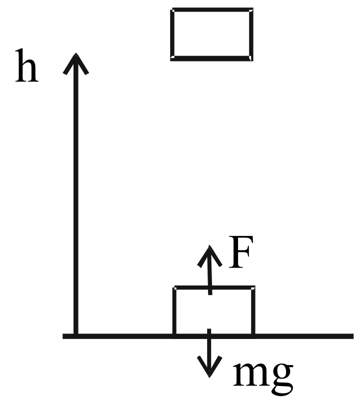
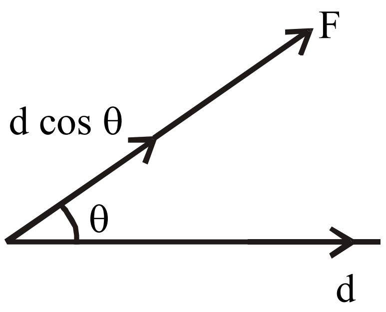
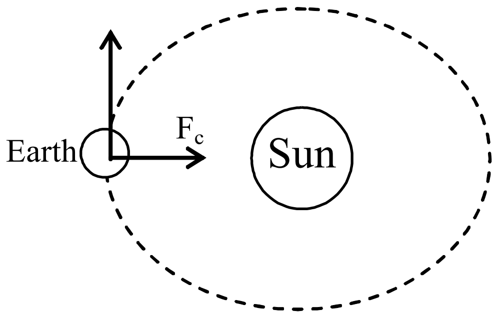
Work, Energy & Simple Machines

Chapter Out Line

- ✧ **Work**
- ✧ **Measurement of work**
- ✧ **Conservative and non-conservative force**
- ✧ **Energy**
- ✧ **Potential and kinetic energy**
- ✧ **Work energy theorem**
- ✧ **Power**
- ✧ **Simple Machines**







F_c = Gravitational force

INTRODUCTION

In everyday language, the word **work** is used to describe any activity in which muscular or mental effort is exerted. In physics, the word work has a special meaning. Work is done only when the force acting on a body produces motion in it in the direction of force (or in the direction of component of force). Thus a boy pushing the wall is doing no work from physics' point of view. It is because the force exerted by the boy is not producing motion of the wall. The speed at which work can be done is an indication of the power of the body doing work. For example, a boy may carry a suitcase upstairs in 3 minutes while a man may do it in 1 minute. Obviously, the power of the man is more than the power of the boy. Thus, time factor is important for power. A body which has the capacity to do work is said to possess energy. The greater the capacity of a body to do work, the greater the energy it has. Thus work, energy and the power are related to each other. In this topic we shall deal with these three important concepts of physics.

WORK

In our day to day life, the word **work** means any kind of mental and physical activity. For example, we say that we are doing work while,

- reading a book,
- cooking the food,
- walking on a level road with a box on our head,
- pushing a wall of a house but fails to do so.

In all these cases, either mental or a physical activity is involved.

But in physics, the term work has entirely a different meaning. In physics work is said to be done if body is displaces due to the application of force.

Conditions which must be satisfied for the work done are :

- A force must act on the body.
- The body must be displaced from one position to another position.

Definition :

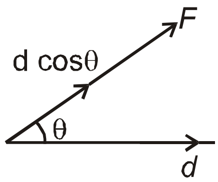
Work is said to be done by a force on a body or an object if the force applied causes a displacement in the body or object.

Eg. : Work is done, when a box is dragged on the floor from one position to another. In this case, force is on box to drag it on the floor and the box moves through a certain distance between one position to another position.

Measurement of Work :

Work is measured by the product of force and the displacement in the direction of force. Work is a scalar quantity.

Work = Force . displacement in the direction of force



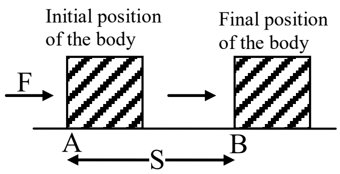
W = F(d cosθ)

- The work done by a force on a body depends on two factors :
  (i) Magnitude of the force, and
  (ii) Distance through which the body moves (in the direction of force)

Unit of Work

S.I. unit of work done is newton × metre = joule.

When a force of 1 newton moves a body through a distance of 1 metre in its own direction, then the work done is known as 1 joule.



Work = Force . Displacement

1 joule = 1 N × 1 m

or 1 J = 1 Nm (In SI unit)

In C.G.S. system the unit of work done is

dyne x cm = erg

Definition of 1 erg :

If F = 1 dyne and d = 1 cm

then, W = 1× 1 = 1 erg

1 joule = 10^7 erg

- **Gravitational unit of work :**
  Work is said to have gravitational unit of work if unit gravitational force displaces the body through unit distance in the direction of force.

(i) In C.G.S. system, gravitational unit of work is gram-weight-centimeter (g wt cm).

Since W = FS

1g wt cm = 1 g wt × 1 cm = 981 dyne × 1cm

1g wt cm = 981 erg.

Thus 1g-wt-cm of work is done when a force of 1g-wt displaces a body through 1cm in its own direction.

(ii) In S.I. system, gravitational unit of work is kilogram weight meter (kg wt m)

1kg wt m = 1kg wt × 1m = 9.81 N × 1 m

1 kg wt m = 9.81 J

Thus, 1 kg wt m of work is done when a force of 1 kg-wt displaces a body through 1 m in its own direction.

Units of work
- Absolute units
  - S.I. Unit Joule(J)
  - C.G.S. unit erg
- Gravitational units
  - S.I. unit kgwtm
  - C.G.S. unit gwtcm

1kg wt m =9.8N-m = 9.8 J
1g wt cm = 981dyne-cm=981erg

ILLUSTRATION

1. How much work is done by a force of 10N in moving an object through a distance of 1 m in the direction of the force ?

Sol. The work done is calculated by using the formula:

W = F . S

Here, Force, F = 10 N

And, Distance, S = 1 m

So, Work done, W = 10 × 1 J = 10 J

Thus, the work done is 10 joules

WORK DONE ANALYSIS

Work done when force and displacement are along same line.

- **Work done by a force :** Work is said to be done by a force if the direction of displacement is the same as the direction of the applied force.
- **Work done against the force :** Work is said to be done against a force if the direction of the displacement is opposite to that of the force.
- **Work done against Gravity :** To lift an object, an applied force has to be equal and opposite to the force of gravity acting on the object. If 'm' is the mass of the object and 'h' is the height through which it is raised, then the upward force

  (F) = force of gravity = mg

  If 'W' stands for work done, then

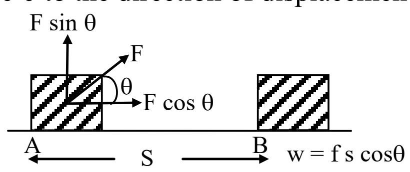
  W = F.h = mg.h

  Thus W = mgh

Therefore we can say that, "The amount of work done is equal to the product of weight of the body and the vertical distance through which the body is lifted.

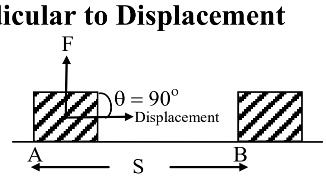
Work done when force and displacement are inclined (Oblique case)

Consider a force 'F' acting at angle θ to the direction of displacement 's' as shown in fig.



W = F.S cosθ º

- **Work done when force is perpendicular to Displacement**



θ = 90º

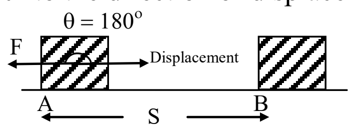
W = F.S cos 90º = F.S × 0 = 0

Thus no work is done when a force acts at right angle to the displacement.

Can you think why ?
- What is work done by static friction?
- Is collision may be possible even without any actual contact?

Work done when force and displacement are at 180°

Consider a force 'F' acting at angle 180° to the direction of displacement 's' as shown in fig.



$\theta = 180^\circ$

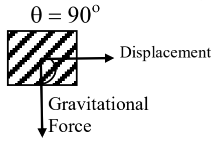
$W = F.S \times \cos 180^\circ = -F.S$

Thus when force is in the opposite direction of displacement then work done is negative.

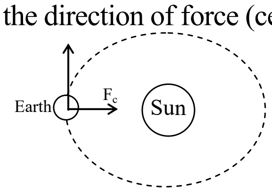
Eg. Work done by kinetic frictional force.

Boost your knowledge
- A weight lifter does work in lifting the weight off the ground, but does not in holding the weight up.

If a body is moving in horizontal direction then work done by the force of gravity will be zero.



Eg. Earth revolves around the sun. A satellite moves around the earth. In all these cases, the direction of displacement is always perpendicular to the direction of force (centripetal force) and hence no work is done.



F_c = Gravitational force

Note :

(i) If F = 0 then work done, W = 0

Eg. A student revising his notes by memory without moving his limbs is doing no physical work.
A meditating saint is doing no physical work though he keeps sitting for hours.

(ii) If displacement, d = 0 then work done, W = 0.

Eg. A foolish labour trying to displace a building has done no work though he may spend the whole day.

(iii) If θ **= 180°**

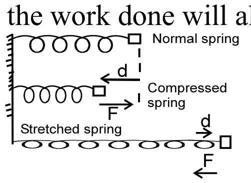
then from equation (i)

$W = Fd \cos 180^\circ \qquad \cos 180^\circ = -1$

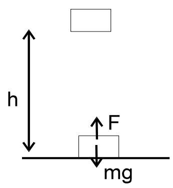
then, $\qquad W = -Fd$

When the force and displacement are in opposite direction then work done will be negative.

Eg. When a spring is compressed then the force applied by the spring and the displacement will be in opposite direction to each other, so work done by the spring will be negative.
When the spring is stretched then the work done will also be negative.



Eg. When a body of mass m is lifted upward a force,
F = mg has to be applied upwards
Work done by the force of gravity will be negative
Work done, W = – mgh



ILLUSTRATION

2. Calculate the work done in pushing a cart, through a distance of 100 m against the force of friction equal to 120 N.

Sol. Force, F = 120 N; Distance, s = 100 m

Using the formula, we have $W = Fs = 120\text{ N} \times 100\text{ m} = 12{,}000\text{ J}$

3. Calculate the work done in raising a bucket full of water and weighing 200 kg through a height of 5 m. (Take $g = 9.8\text{ ms}^{-2}$).

Sol. Force of gravity

$mg = 200 \times 9.8 = 1960.0\text{ N}$

$h = 5\text{ m}$

Work done, $W = mgh$ or $W = 1960 \times 5 = 9800\text{ J}$

4. A boy pulls a toy cart with a force of 100 N by a string which makes an angle of 60º with the horizontal so as to move the toy cart by a distance of 3 m horizontally. Calculate the work done.

Sol. Given F = 100 N, s = 3 m, θ = 60º.

Work done is given by

$$W = Fs\cos\theta = 100 \times 2 \times \cos 60^\circ = 100 \times 3 \times \frac{1}{2} = 150\text{ J} \left(\because \cos 60^\circ = \frac{1}{2}\right)$$

5. An engine does 64,000 J of work by exerting a force of 8,000 N. Calculate the displacement in the direction of force.

Sol. Given W = 64,000 J ; F = 8,000 N

Work done is given by W = Fs

or 64000 = 8000 × s or s = 8 m

6. A student life water from well 25m deep. If the work done by the student is 500J, then find the volume of water. (Take $g = 10\text{ m/s}^2$)

Sol. Given h = 25m ; Acceleration (g) = 10 m/s^2 ; Work done (w) = 500 J

We know $w = mgh \Rightarrow m = \frac{w}{gh} = \frac{500}{10 \times 25} = 2\text{kg}.$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \Rightarrow \rho = \frac{m}{v} \Rightarrow v = \frac{m}{\rho} = \frac{2\text{ kg}}{1000\text{ kg/m}^3} = 2 \times 10^{-3}\text{ m}^3$$

7. A car weighing 1200 kg and travelling at a speed of 20 m/s stops at a distance of 40 m retarding uniformly. Calculate the force exerted by the brakes. Also calculate the work done by the brakes.

Sol. In order to calculate the force applied by the brakes, we first calculate the retardation.

Initial speed, u = 20 m/s; final speed,

v = 0, distance covered, s = 90 m

Using the equation, $v^2 = u^2 + 2as$, we get $0^2 = (20)^2 + 2 \times a \times 40$

or $80a = -400$

or $a = -5\text{ m/s}^2$

Force exerted by the brakes is given by

$F = ma$

Here $m = 1200\text{ kg}; a = -5\text{ m/s}^2$

$\therefore F = 1200 \times (-5) = -6000\text{ N}$

The negative sign shows that it is a retarding force. Now, the work done by the brakes is given by

$W = Fs$

Here F = 6000 N; s = 40 m

$\therefore W = 6000 \times 40\text{ J} = 240000\text{ J}$

$= 2.4 \times 10^5\text{ J}$

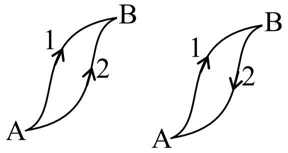
$\therefore$ Work done by the brakes = 2.4×10^5 J

CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative force :

A force is conservative if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end points.

Suppose a particle moves from point A to point B along either path 1 or path 2, as shown in figure 1 (a). If a conservative force F acts on the particle, then the work done on the particle is same along the two paths.



1(a) 1(b)

Mathematically, we can write

W_{AB} (along path 1) = W_{AB} (along path 2) ...(i)

Now suppose the particle moves in a round trip, from point A to point B along path 1 and then back to point A along path 2, as shown in figure 1(b). For a conservative force.

Work done on the particle along the path 2 from A to B = – Work done on the particle along the path 2 from B to A

i.e.,

W_{AB}(along path 2) = $-W_{BA}$ (along path2) …(ii)

From (i) and (ii), we have

W_{AB} (along path 1) = $-W_{BA}$ (along path 2)

or W_{AB} (along path 1) = $-W_{BA}$ (along path2) = 0

or $W_{\text{closed path}} = 0$

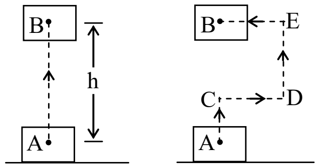
Hence a force is conservative if the work done by the force in moving a particle around any closed path is zero.

e.g. Gravitational force, electrostatic force and elastic force of a spring are all conser-vative forces.

Eg. *Conservative nature of gravitational force.*

(i) As shown in figure 2(a), suppose a body of mass m is raised to a height h vertically upwards from position A to B. The work done against gravity is

$W = mg \times AB = mg\,h$



2 (a) 2 (b)

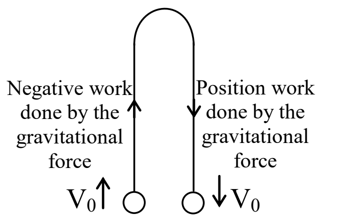
As shown in figure 2(b), now suppose the body is taken from position A to B along the path ACDEB. During the horizontal paths CD and EB, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done only along vertical paths AC and DE, the total work done is

$W = W_{AC} + W_{CD} + W_{DE} + W_{EB} = mg \times AC + 0 + mg \times DE + 0$

$= mg\,(AC + DE) = mg \times h$ or $W = mgh$

Thus the work done in moving a body against gravity is independent of the path taken and depends only on the initial and final positions of the body. Hence gravitational force is a conservative force.

(ii) Suppose a ball is thrown vertically upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the ball descends, the gravitational force does positive work on it, increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and K.E. with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This again shows that the gravitational force is a conservative force.



Conservative nature of gravitational force

Non-Conservative Force :

If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. The work done in moving an object against a non-conservative force along a closed path is not zero.

E.g. Forces of friction and viscosity are non-conservative force.

Conservative Forces	Non-Conservative Force
Work done by a conservative force in a round trip is zero	Work done by a non-conservative force in a round trip is not zero
Work done depends on its initial and final position and not on its path.	Work done is path dependent.

ENERGY

When a man does a work, he feels tired. He feels that he has lost something which he must regain to work more. A weak man gets exhausted after doing only a small amount of work. A strong man can continue to work for longer duration. Something that a working man loses is called energy.

- **Definition :**
  Capacity of doing work or total work done by a man or by an agent, is called the energy of the man or the agent.

(a) Units of energy :

C.G.S. unit of energy is erg and S.I. unit of energy is joule.

(i) KiloWatt × hour (kWh) is commercial unit of energy.

$1 \text{ kWh} = 1000 \text{ watt} \times 60 \times 60 \text{ sec.}$

$= 3.6 \times 10^6 \text{ watt sec.}$

$1 \text{ kWh} = 3.6 \times 10^6 \text{ J.}$

(ii) Electron volt(eV) is also the unit of energy. The energy of an electron, when it is accelerated by a potential difference of 1 volt, is known as one eV

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J.}$

$1 \text{ watt-hour} = 1 \text{ watt} \times 1 \text{ hour} = 1 \text{ watt} \times 60 \times 60 \text{ sec} = 3600 \text{ J}$

$1 \text{ kilowatt-hour (kWh)} = 3.6 \times 10^6 \text{ Joule}$

Heat energy is usually measured in calorie or kilocalorie such that

$1 \text{ calorie} = 4.18 \text{ J}$

A very small unit of energy is electron volt(eV).

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

DIFFERENT FORM OF ENERGIES

(i) Heat energy :

When we burn coal, wood or gas, heat energy is released. Steam possess heat energy that is why in a steam engine, the heat energy of steam is used to get the work done. Sun also radiates heat energy.

(ii) Light energy :

It is a form of energy which gives us the sensation of vision. Natural source of light is the sun. An electric bulb also emits light energy.

(iii) Sound energy :

The energy emitted by a vibrating wire, tuning fork, vibrating membrane etc., that can be sensed by human ears is called sound energy.

Eg. whistle, flute, sitar, all emits sound energy when they are made to vibrate.

(iv) Magnetic energy : A magnet also possess energy known as magnetic energy. When a current is passed through a coil, it stores magnetic energy.

(v) Electrical energy : An electric cell stores electrical energy. Two charges placed at some distance experience a force. They also possess electrical energy.

Eg. A charged body possess electrical energy.

Boost your knowledge

- Wind energy, ocean wave energy, ocean thermal energy, tidal energy, geothermal energy are some natural (non-conventional) sources of energy.

MECHANICAL ENERGY

The energy possessed by a body due to its state of rest and state of motion is called mechanical energy. Mechanical energy is the sum of following energies.

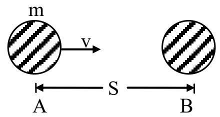
(A) Kinetic Energy (B) Potential Energy.

M.E. = K.E. + P.E.

◈ KINETIC ENERGY :

The energy of a body due to its motion is called kinetic energy. In other words. The ability of a body to do work by virtue of its motion is called its kinetic energy.

- **Expression for Kinetic Energy :** The kinetic energy of a body is measured in terms of the amount of work done by an opposing force that brings the body to rest from its present state of motion.



Suppose a body of mass m is moving with a velocity v and is brought to rest by an opposing force F.

Now retarding force is given by

$$F = ma \qquad ...(1)$$

Now using the equation of motion,

$v^2 - u^2 = 2as$, we get

$0^2 = v^2 - 2as$

$$\therefore \quad s = \frac{v^2}{2a} \qquad ...(2)$$

Kinetic energy of the body = work done by the retarding force

or Kinetic energy = force × displacement

$$= F\,.\,s \qquad ...(3)$$

Substituting the value of F from equation (1) and the value of s from equation (2) in equation (3), we get

$$K.E. = ma \times \frac{v^2}{2a} = \frac{1}{2}mv^2$$

Thus, a body of mass m and moving with a velocity v has the capacity of doing work equal to $\frac{1}{2}mv^2$ before it stops.

- **Factors affecting Kinetic energy :**

(i) The more the mass of a body, the greater its kinetic energy

(ii) The more the velocity of a body, the more its kinetic energy.

> **Boost your knowledge**
>
> ☛ Kinetic energy of a body is always positive. This is because mass can never the negative and square of a real value is always positive.

Relation between Momentum and Kinetic Energy :

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

On multiplying and dividing by ‘m’ in the R.H.S. of equation, we get

$$K.E. = \frac{m^2v^2}{2m}$$

$$\text{As } p = mv \Rightarrow K.E. = \frac{P^2}{2m}$$

◈ POTENTIAL ENERGY :

The energy possessed by a body by virtue of its position or shape or configuration is known as potential energy.

(i) Water stored in a dam has potential energy due to its position.

(ii) A stone lying on the top of a hill or a mountain has potential energy due to its position.

(iii) A stretched or a compressed spring has potential energy due to its shape. When spring is stretched or compressed, work is done on it. This work done is stored as potential energy of the stretched or compressed spring.

(iv) A wound spring of a watch has potential energy due to its shape.

(v) A stretched bow and arrow has potential energy due to its shape.

Can you think why ?
- A system have mechanical energy but not momentum, is it possible?

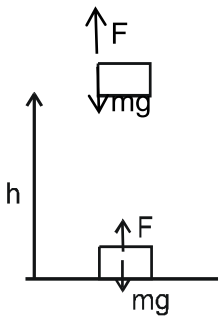
(i) Gravitational potential energy :
The energy possessed by a body by virtue of its position (i.e. height above the surface of the earth) is known as gravitational potential energy.

(ii) Elastic potential energy :
The energy possessed by a body by virtue of its deformed shape (i.e. either stretched or compressed) is known as elastic potential energy.

Expression for Potential Energy of a Body at a Certain Height

The energy possessed by a body due to its position in the gravitational field of the earth is called gravitational potential energy.

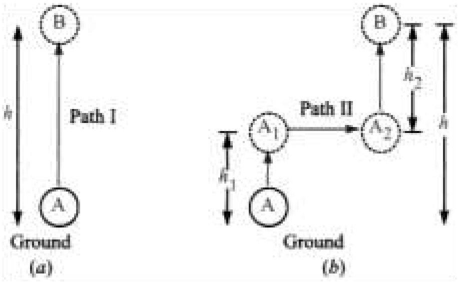
Consider a block of mass m which is to be raised to a height 'h'. The force required to lift the block must be equal to the gravitational force (i.e. weight of the block). Thus, Fg = mg. Let the applied force on the block be F = mg and the block is raised to the height h as shown in the figure.



Work done by the applied force F is given by
$W = Fh \cos 0^o$ [$\cos 0^o = 1$]
or $W = Fh = mgh$
Now, work done by the gravitational force on the block,
$W = Fgh \cos 180$
[$\theta = 180$ between F_G and H]
or $W_g = -F_gh = -mgh$ [$F_g = mg$]
Work done against the gravitational force on the block is known as gravitational potential energy.
$\mathbf{U_g = -(-mgh) = mg}$

ILLUSTRATION

8. Prove that gravitational potential energy depends upon the difference in heights of the initial position and final position of a body but is independent of the path followed by the body while going from initial position to final position.



Sol. Consider a ball of mass m raised through height h from position A to position B along path I as shown in the figure(a). Then gravitational potential energy of the ball at height, h
$U = mgh$ …(i)
Now let the ball is raised through height h_1 from position A to position A_1 as in figure (b). Then gravitational potential energy of the ball at height, h_1
$U_1 = mgh_1$ .... (ii)
When the ball is taken from position A_1 to position A_2, then no work is done against gravity because the force of gravity acts perpendicular to the displacement of the ball. Thus, gravitational potential energy of the ball at position A_1 and at position A_2 is same. Hence, there is no change in gravitational potential energy of the ball in going from position A_1 to position A_2.
Now when the ball is raised through height h_2 from position A_2 to position B, then the gravitational potential energy of the ball at height, h_2
$U_2 = mgh_2$ ..... (iii)
Therefore, the total gravitational potential energy of the ball at position B
$U = U_1 + U_2 = mgh_1 + mgh_2$
$= mg (h_1 + h_2) = mgh$ ....(iv)
$[(h_1 + h_2) = h]$
$U = U_1 + U_2 = mgh$

- **Conclusion :**
  - **(i)** The gravitational potential energy of a body depends upon the difference in height (h) of the initial and final positions of the body
  - **(ii)** The gravitational potential energy of the body does not depend upon the path followed by the body going from initial to final positions.
- **Important Information :**
  - **(i)** Gravitational potential energy of a body on the surface of the earth (i.e. h = 0) is zero.
  - **(ii)** Gravitational potential energy of a body increases if the body moves upward (i.e. h increases).
  - **(iii)** Gravitational potential energy of a body decreases if the body moves downward (i.e. h decreases).
  - **(iv)** Gravitational potential energy depends only on the initial and final position of the body and not on the path followed by the body to go from initial position to final position. It means, the gravitational potential energy of a body at height h will be same if it is either taken straight upward to height h or it is taken along a curved path to height h.

WORK ENERGY THEOREM

This theorem states that the work done by the forces acting on a body is equal to the change in the kinetic energy of the body.

Consider a body of mass m moving with a velocity u. Let a force F be applied on the body, so that it is accelerated with an acceleration 'a'.

Then, F = ma

If s be the distance travelled by the body during its accelerated motion, then the work done by the force F is given by

W = Fs = mas,

Since (F = ma) ......(i)

Let the body acquires velocity v after travelling a distance s, then from

$v^2 - u^2 = 2as$, we have

$$a = \frac{v^2 - u^2}{2s} \qquad \text{......(ii)}$$

Put this value in equation (equation-i), we get

$$W = m\left(\frac{v^2 - u^2}{2s}\right)s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad \text{...(iii)}$$

Here $\frac{1}{2}mv^2$ = Final K.E.

and $\frac{1}{2}mu^2$ = Initial K.E

Now as W = F.s so (equation-iii) can be written as W = Fs

$$F = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mu^2}{s} \qquad \text{......(iv)}$$

Equation (iv) gives us the relationship between force & energy.

The difference between the final and initial kinetic energies is the change in K.E. of the body Δ(K.E.)

W = change in K.E. = Δ (K.E.)

- **Examples of kinetic energy :**

(i) Energy possessed by a moving bicycle.
(ii) Energy possessed by running water
(iii) Energy possessed by a shooting arrow.
(iv) Energy possessed by blowing wind
(v) Energy possessed by a spinning electron round the nucleus

ILLUSTRATION

9. A bullet of mass 100 gm is fired with a velocity 50 m/s from a gun. Calculate the kinetic energy of the bullet.

Sol. Kinetic energy is given by

$$K.E. = \frac{1}{2}mv^2$$

Here m = 100 gm = 0.1 kg; v = 500 m/s

$$K.E. = \frac{1}{2} \times 0.1 \times (50)^2 = \frac{1}{2} \times 0.1 \times 50 \times 50 = 125 \text{ J}$$

10. Calculate the velocity of 4 kg mass with kinetic energy of 128 J.

Sol. The formula for kinetic energy is given by

$$K.E. = \frac{1}{2} mv^2$$

Here K.E. = 128 J; m = 4 kg

$$\therefore\ 128 = \frac{1}{2} \times 4 \times v^2$$

or $v^2 = 64$; or $v = 8$ m/s

11. When the mass and velocity of the **body** are doubled, what happens to its kinetic energy ?

Sol. Let the mas, initial velocity and K.E. of the body be m,v and k respectively.

$$K = \frac{1}{2} mv^2$$

When mass and velocity are doubled, the kinetic energy

$$K' = \frac{1}{2} \times 2m \times (2v)^2 = 8\left(\frac{1}{2} mv^2\right)$$

K' = 8(K)

When the mass and velocity of the body are doubled, the kinetic energy becomes 8 times the original.

12. A body is moving in a straight line with a certain velocity. Another body with double the mass and half the velocity of the first, is moving in the straight line. What is the ratio of kinetic energy of 2nd body with the first body ?

Sol. Let the mass, initial velocity and K.E. of the body be m,v and K respectively.

Kinetic energy of first body, $K_1 = \frac{1}{2} mv^2$

Kinetic energy of second body,

$$K_2 = \frac{1}{2}\ 2m\left(\frac{v}{2}\right)^2 \Rightarrow \frac{k_2}{k_1} = \frac{\frac{1}{2} \times 2m\left(\frac{v}{2}\right)^2 - \frac{1}{2} mu^2}{\frac{1}{2} mv^2} = \frac{\frac{1}{2} \times 2m\left(\frac{v}{2}\right)^2}{\frac{1}{2} mv^2} = \frac{1}{2}$$

The ratio of K.E. of the second body to first body is 1 : 2

13. In the kinetic energy of a body is doubled, then how does its momentum change ?

Sol. Let m, p and K be the mass, initial momentum and kinetic energy of a body respectively.

$$P_1 = \sqrt{2mK}$$

$$p_2 = \sqrt{2m(2K)} = \sqrt{2} \times \sqrt{2mK} = \sqrt{2}\, p_1$$

$$p_2 = \sqrt{2}\, p_1$$

Therefore, the final momentum is $\sqrt{2}$ times of the initial momentum.

INTER CONVERSION OF POTENTIAL AND KINETIC ENERGY

(i) For a freely falling body, potential energy changes into kinetic energy.

Let a body of mass m be at rest at a point at height h from the ground.

At highest point :

Potential energy of the body $U_1 = mgh$

Kinetic energy of the body $K_1 = 0$ [u = 0]

As the body falls freely, it gains velocity and reduces height. Let the body have velocity **v** when it reaches the ground.

At lowest point :

Potential energy of the body, $U_2 = 0$ [h = 0]

Kinetic energy of the body, $K_2 = \frac{1}{2} m v^2$

From third equation of motion, $v^2 = u^2 + 2gh$

We have, $v^2 = 2gh$ [u = 0]

Hence, final kinetic energy $= \frac{1}{2} m v^2 = \frac{1}{2} m(2gh) = mgh$ = Initial potential energy

(ii) For an upward projected body, kinetic energy changes into potential energy.

Let a body of mass **m** be projected upwards with a velocity **u** from a point on the ground.

At lowest point :

Kinetic energy of the body, $K_1 = \frac{1}{2} m u^2$

Potential energy of the body, $U_2 = 0$

As the body rises upward, it gains height and loses velocity.

Let the body reach highest point at height **h** where veliocity becomes zero.

At highest point :

Kinetic energy of the body, $K_2 = 0$

Potential energy of the body $U_2 = mgh$

From third equation of motion,

$v^2 = u^2 + 2gh$

We have, $0 = u^2 - 2gh$ [u = 0]

Or $u^2 = 2gh$

(v = 0 and g is negative for upward motion)

Hence, final P.E. = $mgh = m\frac{u^2}{2}$

P.E. = $\frac{1}{2} m u^2$ = Initial K.E.

LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor be destroyed,
It can only be changed from one form to another form.

Appearing amount of energy in one form is always equal to the disappearing amount of energy in some other form. The total energy thus remains constant.

Examples of Transformation of Energy :

(i) Conversion of Mechanical energy into other forms:

(A) When two stones are rubbed together, fire is produced. Here, mechanical energy is converted into heat energy.
(B) Vibrating bodies produce sound. Here, Mechanical energy is converted into sound energy.
(C) Generation of electricity by rotation of dynamo. Here, Mechanical energy is converted into Electrical energy.

(ii) Conversion of Heat energy into other forms :

(A) Heat energy is converted into mechanical energy in heat engines.
(B) Heat energy is converted into light energy in electric bulbs.
(C) Heat energy is converted into chemical energy during the preparation of chemical compounds.

(iii) Conversion of sound energy into other forms :

(A) Sound energy is converted into electric energy during telephonic conversion.
(B) During marching of soldiers on a bridge, due to resonance,sound energy is converted into mechanical energy, resulting in the collapse of the bridge.

(iv) Conversion of electrical energy into other forms :

(A) Electrical energy is converted into mechanical energy in electrical fans.
(B) Electrical energy is converted into magnetic energy in electro magnets.
(C) Electrical energy is converted into chemical energy in electrolysis.

(v) Conversion of magnetic energy into other forms :

(A) Magnetic energy is converted into electrical energy during electro magnetic induction.
(B) Magnetic energy is converted into mechanical energy in magnetic compass.

(vi) Conversion of chemical energy into other forms :

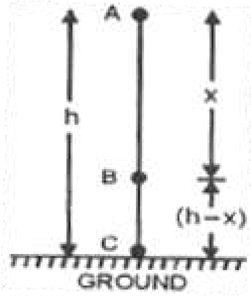
(A) Chemical energy is converted into electrical energy in electrical cells.
(B) Chemical energy is converted into light, heat, sound and mechanical energy during the explosion of atom bonds.
(C) Chemical energy is converted into light energy during burning of magnesium ribbon.

(vii) Conversion of nuclear energy into other forms :

(A) Nuclear energy is converted into electrical energy in nuclear reactor.
(B) Nuclear energy is converted into heat and light energy inside the sun.
(C) Nuclear energy is converted into light and heat energy during nuclear fission and fusion reactions.

Device used	Energy tranformation	
	Form	to
Steam engine	Heat	Mechanical
Electric fan	Electrical	Mechanical
Electric lamp	Electrical	Light and Heat
Electric heater	Electrical	Heat
Microphone	Sound	Electrical
Solar cell	Solar heat	Electrical
Photo-cell	Light	Electrical
Car engine	Chemical	Heat, Mechanical
Electric cell/batteries	Chemical	Electrical

INTER CONVERSION OF ENERGY FOR A FREELY FALLING BODY

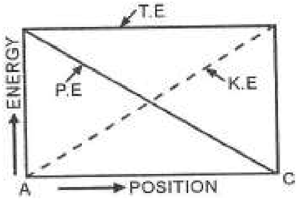


Let a body of mass **m** is at rest at a height **h** from the earth's surface, when it starts falling, after a distance **x** (point B) its velocity becomes **v** and at earth's surface its velocity is **v'**.
Mechanical energy of the body :

- **At point A :**
  E_A = Kinetic energy + Potential energy
  $E_A = \frac{1}{2}m(0) + mgh$
  $E_A = mgh$ .... (i)
- **At point B :**
  $E_B = \frac{1}{2}m v^2 + mg(h-x)$ .... (ii)
  From third equation of motion at points A and B
  $V^2 = u^2 + 2gx$ $u = 0$
  $v^2 = 2gx$
  On putting the value of v^2 in equation (ii)
  $E_B = \frac{1}{2}m(2gx) + mgh - mgx$
  $E_B = mgx + mgh - mgx$
  $\mathbf{E_B = mgh}$ .... (iii)
- **At point C :**
  $E_C = \frac{1}{2}m v'^2 + mg \times o.$
  $E_C = \frac{1}{2}m v'^2$ .... (iv)
  From third equation of motion at points A & C.
  $(v')^2 = u^2 + 2gh$ $u = 0$
  So, $(v')^2 = 2gh$
  On putting the value of $(v')^2$ in equation (iv)
  $E_c = \frac{1}{2}m(2gh)$
  or $\mathbf{E_C = mgh}$ .... (v)
  From equation (i), (iii) and (v)
  $\mathbf{E_A = E_B = E_C}$
  Hence, the mechanical energy of a freely falling body will be constant.

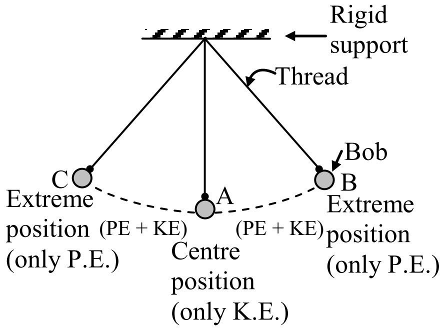
Total energy of the body during free fall, remains constant at all positions. The form of energy, however keeps on changing. At point A, energy is entirely potential energy and at point C, it is entirely kinetic energy In between A and C, energy is partially potential and partially kinetic. This variation of energy is shown in figure. Total mechanical energy stays constant (mgh) throughout.

Thus in an isolated system, where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but the mechanical energy of the system (which is sum of kinetic energy and potential energy) cannot change. We can, therefore, equate the sum of kinetic energy and potential energy at one instant to the sum of kinetic energy and potential energy at another instant without considering intermediate states. This law has been found to be valid in every situation. No violation, whatsoever, of this law has ever been observed.

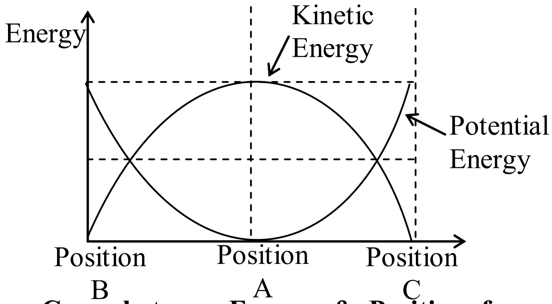


A swinging simple pendulum is an example of conservation of energy :

This is because a swinging simple pendulum is a body whose energy can either be potential or kinetic, or a mixture of potential and kinetic, but its total energy at any instant of time remains the same.



- When the pendulum bob is at position B, it has only potential energy (but no kinetic energy).
- As the bob starts moving down from position B to position A, its potential energy goes on decreasing but its kinetic energy goes on increasing.
- When the bob reaches the centre position A, it has only kinetic energy (but no potential energy).
- As the bob goes from position A towards position C, its kinetic energy goes on decreasing but its potential energy goes on increasing.
- On reaching the extreme position C, the bob stops for a very small instant of time. So at position C, the bob has only potential energy (but no kinetic energy).



Curve between Energy & Position for BOB Pendulum

ILLUSTRATION

14. In lifting a mass of 25 kg to a certain height 1250 J energy is utilized. Calculate to what height it has been lifted ? (Take $g = 10\ m/s^2$)

Sol. In lifting a mass through a height h the work done is given by

$U = mgh$

Here, $U = 1250\ J$; $g = 10\ m/s^2$; $m = 25\ kg$

$\therefore \quad 1250 = 25 \times 10 \times h$

or $\quad h = 5\ m$

POWER

The amount of work done for unit time is called power.
The rate of doing work is called power the amount of power a body depends upon,
(i) The magnitude of the work and
(ii) The time taken by the body
So, by knowing the work and time , we can measure power.
The amount of power is the ratio of work and time

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

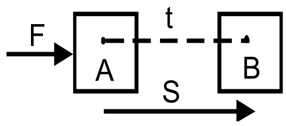
- **Unit :**

S.I. unit of power is watt (**W**).
One watt is the power of a man or a machine capable of doing work at the rate of one joule per second

$1\text{ watt} = \frac{1\text{ joule}}{1\text{ seconds}}$ or $W = J\,s^{-1}$

1 kilowatt = 1 kW = 1000 W
Since watt is a smaller unit, higher units used are
1 Kilowatt (**kW**) = 10^3 watt
1 Megawatt (**MW**) = 10^6 watt
1 Gigawatt (**GW**) = 10^9 W
1 Horse power = 1 H.P. = 746 W (Commercial unit)

(i) **Power to move a body :** Consider a force 'F' acting on a body that is displaced in the direction of force by a distance 'S'.

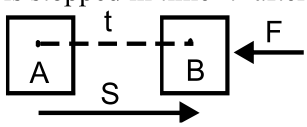


Then the power is given by the following expression :

$$\text{Power(P)} = \frac{\text{Work Done}}{\text{Time}} = \frac{\text{Force . DIsplacement}}{\text{Time}}$$

$$P = \frac{F\,S}{t}$$

(ii) Power to stop a moving body : Consider a force 'F' acting on a body against the direction of the motion of the body. The body is stopped in time 't' after travelling through a distance 'S'.



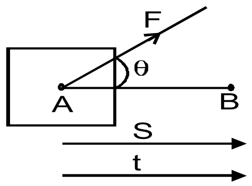
Then the power is given by the following expression :

$$\text{Power (P)} = \frac{\text{Work Done}}{\text{Time}}$$

$$P = \frac{F\,S\cos\theta}{t} \Rightarrow P = \frac{F\,S\cos 180}{t} \Rightarrow P = \frac{-F\,S}{t}$$

Note –ve sign indicates that power is used against the direction of motion of the body.

(iii) Power to pull a body : Consider a force 'F' acting in the direction that makes an angle 'q' with the direction of the motion.



If 'S' is the distance travelled by the body in time (t), then power used by the body can be calculated by the following expression.

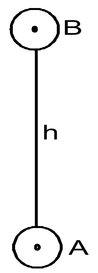
$$\text{Power (P)} = \frac{\text{WorkDone}}{\text{Time}}$$

$$P = \frac{F\,S\cos\theta}{t}$$

(iv) Power to lift a body against gravity : Consider a mass 'm' lifted from the surface to a height 'h' against gravity. If 't' is the time taken to lift the body, then the power can be calculated by the following expression.

Power (P) = $\frac{\text{Work Done}}{\text{Time}}$

(P) = $\frac{\text{weight of Body(Mg) . distance in which it lift}}{\text{time taken}}$



$P = \frac{mgh}{t}$

Power against gravity , P = $\frac{-mgh}{t}$

(v) Power to a body moving with velocity (v) : Consider a body moving with a speed of velocity of 'v' m/s. If 'F' is the force applied to stop the body, then power of the body is given by the following expression.

Power (P) = $\frac{\text{WorkDone}}{\text{Time}}$

$= \frac{\text{Force(F) . Displacement(S)}}{\text{Time(t)}}$

$P = \frac{F.S}{T}$ $\quad\quad \therefore v = \frac{.S}{T}$

Power = Fv

ILLUSTRATION

15. A machine raises a load of 750 N through a height of 15 m in 5s. Calculate :

(i) the work done by the machine.

(ii) the power at which the machine works.

Sol. (i) Work done is given by W = F.s

Here F = 750 N; s = 15 m

$\therefore$ W = 750 × 15 = 11250 J

= 11.250 kJ

(ii) Now, power of the machine is given by

$P = \frac{W}{t}$

Here, W = 11250 J; t = 5 s

$\therefore$Power P = $\frac{11250J}{5s}$ = 2250 W = 2.250 kW

ILLUSTRATION

16. A man carries a load of 50 kg through a height of 40 m in 25s. If the power of the man is 1568W, then find his mass? (Take g =$10ms^{-2}$)

Sol. Given, Mass of the load (mt) = 50 kg

Height (h) = 40 m

Time (t) = 25 s

Power (P) = 1568 W

Acceleration due to gravity (g) = 10 m/s^2

Mass of the man = m_m = ?

Selection of formula :

Power(P) = Work done (W) / Time(t)

How to get W ?

We know, work done in lifting a body against gravity,

W = mgh

$P = \frac{mgh}{t}$ Here, $m = m_L + m_m$

$P = \frac{(m_l + m_m)gh}{t}$ ... (i)

Substituting the values in (1), we get

$1568 = \frac{(50 + m_m)10 \times 40}{25}$

$m_m + 50 = 1568 \times \frac{25}{400} \rightarrow m_m + 50 = 98$

$m_m = 98 - 50 = 48$ kg

Therefore, the mass of man is 48 kg.

17. How many 2.5kg bricks can a man carry up a stair case 3.6m high in one hour, if he works at tlie average rate of 9.8 watt?

Sol. Given,

Mass, m = 2.5 kg, Height, h = 3.6 m

Time, t =1 hr = 3600 s

Power, P = 9.8 W

Acceleration due to gravity, g = 9.8 ms^{-2}

No. of bricks, (n) =?

We know,

The power to carry I brick, $P_1 = \frac{W}{t}$

The power to carry 'n' bricks,

$P_n = \frac{n \times W}{t} = \frac{nmgh}{t}$

$(\therefore\ W = mgh)$

$n = \frac{p \times t}{mgh} = \frac{9.8 \times 3600}{2.5 \times 9.8 \times 3.6} = 400$

Therefore, a man can carry 400 bricks.

18. The heart does 2.5 J of work in each heartbeat. How many times per minute does it beat, if its power is 4 watt?

Sol. Given,

Work done by heart for 1 beat, W = 2.5J

Time, t = 1 min = 60s

Power, P = 4W

Number of heart beats in 1 min., n = ?

$\text{Power, } P = \frac{\text{Work done}}{t}$

Work done for 1 beat = W

Work done for n beat = n × w

$P_n = \frac{n \times W}{t}$ = no. of heat beats,

$n = \frac{P \times t}{W} = \frac{4 \times 60}{2.5}$ n = 96 times

Therefore, number of heart beats in 1 minute is 96.

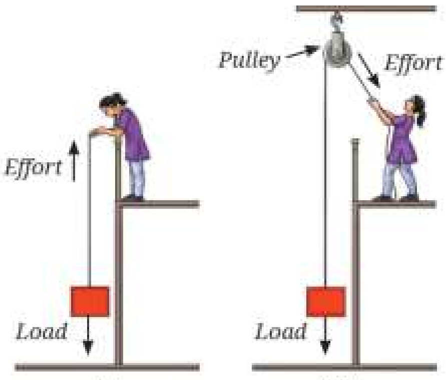
SIMPLE MACHINES

In everyday life, we often need to do work against gravity or other forces, such as lifting or moving heavy objects. Although the total work required for a task cannot be reduced, it can be made easier by changing the magnitude or direction of the applied force. The devices that help us do this are called simple machines.

In this section, we will study three simple machines-a pulley, an inclined plane, and a lever - to understand how these make the task feel easier. The force we apply to a machine is called the effort, and the force that needs to be overcome is called the load. To describe how a machine changes the magnitude of the applied force, we define mechanical advantage as the ratio of the load to the effort. It can be written as

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

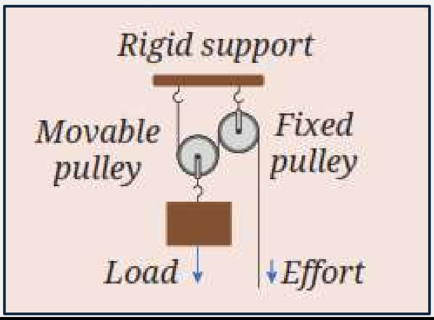
◈ PULLEY



(a) (b)

You may have seen a flag or a load being raised where a rope is pulled in the downward direction, while the flag or the load moves in the upward direction. This is done with the help of a pulley fixed at the top. A pulley is a wheel with a groove that guides a rope (Fig). A fixed pulley does not reduce the magnitude of the force required; it only changes its direction. It is easier for us to pull downward than to lift a load by applying an upward force directly (Figs. a and b). Thus, the pulley provides convenience by changing the direction of the effort. Since the effort and the load are equal in magnitude, the mechanical advantage of a fixed pulley is 1.

NOTE:
Movable pulleys or a system of pulleys can have a mechanical advantage greater than 1 and can lift much heavier objects with much smaller effort. In a movable pulley system, the load is attached to the movable pulley. One end of the rope is fixed to a point, while the other end is free to apply effort. Pulleys are widely used in real life, such as in elevators and cranes, given the convenience they provide us.



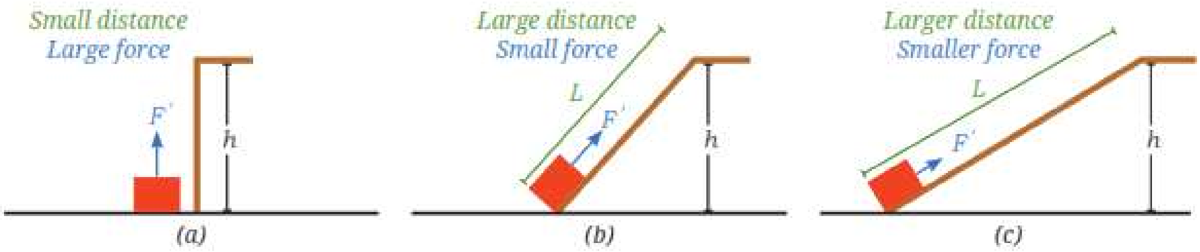
◈ INCLINED PLANE

Suppose you want to lift a heavy box onto a platform. Lifting it vertically (Fig. a) requires an upward force F equal to its weight. But what if the box is too heavy for you to lift directly?


(a) (b)

You can place the box on a smooth inclined plank as shown in (Fig. b), and push the box up the plank to the platform. Does this method require a smaller force?

The force required to pull up the cart upto the height of the pile of books or stool decreases as the plank becomes less steep. However, you have to apply the force over a larger distance to bring it to the same height.



An inclined plane is a simple machine that helps move a heavy load to a higher (or lower) level. Let us now find its mechanical advantage.

Let the mass of the object be m. Then, the load is the weight mg of the object to be lifted. Let $F^{'}$ be the force required to raise the object to a height h (Fig. a). Then, $F^{'}$ is the effort. Let the length of the inclined plane be L (Fig. b).

If you move the object at constant speed up the inclined plane, then using $W = F \times s$,

total work done by you on the object t $= F^{'} \times L$

And using $U = mgh$,

potential energy gained by the object $= mgh$

Thus, from the work-energy theorem, we obtain (ignoring friction)

work done on an object = change in its energy

$F^{'} \times L = mgh$

or

$\frac{mg}{F^{'}} = \frac{L}{h}$

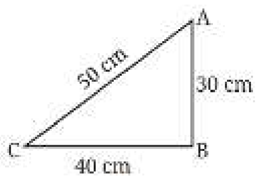
Now, mg is the load and $F^{'}$ is the effort, So

mechanical advantage $= \frac{\text{load}}{\text{effort}} = \frac{mg}{F^{'}} = \frac{L}{h}$

Since L is larger than h, the force $F^{'}$ is less than mg, and the mechanical advantage of the inclined plane is greater than 1. By further increasing L, for example, by making the inclined plane longer with a shallower angle (Fig. c), we can further reduce the effort $F^{'}$ required to move the object.

ILLUSTRATION

19. A person uses an inclined ramp to raise an object over a step 30 cm high. The ramp has a width of 40 cm. What is the mechanical advantage of the ramp that helps the person achieve the task?

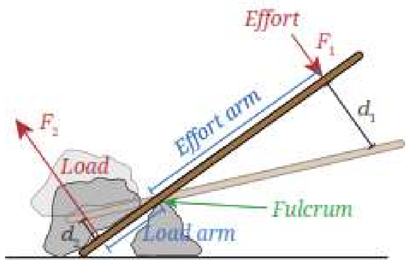


Sol. As shown in Fig, when the height AB is 30 cm, and the width BC is 40 cm, the length AC of the ramp is 50 cm (right-angled triangle property).

mechanical advantage $= \frac{L}{h} = \frac{50\,\text{cm}}{30\,\text{cm}} = 1.67$

◈ LEVER

A lever is a rigid bar, such as your scale, that can rotate about a fixed point, such as the point of contact of the scale with the pencil.



In everyday life, levers are often used to lift heavy objects. A lever has three main parts (Fig):

(i) Fulcrum: a fixed point about which the lever rotates, (ii) Load: the force to be overcome, and (iii) Effort: the force applied. The distance of the load from the fulcrum is called the load arm, and the distance of the effort from the fulcrum is called the effort arm.

By applying a small force at one end of the lever, a larger force can be applied on the object on the other end of the lever. How is this possible?

The end of the lever on which the smaller force (F_1) is applied, moves a larger distance (d_1), while the other end, which applies a larger force (F_2) to lift a heavier object, moves a smaller distance (d_2). The work done on one end of the lever is transferred to the other end.

NOTE:
A lever reduces the force required to perform a task but not the total work done.

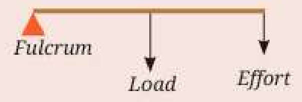
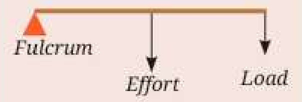
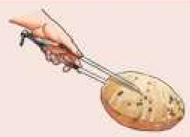
$$F_1 \times d_1 = F_2 \times d_2$$

Thus, by increasing the effort arm, the lever applies a larger force F_2 to the load compared to the applied effort F_1.

Ready to Go Beyond

Levers can be of three classes depending upon the relative positions of effort, fulcrum and load, as shown in Table 7.2.

Classes of Lever

Levers		
Class I	**Class II**	**Class III**
Fulcrum in between	Load in between	Effort in between
		
		
Tongs, scissors, crowbar, pliers, balance scale, seesaw	Lemon squeezer, wheel barrow, bottle opener	Tongs, tweezers, broom, hammer, oar

ILLUSTRATION

20. For a seesaw having four seats A, B, D, E and fulcrum at C, $AC = EC = 2\,m$ and $BC = DC = 1\,m$. On which seats should children of masses 15 kg and 30 kg sit to make the seesaw balanced?

Sol. Suppose the child of mass 15 kg sits on seat A and the other child sits at a distance L from the fulcrum. we obtain

$15\,kg \times 2\,m = 30\,kg \times L$

$L = 1\,m$

Thus, the other child should sit at seat D.