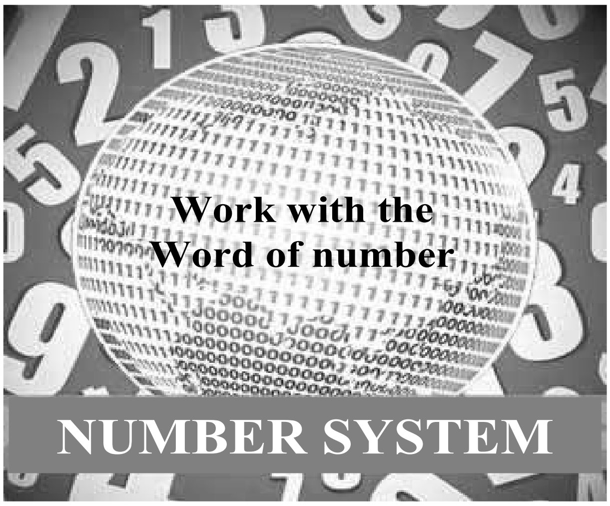
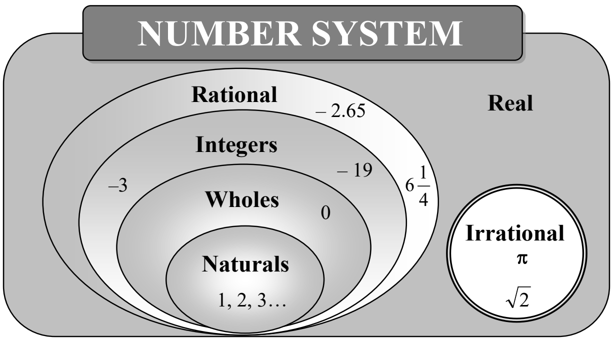
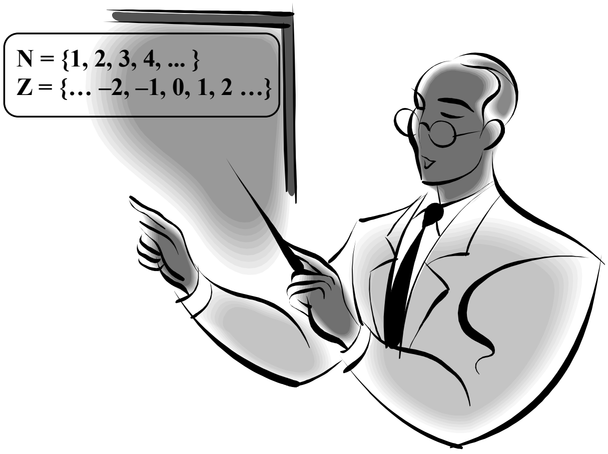
The World of Numbers

Chapter Outline

- ✧ **Introduction to Rational Numbers**
- ✧ **Introduction to Irrational Number**
- ✧ **Real Number**
- ✧ **Decimal Representation of Rational & Irrational number**
- ✧ **Representing Rational and Irrational Numbers on the Number Line**
- ✧ **Proof of irrationality of $\sqrt{2}$ and $\sqrt{3}$**
- ✧ **Surds or Radicals & Exponents**
- ✧ **Operations on Surds & Exponents**
- ✧ **Rationalisation of Surds**







NUMBER SYSTEM

◈ Natural Numbers

Counting numbers are known as natural numbers.

N = { 1, 2, 3, 4, ... }.

- **Even Numbers :** All natural numbers which are multiples of 2 are even numbers (i.e.) 2, 4, 6, 8… are even numbers.
- **Odd numbers :** All natural numbers which are not multiples of 2 are odd numbers.
- **Prime and composite Numbers :** A natural number which has exactly two factors, 1 and the number itself is called a **prime number**.

  All natural numbers other then 1 which are not prime are **composite numbers**.

  Note : 1 is neither a prime nor a composite number.

  2, 3, 5, 7, 11, 13, 17… are prime numbers. 4, 6, 8, 9, 10, 12 …… are **composite numbers.**
- **Co-prime:** Any natural numbers a and b are said to be co-prime if HCF (a, b) = 1. For example 4, 9 are co-prime numbers as H.C.F. (4, 9) = 1

◈ Whole numbers

The natural numbers along with the zero form the set of whole numbers i.e. numbers 0, 1, 2, 3, 4 are whole numbers. W = {0, 1, 2, 3, 4…}

◈ Integers

The natural numbers, their negatives and zero make up the integers.

Z = {…–4, –3, –2, –1, 0, 1, 2, 3, 4…}

The set of integers contains positive numbers, negative numbers and zero.

◈ Rational Numbers

(i) A rational number is a number which can be put in the form p/q, where p and q are both integers and $q \neq 0$.

(ii) A rational number is either a terminating or non-terminating but recurring (repeating) decimal.

(iii) A rational number may be positive, negative or zero.

◈ Irrational Number

All real numbers are irrational if and only if their decimal representation is non-terminating and non-repeating.

e.g. $\sqrt{2}$, $\sqrt{3}$, π… etc.

◈ Real Numbers

Rational number and irrational number taken together form the set of real numbers.

Note :

(1) If a and b are two real numbers, then either (i) $a > b$ or (ii) $a = b$ or (iii) $a < b$

(2) Negative of an irrational number is an irrational number.

(3) The sum of a rational number with an irrational number is always irrational number.

(4) The product of a non-zero rational number with an irrational number is always an irrational number.

(5) The sum of two irrational numbers is not always an irrational number.

(6) The product of two irrational numbers is not always an irrational number.

(7) $\pi = 3.14159265358979\ldots$

while $\frac{22}{7} = 3.1428571428\ldots$

$\therefore\ \pi \neq \frac{22}{7}$ but for calculation we can take $\pi \approx \frac{22}{7}$.

◈ The Absolute Value (or modulus) of a real Number

If a is a real number, modulus a is written as |a|; |a| is always positive or zero. It means positive value of 'a' whether a is positive or negative

$|3| = 3$ and $|0| = 0$, Hence $|a| = a$; if $a = 0$ or $a > 0$ (i.e.) $a \geq 0$

$|-3| = 3 = -(-3)$. Hence $|a| = -a$ when $a < 0$

Ex.1 Is zero a rational number? Can you write it in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. It can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc. where denominator $q \neq 0$, it can be negative also.

DENSITY OF RATIONAL NUMBERS

What Does "Density" Mean?

Density here means:

- Between any two rational numbers, there are infinitely many rational numbers.
  In simple words:
  No matter how close two numbers are, you can always find another rational number between them.

Ex.2 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. A rational number between two rational numbers r and s is $\frac{r+s}{2}$.

A rational number between

$\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right) = \frac{7}{10}$.

And a rational number between

$\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right) = \frac{13}{20}$

Similarly; $\frac{5}{8}, \frac{27}{40}, \frac{31}{40}$ are between $\frac{3}{5}$ and $\frac{4}{5}$.

So, five rational numbers between

$\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{31}{40}, \frac{27}{40}$

Ex.3 Find 3 irrational numbers between 3 & 5.

Sol. $\because$ 3 and 5 both are rational

The irrational are 3.1010010001…;
3.2020020002…; 3.3030030003…

Ex.4 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, and if $\sqrt{ab}$ is irrational then $\sqrt{ab}$ is an irrational number lying between a and b, if ab is not a perfect square

$\therefore$ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2}\times 6^{1/4}} = 2^{1/4} \times 6^{1/8} = 2^{2/8} \times 6^{1/8}$
$= (4 \times 6)^{1/8} = 24^{1/8}$

Hence required irrational numbers are $6^{1/4}$ and $24^{1/8}$

DECIMAL REPRESENTATION OF RATIONAL NUMBERS

(A) Terminating

Ex.5 $\frac{6}{5}, \frac{8}{5}, \frac{7}{4}$ … are equal to 1.2, 1.6, 1.75 respectively, so these are terminating

Ex.6 Express $\frac{-17}{8}$ in decimal form by long division method.

Sol. In order to convert $\frac{-17}{8}$ in the decimal form, we first express $\frac{17}{8}$ in the decimal form and the decimal form of $\frac{-17}{8}$ will be negative of the decimal form of $\frac{17}{8}$

we have,

$$\begin{array}{r|l} 8 \,)\, 17.000 & 2.125 \\ 16 & \\ \hline 10 & \\ 8 & \\ \hline 20 & \\ 16 & \\ \hline 40 & \\ 40 & \\ \hline 0 & \end{array}$$

$\therefore \ \frac{-17}{8} = -2.125$

(B) Non terminating recurring (repeating)

Ex.7 $\frac{10}{3} = 3.333\ldots \text{or } 3.\overline{3}$

$\frac{1}{7} = 0.14285714285\ldots \text{or } 0.\overline{142857}$

$\frac{2320}{99} = 23.434343\ldots \text{or } 23.\overline{43}$

These expansions are not terminated but digits are continuously repeated so we use a line on those digits, called bar ($\overline{\ }$).

So we can say that rational numbers are of the form either terminating, or non terminating repeating (recurring).

CONVERSION OF DECIMAL NUMBERS INTO RATIONAL NUMBERS OF THE FORM $\frac{m}{n}$

Case I : When the decimal number is of terminating nature.

Ex.8 Express each of the following numbers in the form $\frac{p}{q}$.

(i) 0.675 (ii) –25.6875

Sol. (i) $0.675 = \frac{675}{1000} = \frac{675 \div 25}{1000 \div 25} \Rightarrow = \frac{27}{40}$

(ii) $-25.6875 = \frac{-256875}{10000}$

$= \frac{-256875 \div 625}{10000 \div 625} = \frac{-411}{16}$

Case II : When decimal representation is of non-terminating repeating nature.

- **Conversion of a pure recurring decimal to the form $\frac{p}{q}$**

Ex.9 Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\overline{6}$

Sol. (i) Let x = $0.\overline{6}$

then, x = 0.666… ....(i)

Here, we have only one repeating digit, So, we multiply both sides of (i) by 10 to get

10 x = 6.66… ....(ii)

Subtracting (i) from (ii), we get

10 x – x = (6.66…) – (0.66…)

$\Rightarrow \ 9x = 6 \qquad \Rightarrow \ x = \frac{6}{9}$

$\Rightarrow \ x = \frac{2}{3} \qquad$ Hence $0.\overline{6} = \frac{2}{3}$

- **Conversion of a mixed recurring decimal to the form $\frac{p}{q}$**

Ex.10 Express the following decimals in the form

(i) $0.3\overline{2}$

Sol. (i) Let x = $0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore \quad 10x = 3.\overline{2}$

$\Rightarrow \quad 10x = 3 + 0.\overline{2} \qquad \left[\because 0.\overline{2} = \frac{2}{9}\right]$

$\Rightarrow \quad 10x = 3 + \frac{2}{9} \quad \Rightarrow \quad 10x = \frac{9\times3+2}{9} \quad \Rightarrow \quad 10x = \frac{29}{9} \quad \Rightarrow \quad x = \frac{29}{90}$

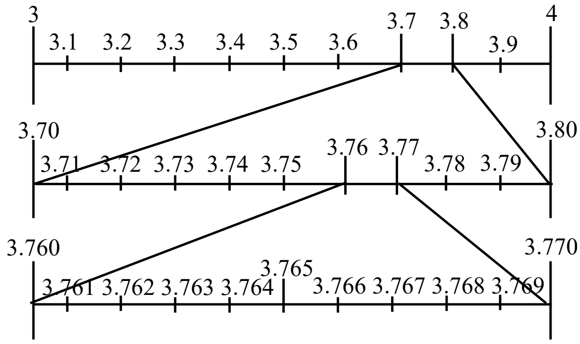
REPRESENTING RATIONAL NUMBERS ON THE NUMBER LINE

Number line is a geometrical straight line with arbitrarily defined zero.

We understand the concept with the help of following examples.

Ex.11 Represent 3.765 on the number line.

Sol. This number lies between 3 and 4. The distance 3 and 4 is divided into 10 equal parts. Then the first mark to the right of 3 will represent 3.1 and second 3.2 and so on. Now, 3.765 lies between 3.7 and 3.8. We divide the distance between 3.7 and 3.8 into 10 equal parts 3.76 will be on the right of 3.7 at the sixth mark, and 3.77 will be on the right of 3.7 at the 7th mark and 3.765 will lie between 3.76 and 3.77 and so on.

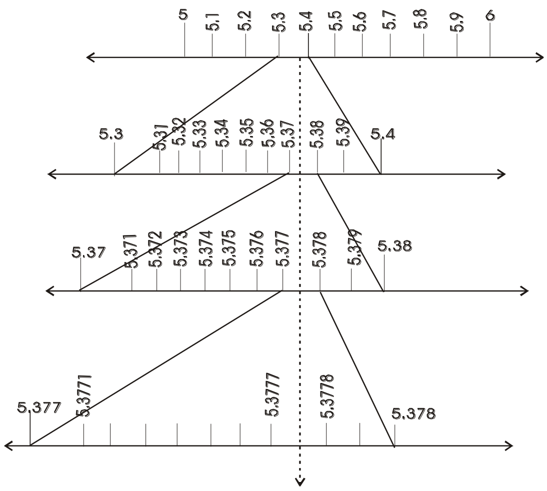


To mark 3.765 we have to use magnifying glass method

Ex.12 Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal places.

Sol. We have, $5.3\overline{7}$ = 5.3777….

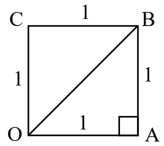
This number lies between 5.3 and 5.4. The distance between 5.3 and 5.4 is divided into 10 equal parts. Then the first mark to the right of 5.3 will represent 5.31 and second 5.32 and so on. Now, 5.3777 lies between 5.37 and 5.38. We divide the distance between 5.37 and 5.38 into 10 equal parts and so on.



REPRESENTING IRRATIONAL NUMBERS ON THE NUMBER LINE

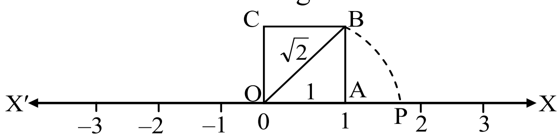
❖ Represent $\sqrt{2}$ & $\sqrt{3}$ on the number line

Greeks discovered this method. Consider a unit square OABC, with each side 1 unit in length. Then by using pythagoras theorem



$OB = \sqrt{1+1} = \sqrt{2}$

Now, transfer this square onto the number line making sure that the vertex O coincides with zero



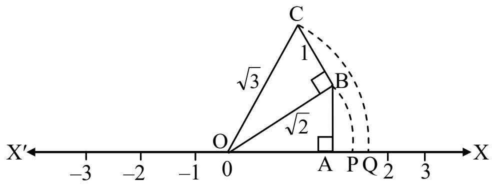
With O as centre & OB as radius, draw an arc, meeting OX at P. Then

$OB = OP = \sqrt{2}$ units

Then, the point P represents $\sqrt{2}$ on the number line.

Now draw, $BC \perp OB$ such that BC = 1 unit, join OC. Then

$OC = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{3}$ units



With O as centre & OC as radius, draw an arc, meeting OX at Q. Then

$OQ = OC = \sqrt{3}$ units

Then, the point Q represents $\sqrt{3}$ on the number line.

Remark : In the same way, we can locate $\sqrt{n}$ for any positive integer n, after $\sqrt{n-1}$ has been located.

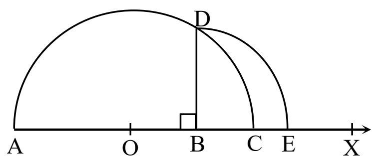
❖ Existence of $\sqrt{n}$ for a positive real number
(Geometrical Representation)

Represent the value of $\sqrt{4.3}$ geometrically :-

Draw a line segment AB = 4.3 units and extend it to C such that BC = 1 unit.

Find the midpoint O of AC by drawing the perpendicular bisector of AC.

With O as centre and OA a radius, draw a semicircle.



Now, draw $BD \perp AC$, intersecting the semicircle at D. Then, $BD = \sqrt{4.3}$ units.

With B as centre and BD as radius, draw an arc, meeting AC produced at E.

Then, $BE = BD = \sqrt{4.3}$ units.

PROOF OF IRRATIONALITY OF $\sqrt{2}$ AND $\sqrt{3}$

Theorem-1 : Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Proof : Let the prime factorisation of a be as follows :

$a = p_1p_2.....p_n$, where $p_1, p_2, ...p_n$ are primes, not necessarily distinct.

Therefore, $a^2 = (p_1p_2.....p_n)(p_1p_2 ..... p_n) = p_1^2p_2^2 ..... p_n^2$.

Now, we are given that p divides a^2. Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are $p_1, p_2, ..., p_n$. So p is one of $p_1, p_2, ..., p_n$.

Now, since $a = p_1 p_2 ...... p_n$, p divides a.

Ex.13 Prove that $\sqrt{2}$ is irrational number

Sol. Let us assume, to contrary, that $\sqrt{2}$ is rational. That is, we can find integers a and b (a, b $\neq$ 0) such that

$\sqrt{2} = \frac{a}{b}$ .Where a and b are coprime i.e. their H.C.F. = 1

So, $b\sqrt{2} = a$.

Squaring on both sides, and rearranging, we get $2b^2 = a^2$.

Therefore, a^2 is divisible by 2, and by Theorem 1, it follows that a is also divisible by 2.

So, we can write a = 2c for some integer c.

Substituting for a, we get $2b^2 = 4c^2$,

That is, $b^2 = 2c^2$.

This means that b^2 is divisible by 2, and so b is also divisible by 2.

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

SURDS OR RADICALS

◈ Surds

An irrational number of the form $\sqrt[n]{a}$ is called Surd, where 'a' is called radicand and it should always be a rational number. The symbol $\sqrt[n]{\ }$ is called the radical sign and the index n is called order of the surd. $\sqrt[n]{a}$ is read as 'nth root of a' and can also be written as $a^{\frac{1}{n}}$, where n is a natural number other then 1.

◈ Laws of Radicals

For any positive integer 'n' and a positive rational number 'a'.

(a) $\left(\sqrt[n]{a}\right)^n = a$

(b) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [one of either a or b should be non-negative integer]

(c) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ [one of either a or b should be non-negative integer]

(d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(e) $\sqrt[m]{a^n} = \sqrt[mp]{a^{np}}$

Note :

(i) A surd consisting of one term only is called a **monomial surd**.

(ii) An expression consisting of the sum or difference of two monomial surds or the sum or difference of a monomial surd and a rational number is called **binomial surd**. e.g.

$\sqrt{2} + \sqrt{5}, \sqrt{3} + 2, \sqrt{2} - \sqrt{3}$ etc. are binomial surds.

(iii) The binomial surds which differ only in sign (+ or –) between the terms connecting them, are called **conjugate surds** e.g. $\sqrt{3} + \sqrt{2}$ and $\sqrt{3} - \sqrt{2}$ or $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugate surds.

◈ Pure And Mixed Surds

(i) Pure Surd :

A surd which has unity only as rational factor, the other factor being irrational, is called a **pure surd**.

E.g. $\sqrt{3}, \sqrt[5]{2}, \sqrt[4]{3}$ are pure surds.

E.g. $\sqrt{6}, \sqrt[3]{12}$ are pure surds.

(ii) Mixed Surd :

A surd which has a rational factor other than unity, the other factor being irrational, is called a **mixed surd**.

E.g. $2\sqrt{3}, 5\sqrt[3]{12}, 2\sqrt[4]{5}$ are mixed surds.

Type-I : On expressing of mixed surds into pure surds

Ex.14 Express each of the following as a pure surd.

(i) $2\sqrt{3}$ (ii) $2.\sqrt[3]{4}$ (iii) $\frac{3}{4}\sqrt{32}$ (iv) $\frac{3}{2}\sqrt[4]{\frac{32}{243}}$

Sol. (i) $2\sqrt{3} = 2\times3^{1/2} = (2^2)^{1/2}\times3^{1/2} = 4^{1/2}\times3^{1/2} = (4\times3)^{1/2} = 12^{1/2} = \sqrt{12}$

(ii) $2.\sqrt[3]{4} = 2\times4^{1/3} = (2^3)^{1/3}\times4^{1/3} = 8^{1/3}\times4^{1/3} = (8\times4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

(iii) $\frac{3}{4}\sqrt{32} = \sqrt{\left(\frac{3}{4}\right)^2}\times\sqrt{32} = \sqrt{\left(\frac{3}{4}\right)^2\times32} = \sqrt{\frac{9}{16}\times32} = \sqrt{18}$

(iv) $\frac{3}{2}.\sqrt[4]{\frac{32}{243}} = \frac{3}{2}\times\left(\frac{32}{243}\right)^{1/4} = \left(\left(\frac{3}{2}\right)^4\right)^{1/4}\times\left(\frac{32}{243}\right)^{1/4}$

$= \left(\frac{81}{16}\right)^{1/4}\times\left(\frac{32}{243}\right)^{1/4} = \left(\frac{81}{16}\times\frac{32}{243}\right)^{1/4} = \left(\frac{2}{3}\right)^{1/4} = \sqrt[4]{\frac{2}{3}}$

Ex.15 Express each of the following as pure surd :

(i) $a\sqrt{a+b}$ (ii) $a\sqrt[3]{b^2}$ (iii) $2ab\sqrt[3]{ab}$

Sol. (i) $a\sqrt{a+b} = a\times(a+b)^{1/2} = (a^2)^{1/2}\times(a+b)^{1/2} = [a^2\times(a+b)]^{1/2} = (a^3+a^2b)^{1/2} = \sqrt{a^3+a^2b}$

(ii) $a\sqrt[3]{b^2} = (a^3)^{1/3}\times(b^2)^{1/3} = (a^3\times b^2)^{1/3} = \sqrt[3]{a^3b^2}$

(iii) $2ab.\sqrt[3]{ab} = \left((2ab)^3\right)^{1/3}\times(ab)^{1/3} = (8\,a^3b^3.ab)^{1/3} = (8\,a^4b^4)^{1/3} = \sqrt[3]{8a^4b^4}$

Type-II : On expressing given surds as mixed surds in the simplest form.

Ex.16 Express each of the following as mixed surd in its simplest form:

(i) $\sqrt[3]{72}$ (ii) $5.\sqrt[3]{135}$

Sol. (i) $\sqrt[3]{72} = \sqrt[3]{8\times9} = \sqrt[3]{2^3\times9} = \sqrt[3]{2^3}\times\sqrt[3]{9} = 2\sqrt[3]{9}$

(ii) $5.\sqrt[3]{135} = 5\sqrt[3]{27\times5} = 5\sqrt[3]{3^3\times5} = 5\sqrt[3]{3^3}\times\sqrt[3]{5} = 5\times3\times\sqrt[3]{5} = 15\sqrt[3]{5}$.

◈ Operations on surds

- **Addition and Subtraction of Surds :**

  Addition and subtraction of surds are possible only when order and radicand are same i.e. only for like surds.

Ex.17 Simplify : $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

Sol. $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54} = 5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

$= 5\times5\sqrt[3]{2} + 7\times2\sqrt[3]{2} - 14\times3\times\sqrt[3]{2} = (25+14-42)\sqrt[3]{2} = -3\sqrt[3]{2}$

- **Multiplication and Division of Surds :**

Ex.18 State with reasons which of the following are surds and which are not :

(i) $\sqrt{64}$ (ii) $\sqrt{45}$ (iii) $\sqrt{20}\times\sqrt{45}$

Sol. (i) $\sqrt{64} = 8$

8 is a rational number, hence $\sqrt{64}$ is not a surd.

(ii) $\sqrt{45} = \sqrt{9\times5} = 3\sqrt{5}$

Thus $\sqrt{45}$ is an irrational number.

Because the rational number 45 is not the square of any rational number, hence $\sqrt{45}$ is a surd.

(iii) We have $\sqrt{20}\times\sqrt{45} = \sqrt{900} = \sqrt{30\times30} = \left(\sqrt{30}\right)^2 = 30$

Which is a rational number and therefore $\sqrt{20}\times\sqrt{45}$ is not a surd.

Ex.19 Simplify each of the following:

(i) $\sqrt[3]{3} \times \sqrt[3]{4}$ (ii) $\sqrt[3]{128}$

Sol. (i) $\sqrt[3]{3} \times \sqrt[3]{4} = \sqrt[3]{3 \times 4} = \sqrt[3]{12}$

(ii) $\sqrt[3]{128} = \sqrt[3]{64 \times 2} = \sqrt[3]{64}\ \sqrt[3]{2} = \sqrt[3]{4^3} \cdot \sqrt[3]{2} = 4\sqrt[3]{2}$

Ex.20 Simplify each of the following:

(i) $\sqrt[3]{\frac{8}{27}}$ (ii) $\frac{\sqrt[4]{3888}}{\sqrt[4]{48}}$

Sol. (i) $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{3^3}} = \frac{2}{3}$ (ii) $\frac{\sqrt[4]{3888}}{\sqrt[4]{48}} = \sqrt[4]{\frac{3888}{48}} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$

◈ **Rationalization of surds**

Rationalizing factor : If product of two surds is a rational number then each of them is called the **rationalizing factor (R.F.)** of the other. The process of converting a surd to a rational number by using an appropriate multiplier is known as **rationalization.**

When the denominator of an expression contains a term with a square root (or a number with radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called **rationalizing** the **denominator**.

Ex.21 Rationalize the denominator : $\frac{1}{\sqrt{162}}$

Sol. $\frac{1}{\sqrt{162}} = \frac{1}{\sqrt{81 \times 2}} = \frac{1}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{18}$

Ex.22 Rationalize the denominator. $\frac{1}{7+5\sqrt{3}}$

Sol. $\frac{1}{7+5\sqrt{3}} = \frac{1}{7+5\sqrt{3}} \times \frac{7-5\sqrt{3}}{7-5\sqrt{3}} = \frac{7-5\sqrt{3}}{49-75} = \frac{7-5\sqrt{3}}{-26}$.

SOME RULES FOR EXPONENTS

Let a, b > 0 be real numbers & p, q are rational numbes then

(i) $a^p.a^q = a^{p+q}$ (ii) $(a^p)^q = a^{pq}$ (iii) $\frac{a^p}{a^q} = a^{p-q}$

(iv) $(ab)^p = a^pb^p$ (v) $a^0 = 1$ (vi) $a^p = \frac{1}{a^{-p}}$ or $a^{-p} = \frac{1}{a^p}$

Ex.22 Find the value of the following:

(i) $\frac{(7)^3 \times 21}{3}$ (ii) $\frac{(121)^3 \times 33}{3}$

Sol. (i) $\frac{(7)^3 \times 21}{3} = 7^3 \times 7 = 7^{3+1} = 7^4$ (ii) $\frac{(121)^3 \times 33}{3} = (11^2)^3 \times 11 = 11^6 \times 11^1 = 11^7$

Ex.23 Find the value of x : $\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$

Sol. $\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$

$\left(\frac{5}{3}\right)^{-x} \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$

$\left(\frac{5}{3}\right)^{2x-x} = \frac{125}{27}$

$\left(\frac{5}{3}\right)^{x} = \left(\frac{5}{3}\right)^{3}$

Because the base is same, so comparing the powers x = 3.