8 + 7 = 20 balls which are not green.

$\therefore$ Favourable number of outcomes = 20

Hence, P (Not getting a green ball) = $\frac{20}{24} = \frac{5}{6}$

Ex.8 A game of chance consists of spinning an arrow which comes to rest pointing at one of the number 1, 2, 3, 4, 5, 6, 7, 8 (see fig), and these are equally likely outcomes. What is the probability that it will point at



(i) 8
(ii) an odd number
(iii) a number greater than 2
(iv) a number less than 9

Sol. Total number of possible outcomes in the game = 8

(i) Number of cases in favour of arrow to rest on 8 = 1
So, number of favourable outcomes = 1

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(8) = \frac{1}{8}$$

(ii) In the game the number of odd number
1, 3, 5, 7 = 4
Number of favourable outcomes of odd number = 4

$$P = (\text{odd number}) = \frac{4}{8} = \frac{1}{2}$$

(iii) Numbers greater than 2 are 3, 4, 5, 6, 7, 8
Number of favourable outcomes for number greater than 2 = 6

$$P(\text{greater than } 2) = \frac{6}{8} = \frac{3}{4}$$

(iv) Number less than 9 are 1, 2, 3 …. 8
Number of favourable = 8

$$P(\text{less than } 9) = \frac{8}{8} = 1$$

Ex.9 Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number when each of the given number is equally likely to be selected.

Sol. Total number (1, 2, 3, 4, ... 25) = 25.
Out of 25 numbers, prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23.
So, remaining not a prime number are 25 – 9 = 16
Total number of possible outcomes = 25
and number of favourable outcomes = 16

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(\text{not a prime}) = \frac{16}{25}$$

Ex.10 It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday ?

Sol. Probability of 2 students from a group of 3 students, not having the same birthday
P(A) = 0.992
Probability of 2 students from a group of 3 students, having the same birthday
$[\because p(\overline{A}) = 1 - P(A)] = 1 - 0.992 = 0.008$