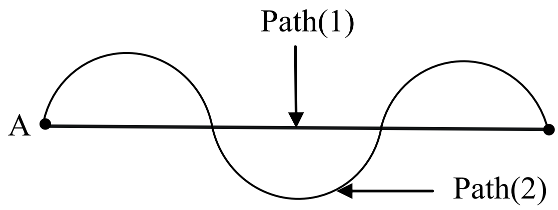
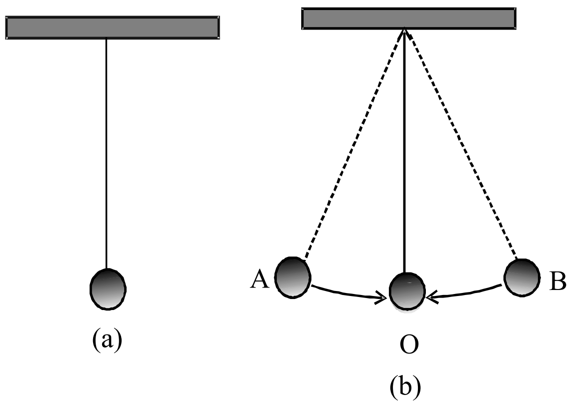
Describing Motion Around Us

Chapter Outline

- ✧ **Rest and motion**
- ✧ **Types of motion**
- ✧ **Distance and displacement**
- ✧ **Speed, velocity and acceleration**
- ✧ **Equation's of motion**
- ✧ **Distance, displacement, time-graph**
- ✧ **Circular motion**





PHYSICS

It is the branch of science in which we observe, measure and describe nature and natural phenomena.

Mechanics :

It is the branch of physics which deals with the study of object in the condition of rest or motion. It is divided into two parts

(i) Statics (ii) Kinematics and Dynamics

(i) Statics :

It deals with the study of objects at rest or in equilibrium, even when they are under the action of several forces.

(ii) Kinematics and Dynamics :

Kinematics: It deals with the study of motion of objects without considering the cause of motion .

eg. equations of motion

Dynamics : It deals with the study of motion of objects with considering the cause of motion .

Eg. Newton's laws of motion

REST & MOTION

Rest : An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time.

Eg. : The chair, black board, table in the class room are at rest w.r.t. the students.

Motion : A body is said to be in motion if its position changes continuously w.r.t. the surroundings (or with respect to an observer) with the passage of time.

Eg. : A car moving on the road will be in motion w.r.t. to the person stading on the road

Rest and motion are relative terms, there is nothing like absolute motion or rest.

Eg.:A train is moving on the track, the passengers are seated, will be stationary with respect to each other but in moving condition with respect to station.

Therefore, all the motions are relative. There is nothing like absolute motion.

Can you think why ?

- Rest and Motion are relative terms?

To study the motion of an object, following points are essential :

Concept of a Point Object : In mechanics while studying the motion of an object, sometimes its dimensions are not important and the object may be treated as a point object without much error. When the size of the object is much less in comparison to the distance covered by the object then the object is considered as a point object.

Eg. : Earth can be considered as a point object for studying its motion around the sun. Because length of the path covered by the earth in one revolution is very large in comparision to the size of earth, so that earth can be considered as a point object.

Frame of Reference : A fixed point or a fixed object with respect to which the given body changes its position is known as reference point or orgin

To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axis and name them x-y-z axis. The co-ordinates (x, y, z) of the particle then specify the position of object w.r.t. that frame. If any one or more co-ordinates change with time, then we say that the object is moving w.r.t. this frame.

Boost your knowledge

- We need Frame of reference to define dimensions of motion.

MOTION IN ONE, TWO AND THREE DIMENSIONS

As position of the object may change with time due to change in one or two or all the thee co-ordinates, so we have classified motion as follows:

◈ **Motion in 1-D :**

If only one of the three co-ordinates specifying the position of object changes w.r.t. time then its motion is known as 1D motion . In such a case the object moves along a straight line. This motion is also known as rectilinear or linear motion.

Eg.(i) Motion of train along straight railway track.

(ii) An object falling freely under gravity.

◈ **Motion in 2-D :**

If two of the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called two dimensional. In such a motion the object moves in a plane.

Eg.(i) Motion of queen on carom board.
(ii) An insect crawling on the floor of the room.
(iii) Motion of object in horizontal and vertical circles etc.
(iv) Motion of planets around the sun.
(v) A car moving along a zig-zag path on a level road.

◈ **Motion in 3-D :**
If all the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called 3-D. In such a motion the object moves in a space.

Eg. (i) A bird flying in the sky (also kite).
(ii) Random motion of gas molecules.
(iii) Motion of an aeroplane in space.

TYPES OF MOTION

◈ **Linear motion (or translatory motion) :**
The straight line motion is called linear motion.
Eg. : The motion of a car moving on straight road, a running person, a stone being dropped, motion of a train on a straight track

◈ **Rotational motion :**
Motion of a body around a fix axis is called rotational motion.
Eg. The motion of an electric fan, motion of earth about its own axis.

◈ **Oscillatory motion :**
The to and fro periodic motion of a body around a fix point is called oscillatory motion.
Eg. : The motion of a simple pendulum, a body suspended from a spring.

SCALAR AND VECTOR QUANTITY

Physical quantities (i.e. quantities of physics) can be divided into two types :

◈ **Scalar quantity :**
Any physical quantity, which can be completely specified by its magnitude, is know as scalar quantity or a scalar.
Eg. Charge, distance, area, speed, time, temperature, density, volume, work, power, energy, pressure, potential etc.

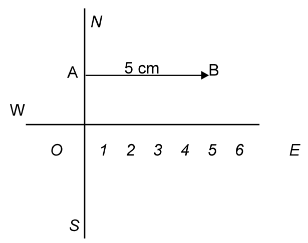
◈ **Vector quantity :**
The quantity which can be determined by its magnitude and the direction and also can be added or subracted by vector algebra, is called a vector quantity.
Eg. Displacement, velocity, acceleration, force, momentum, weight and electric field etc.

- **Representation of a vector :**
A vector is represented by a directed line segment drawn in the given direction on a certain scale.The length of line shows its magnitude and arrow shows direction.

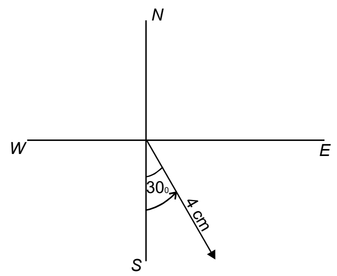
Tail ⟶ *head*
(symbolic representation)

ILLUSTRATIONS

1. Represent a displacement of 50 m towards east.
Sol. Take the scale 10 m = 1 cm



2. Represent a velocity of 20 km/h towards 30° east of south.
Sol. Take scale 5 km/h = 1 cm.



Difference between scalar and vector :

Scalar	Vector
1. They have magnitude only.	1. They have magnitude as well as direction.
2. They are added or subtracted arithmetically like 3 kg + 5 kg = 8 kg	2. They are added or subtracted by the process of vector addition.

DISTANCE AND DISPLACEMENT

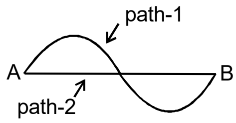
Distance :

It is the actual length of path covered by a moving particle . It is a scalar quanitiy. Its S.I. unit is metre (m).

Displacement :

It is the shortest distance between the initial and final position of the particle. It is a vector quantity. Its S.I. unit is metre (m).

Eg.: Consider a body moving from a point A to a point B along the path shown in figure. Then total length path covered is called distance (path-1). While the length of straight line AB in the direction from A to B is called displacement (path-2).



NOTE : If a body travels in such a way that it comes back to its starting position, then the displacement is zero. However, distance travelled is never zero in case of moving body.

Some important points :

(i) When an object moves towards right from origin, its displacement consider as positive.

(ii) When an object moves towards left from origin its displacement consider as negative.

(iii) When an object remains stationary or it moves first towards right and then an equal distance towards left, its displacement is zero.

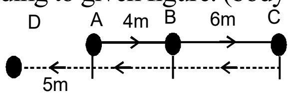
(iv) Shifting origin causes no change in displacement.

(v) If body moves along the circumference of the circle of radius r then distance travelled by it is given by $2\pi r$ and displacement is given by zero, for one complete revolution.

Can you think why ?
- Distance cannot decrease?

ILLUSTRATIONS

3. A body satrts from A and moves according to given figure. (body retraces the path after C then reaches to D)



The distance and displaemcent are as follows for different path.

Sol.

Path	Distance	Displacement
AB	4 m	4 m
ABC	10 m	10 m
ABCB	16 m	4 m
ABCA	20 m	0 m
ABCAD	25 m	–5 m

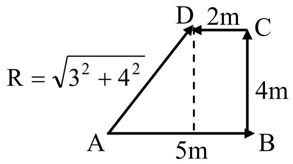
Difference between distance and displacement :

Distance	Displacement
1. Distance is the length of the path actually traveled by a body in any direction.	1. Displacement is the shortest distance between the initial and the final positions of a body in the direction of the point of the final position.
2. Distance between two given points depends upon the path chosen.	2. Displacement between two points is measured by the straight path between the points.
3. Distance is always positive.	3. Displacement may be positive as well as negative and even zero.
4. Distance is a scalar quantity.	4. Displacement is a vector quantity.
5. Distance will never decrease.	5. Displacement may decrease.

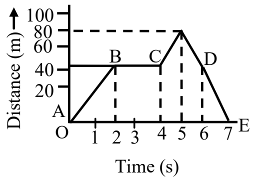
ILLUSTRATIONS

4. A person travels a distance of 5 m towards east, then 4 m towards north and then 2 m towards west.
(i) Calculate the total distance travelled. (ii) Calculate the resultant displacement.

Sol. (i) Total distance travelled by the person = 5 m + 4 m + 2 m = 11 m
(ii) The resultant displacement is calculated by joining the initial position A to the final position D. Hence, the displacement of the person = 5m towards AD.



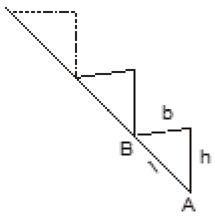
5. A body is moving in a straight line. Its distances from origin are shown with time in Fig. A, B, C, D and E represent different parts of its motion. Find the following :
(i) Displacement of the body in first 2 seconds. (ii) Total distance travelled in 7 seconds.
(iii) Displacement in 7 seconds



Displacement-time graph

Sol. (i) Displacement of the body in first 2s = 40m
(ii) From t = 0 to t = 7 s, the body has moved a distance of 80 m from origin and it has again come back to origin. Therefore, the total distance covered = 80 × 2 = 160 m
(iii) Since the body has come back to its initial position, the displacement is zero.

6. There are n steps each of dimension l,b & h if a man climbs n steps what is his displacement and distance.

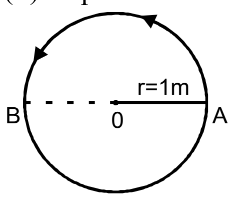


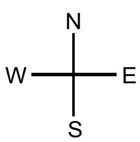
Sol.

By pythagorian theorem AB = $\sqrt{b^2 + h^2}$
Similarly for each step
Displacement = $\sqrt{b^2 + h^2}$
So that total displacement = n × $\sqrt{b^2 + h^2}$
Distance = n (h+b)

7. A person moves in a circular path centered at O. He starts from A and reaches diametrically opposite point B. Then find :
(i) distance between A and B
(ii) displacement between A and B

Sol.





(i) Distance = Length of actual circular path from A to B = Half the circumference

i.e. Distance = $\frac{2\pi r}{2} = \pi r$

r = 1m

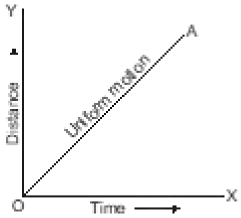
∴ Distance = π m

(ii) Displacement = 2r along west = 2m along west

UNIFORM AND NON-UNIFORM MOTION

Uniform Motion :

A body has a uniform motion if it travels equal distances in equal intervals of time, no matter how small these time intervals may be. For example, a car running at a constant speed say, 10 metre per second, will cover equal distances of 10 metre every second, so its motion will be uniform. Please note that the distance-time graph for uniform motion is a straight line (as shown in the figure).



Non-Uniform Motion :

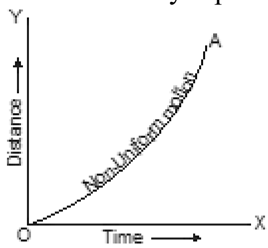
A body has a non-uniform motion if it travels unequal distances in equal intervals of time. For example, if we drop a ball from the roof of a building, we will find that it covers unequal distances in equal intervals of time. It covers :

4.9 metre in the 1st second,

14.7 metre in the 2nd second,

24.5 metre in the 3rd second, and so on.

Thus, a freely falling ball covers smaller distance in the initial '1 second' interval and larger distance in the later '1 second' interval. From this discussion we conclude that the motion of a freely falling body is an example of non-uniform motion. The motion of a train starting from the railway station is also an example of non-uniform motion. This is because when the train starts from a station, it moves a very small distance in the 'first' second. The train moves a little more distance in the '2nd' second and so on. And when the train approaches the next station, the distance travelled by it per second decreases.



Please note that the distance-time graph for a body having non-uniform motion is a curved line (as shown in the figure). Thus, in order to find out whether a body has uniform motion or non-uniform motion, we should draw the distance-time graph for it. If the distance time graph is a straight line, the motion will be uniform and if the distance-time graph is a curved line, the motion will be non-uniform. It should be noted that non-uniform motion is also called accelerated motion.

SPEED

The distance travelled by a body in unit time is called its speed. Therefore,

$$\text{speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad s = \frac{d}{t}$$

S.I. unit of speed or average speed is m/sec. It is a scalar quantity.

TYPES OF SPEED

Average Speed :

For an object moving with variable speed, it is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

- Let initial speed of an object is v_1, final speed is v_2 and acceleration is constant, then

  $$\text{average speed} = \frac{v_1 + v_2}{2}$$

- A body covers a distance s_1 in time t_1, s_2 in time t_2 and s_3 in time t_3.

  Then, average speed, $V_{av} = \dfrac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

- A body travels with speed v_1 for a time t_1 v_2 for time t_2 and v_3 for the time t_3.

Then, average speed, $V_{av} = \dfrac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3}$

$s_1 = v_1t_1$, $s_2 = v_2t_2$ and $s_3 = v_3t_3$

if $t_1 = t_2 = t_3 = t$

$$V_{av} = \frac{t(v_1 + v_2 + v_3)}{3t}$$

$$V_{av} = \frac{(v_1 + v_2 + v_3)}{3}$$

- A body covers a distance s_1 with speed v_1 , s_2 with speed v_2 and s_3 with speed v_3.

  Then, average speed, $V_{avg} = \dfrac{(s_1 + s_2 + s_3)}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}}$

$t_1 = \dfrac{s_1}{v_1}$, $t_2 = \dfrac{s_2}{v_2}$, $t_3 = \dfrac{s_3}{v_3}$

- A boy goes form home to school with speed v_1 and come back to home with speed v_2.

Here distance covered by the boy is same

time taken by the boy, from home to school,

$$t_1 = \frac{s}{v_1}$$

and time taken by the boy, from school to home,

$$t_2 = \frac{s}{v_2}$$

Then, average speed, $V_{av} = \dfrac{s + s}{t_1 + t_2} = \dfrac{2s}{\frac{s}{v_1} + \frac{s}{v_2}}$

$$V_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If an object covered $1/3^{rd}$ distance with speed u, next $1/3^{rd}$ with speed V and last $1/3^{rd}$ distance with speed w then,

$$V_{avg} = \frac{3uvw}{uv + vw + wu}$$

Uniform Speed (or Constant Speed) :

When an object covers equal distance in equal intervals of time, it is said to move with uniform speed. **Eg.** A car moves 10 m in every one second so its motion is uniform.

Variable Speed (Non-Uniform Speed) :

If a body covers unequal distance in equal intervals of time, its motion is said to be non-uniform.

Eg. Falling of an apple from a tree, a cyclist moving on a rough road, an athlete running a race, vehicle starting from rest, the motion of a freely falling body etc.

Instantaneous speed :

The speed of object at a particular instant is called instantaneous speed. The limiting value of average speed when the time interval approaches zero, Thus

Instantaneous speed= $\lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

Can you think why ?

- Is average speed define for a time interval or at an instant?

VELOCITY

It is define as the rate of change of displacement.

Therefore, velocity = $\frac{\text{displacement}}{\text{time}}$ or it is the distance travelled in unit time in a given direction.

$$\text{velocity} = \frac{\text{distance travelled in a given direction}}{\text{time taken}}$$

S.I. unit of velocity is m/s. It is a vector quantity.

(Magnitude of the velocity is known as speed)

Note :

- To convert m/s into km/h we multiply by 18/5
- To convert km/h into m/s we multiply by 5/18

TYPES OF VELOCITY

Uniform Velocity (Constant Velocity) :

If a body covers equal distance in equal intervals of time in a given direction then it is said to be moving with constant velocity.

Non-Uniform Velocity :

When a body does not cover equal distances in equal intervals of time, in a given direction (in this case speed is not constant), then it is known as non uniform velocity.

In uniform circular motion speed is constant but velocity is not constant.

Average Velocity :

It is defined as the ratio of total displacement to the total time taken for this displacement. It is denoted by $\vec{V}_{av}$ or $\vec{v}$. It is a vector quantity.

$$\therefore \text{ Average velocity} = \frac{\text{Total displacement}}{\text{total time}}$$

i.e. $\vec{v} = \frac{\vec{s}}{t}$

It is a vector quantity, and its direction is in the direction of displacement.

Note :

- If for a straight line motion displacement is positive then $\vec{v}$ is positive.
- if displacement is negative then $\vec{v}$ is also negative.

Instantaneous Velocity :

The velocity of an object at a particular instant of time is called instantaneous velocity. It is equal to the limiting value of average velocity of the object when the time interval approaches zero , thus

$$\text{Instantaneous velocity } \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

It is a vector quantity.

More about speed and velocity :

(i) The instantaneous velocity in magnitude is equal to instantaneous speed.

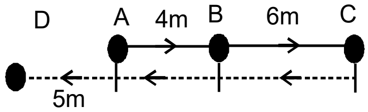
(ii) A particle may have constant speed but variable velocity. In uniform circular motion speed remains constant while velocity changes because of change in direction in motion.

(iii) If particle is moving along a straight line without changing the direction, its average velocity will be equal to its average speed. Otherwise average velocity will be less than average speed

$$\left|\frac{\text{Average velocity}}{\text{average speed}}\right| \le 1$$

ILLUSTRATIONS

8. A body satrts from A and moves according to given figure. Time four each interval is :
t_{AB} = 2s, t_{BC} = 3s, t_{CB} = 2s, t_{BA} = 3s, t_{AD} = 4s



The distance, displacement, speed and velocity are as follows for different path.

Sol.

Path	Dist-ance	Displace-ment	Speed	Velocity
AB	4m	4m	4/2 m/s	4/2 m/s
ABC	10m	10m	10/5 m/s	10/5 m/s
ABCB	16m	4m	16/7 m/s	4/7 m/s
ABCA	20m	0m	20/10 m/s	0/10 m/s
ABCAD	25m	–5m	25/14 m/s	–5/14 m/s

Difference between Speed and Velocity :

Speed	Velocity
1. It is rate of change of position of an object.	1. It is rate of change of position of an object in a specific direction.
2. Speed= $\frac{\text{distance travelled}}{\text{time}}$	2. Velocity = $\frac{\text{displacement}}{\text{time}}$
3. It is a scalar quantity.	3. It is a vector quantity
4. Speed will always be positive	4. It will be positive or negative depending on the direction of motion
5. For moving body, it will never be zero	5. It may be zero
6. It will never decrease with time	6. It may increase or decrease with time

FEATURES OF UNIFORM MOTION

(i) The velocity in uniform motion does not depend on the choice of origin.

(ii) The velocity in uniform motion does not depend on the choice of the time interval.

(iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

(iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(v) For an object in uniform motion no force is required to maintain its motion.

(vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant, at each point of the path.

ILLUSTRATIONS

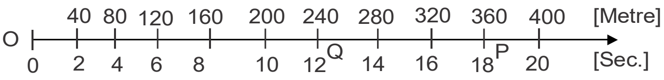
9. When the average speed of an object is equal to the magnitude of its average velocity ? Give reason also.

Sol. As average speed = $\frac{\text{total path length}}{\text{time interval}}$

also, average velocity =. $\frac{\text{Displacement}}{\text{time interval}}$

When an object moves along a straight line or in the same direction its total path length is equal to the magnitude of its displacement. Hence average speed is equal to the magnitude of its average velocity.

10. A car is moving along x-axis. As shown in figure it moves from O to P in 18 s and returns from P to Q in 6 second. What is the average velocity and average speed of the car in going from (i) O to P and (ii) from O to P and back to Q.



Sol. **(i)** Average velocity $= \dfrac{\text{Displacement}}{\text{time interval}} = \dfrac{360}{18} = 20\text{ms}^{-1}$

Average speed $= \dfrac{\text{path length}}{\text{time interval}} = \dfrac{360}{18} = 20 \text{ ms}^{-1}$

(ii) From O to P and back to Q

Average velocity $= \dfrac{OQ}{18+6} = \dfrac{240\text{ m}}{24} = 10 \text{ ms}^{-1}$

Average speed $= \dfrac{\text{path length}}{\text{time interval}} = \dfrac{OP + PQ}{18+6} = \dfrac{360+120}{24} = 20 \text{ ms}^{-1}$

11. A car covers the 1^{st} half of the distance between two places at a speed of 40 km h^{-1} and the 2^{nd} half with 60 km h^{-1}. What is the average speed of the car?

Sol. Suppose the total distance covered is 2S.

Then time taken to cover the distance 'S' with speed 40 km/h,

$t_1 = \dfrac{S}{40} h$

Time taken to cover the next distance 'S' with speed 60 km/hrs,

$t_2 = \dfrac{S}{60} h$

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{S+S}{\left(\frac{S}{40} + \frac{S}{60}\right)}$$

$$V_{av} = \frac{2S}{\left(\frac{3S+2S}{120}\right)} = \frac{2S}{5S} \times 120$$

$\Rightarrow V_{av} = 48 \text{ km/h}$

ACCELERATION

Mostly the velocity of a moving object changes either in magnitude or in direction or in both when the object moves. The body is then said to have acceleration. So it is the rate of change of velocity i.e. change in velocity in unit time is said to be acceleration. It is a vector quantity and

Its S.I unit is m/s^2 and C.G.S. unit is cm/s^2

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{v - u}{t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

Uniform Acceleration (Uniformly Accelerated Motion) :

If a body travels in a straight line and its velocity increases in equal amounts in equal intervals of time. Its motion is known as uniformly accelerated motion.

Eg.1: Motion of a freely falling body is an example of uniformly accelerated motion (or motion of a body under the gravitational pull of the earth).

Eg.2: Motion of a bicycle going down the slope of a road when the rider is not pedaling and wind resistance is negligible.

Non-Uniform Acceleration :

If during motion of a body its velocity increases by unequal amounts in equal intervals of time, then its motion is known as non uniform accelerated motion.

Eg.1: Car moving in a crowded street.

Eg.2: Motion of a train leaving or entering the platform.

TYPES OF ACCELERATION

- **Positive acceleration:**
  If the velocity of an object increases with respect to time in the same direction, the object has a positive acceleration.
- **Negative acceleration (retardation):**
  If the velocity of a body decreases with respect to time in the same direction, the body has a negative acceleration or it is said to be retarding.
  Eg. A train slows down, then its acceleration will be negative.

EQUATIONS OF UNIFORMALY ACCELERATED MOTION

There are three equation of uniformly accelerated motion. They show the relation between initial velocity u, final velocity v, acceleration a, time t and displacement s

- **1st Equation of Motion :**
  Consider a body moving with initial velocity u and its velocity changes from u to v in time t. Then

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}} \quad \Rightarrow a = \frac{v - u}{t}$$

So $at = v - u$ and $v = u + at$

1st equation of motion : $v = u + at$

- **2nd Equation of Motion :**
  We know
  Distance covered = (Average velocity) × (Time)

or $$s = \frac{u+v}{2}t$$

But $v = u + at$

Substituting the value of v in the equation above, we have

$$s = \frac{u + (u + at)}{2}t$$

or $$s = \left(\frac{2u + at}{2}\right)t = \left(u + \frac{at}{2}\right)t$$

or $$s = ut + \frac{1}{2}at^2$$

2nd equation of motion : $s = ut + \frac{1}{2}at^2$

- **3rd Equation of Motion :**
  We know that

$$v = u + at$$

or $$t = \frac{v - u}{a}$$

Distance travelled = (Average velocity) × (time)

$$s = \frac{v+u}{2}t = \frac{v+u}{2}\,\frac{v-u}{a}$$

or $s = \dfrac{v^2 - u^2}{2a}$ or $v^2 - u^2 = 2as$

3rd equation of motion : $v^2 - u^2 = 2as$

- **Distance covered in nth second :**
  Distance travelled in nth second = Distance travelled in n sec – Distance travelled in (n –1) sec.
  So, $S_{nth} = S_n - S_{(n-1)}$

$$= \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

[Putting t = n and t = (n – 1) respectively in equation (ii)]

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 - 2n + 1)$$

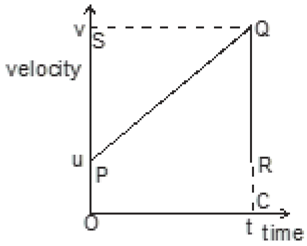
We have, $$\mathbf{S_{n^{th}} = u + \frac{a}{2}(2n - 1)}$$

GRAPHICAL DERIVATION OF EQUATIONS OF MOTION

◈ **First Equation :**

v = u + at

It can be derived from v– t graph , as shown in figure



From line PQ, the slope of the line = acceleration

$$a = \frac{QR}{RP} = \frac{SP}{RP}$$

$\because$ SP = v – u and RP = t

So $a = \frac{v-u}{t}$ or $\mathbf{v = u + at}$

◈ **Second Equation :**

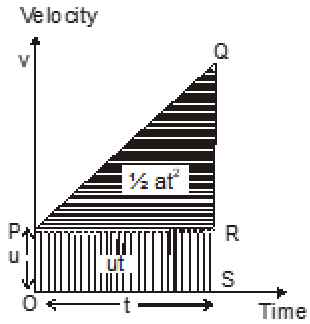
$\mathbf{S = ut + \frac{1}{2} at^2}$

It can also be derived from v - t graph as shown in figure.

From relation,

Distance covered = Area under v - t graph

s = Area of trapezium OPQS = Area of rectangle OPRS + Area of triangle PQR



$= OP \times PR + \frac{RQ \times PR}{2}$

Putting values,

$s = u \times t + \frac{1}{2}(v-u) \times t$ $\quad(\because RQ = v - u \ \& \ PR = OS = t)$

$= u \times t + \frac{1}{2} at \times t$ $\quad(\because v - u = at)$

or $\mathbf{s = ut + \frac{1}{2} at^2}$

◈ **Third Equation:**

$v^2 = u^2 + 2aS$

From above graph OP=u, SQ= v, OP + SQ= u + v

$a = \frac{QR}{PR}$ Or $PR = \frac{QR}{a} = \frac{v-u}{a}$

$S = \text{Area of trapezium OPQS} = \frac{OP + SQ}{\mathbf{2}} \times PR$

On putting the values,

$S = \frac{u+v}{\mathbf{2}} \times \frac{v-u}{a} = \frac{v^2 - u^2}{\mathbf{2}a}$ or $\mathbf{v^2 = u^2 + 2as}$

EQUATIONS OF MOTION FOR FREELY FALLING OBJECT

Since the freely falling bodies fall with uniformly accelerated motion, the three equations of motion derived earlier for bodies under uniform acceleration can be applied to the motion of freely falling bodies. For freely falling bodies, the acceleration due to gravity is **'g'**, so we replace the acceleration **'a'** of the equations by **'g'** and since the vertical distance of the freely falling bodies is known as height **'h'**, we replace the distance **'s'** in our equations by the height **'h'**. This gives us the following modified equations for the motion of freely falling bodies.

General equations of motion		**Equations of motion for freely falling bodies**
(i) $v = u + at$	changes to	$v = u + gt$
(ii) $s = ut + \frac{1}{2}at^2$	c hanges to	$h = ut + \frac{1}{2}gt^2$
(iii) $v^2 = u^2 + 2as$	changes to	$v^2 = u^2 + 2gh$

We shall use these modified equations to solve numerical problems. Before we do that, we should remember the following important points for the motion of freely falling bodies.

(i) When a body is dropped freely from a height, its initial velocity **'u'** becomes zero.

(ii) When a body is thrown vertically upwards, its final velocity **'v'** becomes zero.

(iii) The time taken by a body to rise to the highest point is equal to the time it takes to fall from the same height.

(iv) The distance travelled by a freely falling body is directly proportional to the square of time of fall.

Sign Conventions :

(i) g is taken as positive when it is acting in the same direction as that of motion and g is taken as negative when it is opposing the motion.

(ii) Distance measured upward from the point of projection is taken as positive, while distance measured downward from the point of projection is taken as negative.

(iii) Velocity measured away from the surface of earth (i.e. in upward direction) is taken as positive, while velocity measured towards the surface of the earth is taken as negative.

TO SOLVE NUMERICAL PROBLEMS

(i) If a body is dropped from a height then its initial velocity u = 0 but has acceleration (acting). If a body starts from rest its initial velocity u = 0 .

(ii) If a body comes to rest ,its final velocity v = 0 or, if a body reaches the highest point after being thrown upwards its final velocity v = 0 but has acceleration (acting).

(iii) If a body moves with uniform velocity, its acceleration is zero i.e. a = 0.

(iv) Motion of a body is called free fall if only force acting on it is gravity (i.e. earth's attraction).

MOTION UNDER GRAVITY (UNIFORMLY ACCELERATED MOTION)

The acceleration with which a body travels under gravity is called acceleration due to gravity 'g'. Its value is 9.8 m/s^2 (or ≈ 10 m/s^2). If you have to take g = 10 m/s^2 then it must be mentioned in the question otherwise take g = 9.8 m/s^2.

(i) If a body moves upwards (or thrown up) g is taken negative (i.e. motion is against gravitation of earth). So equation of motion becomes.

$$v = u - gt,\ s = ut - \frac{1}{2}gt^2,\ v^2 = u^2 - 2gh.$$

(ii) If a body travels downwards (towards earth) then g is taken + ve. So equations of motion becomes

$$v = u + gt\ ,\ s = ut + \frac{1}{2}gt^2\ ,\ v^2 = u^2 + 2gh.$$

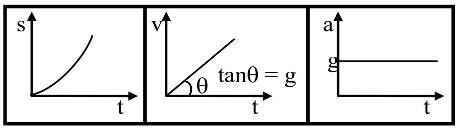
(iii) If a body is projected vertically upwards with certain velocity then it returns to the same point of projection with the same velocity in the opposite direction.

(iv) The time for upward motion is the same as for the downward motion.

BODY FALLING FREELY UNDER GRAVITY

Assuming u = 0 for a freely falling body :

t is given	h is given	v is given
$v = gt$ $h = \frac{1}{2}gt^2$	$t = \sqrt{\frac{2h}{g}}$ $v = \sqrt{2gh}$	$t = \frac{v}{g}$ $h = \frac{v^2}{2g}$



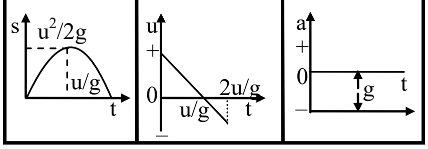
- Body is projected vertically up :

Taking initial position as origin and direction of motion (i.e. vertically up) as positive.

(a) At the highest point v = 0

(b) a = – g

t is given	h is given	u is given
$u = gt$ $h = \frac{1}{2}gt^2$	$t = \sqrt{2h/g}$ $u = \sqrt{2gh}$	$t = \frac{u}{g}$ $h = \frac{u^2}{2g}$



- It is clear that in case of motion under gravity

(a) Time taken to go up is equal to the time taken to fall down through the same distance.

(b) The speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

(c) The body returns to the starting point with the same speed with which it was thrown.

ILLUSTRATIONS

12. A stone drops from the edge of a roof. It passes a window 2 metre high in 0.1 second. How far is the roof above the top of the window ?

Sol. Let the distance between the top of the window and the roof be h. This problem can be solved in two stages.

(a) For the journey across the window i.e., from B to C

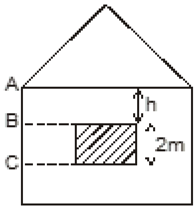
Let, Velocity at B = u m/s

Distance travelled, s = 2 m

Time taken, t = 0.1s

Acceleration, a = g = 9.8 m/s^2

Using the relationship,



$$s = ut + \frac{1}{2}gt^2$$

$$2 = u \times 0.1 + \frac{1}{2} \times 9.8 \times (0.1)^2$$

$$2 = 0.1\,u + 4.9 \times 10^{-2} \text{ or, } u = \frac{(2 - 0.049)}{0.1} = 19.51 \text{ m/s}$$

The velocity of the stone at the top of the window is 19.51 m/s.

(b) For journey from roof to the top of the window i.e., from A to B

The velocity at the top of the window is the velocity of the stone at the end of falling through 'h'. So, for this part of the journey,

Initial velocity, u = 0 m/s

Final velocity, v = 19.51 m/s

Acceleration due to gravity, g = 9.8 m/s^2

Then by using the equation, $v^2 - u^2 = 2\,gh$, one gets

$$(19.51)^2 - 0 = 2 \times 9.8 \times h \Rightarrow h = \frac{(19.51)^2}{19.6} \text{ m} = 19.42 \text{ m}$$

Thus, the roof is 19.42 m from the upper end (top) of the window.

13. A ball is thrown vertically upwards with a velocity 'u'. Calculate the velocity with which it falls to the earth again.

Sol. For a ball thrown vertically upwards,

Initial velocity = u

Final velocity = v = 0

For the vertically upward motion, the equation of motion is

$v = u - g\,t$ So, $0 = u - g\,t$

$$\text{or } t = \frac{u}{g} \qquad \text{....(i)}$$

For the return journey, when the body falls vertically downwards, the equation of motion is

$v = u + g\,t \Rightarrow$ Since, u = 0

Hence $v = 0 + g\,t$

$$\text{or} \quad t = \frac{v}{g} \qquad \text{.... (ii)}$$

From (i) and (ii),

Thus, the body falls back to the earth with the same velocity with which it was thrown vertically upwards.

14. A car is moving at a speed of 50 km/h after two seconds it is moving at 60 km/h. Calculate the acceleration of the car.

Sol. Here $u = 50 \text{ km/h} = 50 \times \frac{5}{18} \text{ m/s} = \frac{250}{18}$ m/s and $v = 60 \text{ km/h} = 60 \times \frac{5}{18} = \frac{300}{18}$ m/s

$$\text{Since } a = \frac{v - u}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{\frac{50}{18}}{2} = \frac{50}{36} = 1.39 \text{ m/s}^2$$

15. A car attains 54km/h in 20 s after it starts. Find the acceleration of the car.

Sol. u = 0 (as car starts from rest)

$$v = 54 \text{ km/h} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

$$\text{As, } a = \frac{v - u}{t} \therefore a = \frac{15 - 0}{20} = 0.75 \text{ m/s}^2$$

GRAPHICAL REPRESENTATION OF MOTION

❖ **Distance–Time Graph :**

A moving body changes its position continuously with time. The simplest way to describe the motion of a moving body is to draw its distance–time graph.

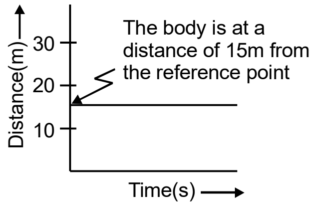
The distance–time graphs of a body under the following three conditions are described below:

(i) When the body is at rest.

(ii) When the body is moving with a uniform speed

(iii) When the body is moving with a non–uniform speed

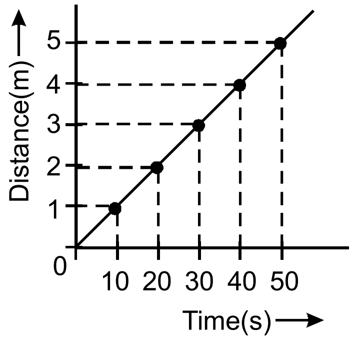
(i) Distance–time graph for a body at rest



The distance–time graph for a body at rest is a straight line parallel to the time axis

(ii) Distance–time graph for a body moving with a uniform speed :

When a body covers equal distances in equal intervals of time, it is said to have uniform speed.



Distance–time graph of a body moving with uniform (constant) speed

graph shows that the distance travelled by a body moving with uniform speed is directly proportional to time.

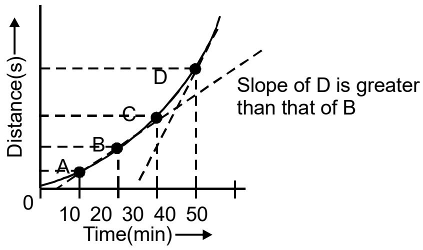
(iii) Distance–time graph for a body moving with a non–uniform speed

A body moving with a non–uniform speed covers unequal distances in equal intervals of time. Therefore, **the distance–time graph of a body moving with a non–uniform speed is a curve.**

The shape of the distance–time graph for a body moving with non–uniform speed depends upon the way speed of the body changes with time. Two typical cases are described below :

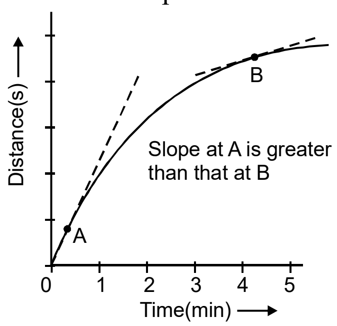
(A) When the speed increases with time :

When the speed of a body increases with time, the distance covered by it in one unit of time also increases with time. Therefore, the distance–time graph for a body moving with an increasing non–uniform speed is a curve whose slope increases with time (Figure).



(B) When the speed decreases with time :

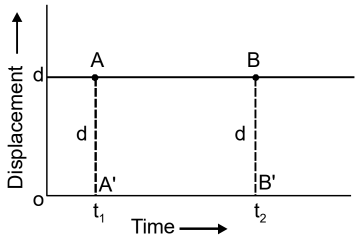
When the speed of a body decreases with time, the distance covered by it in one unit of time also decreases with time. Therefore, the distance–time graph for a body moving with a decreasing non–uniform speed is a curve whose slope decreases with time (Figure).



DISPLACEMENT-TIME GRAPH

◈ **Displacement – time graph of a body at rest:**

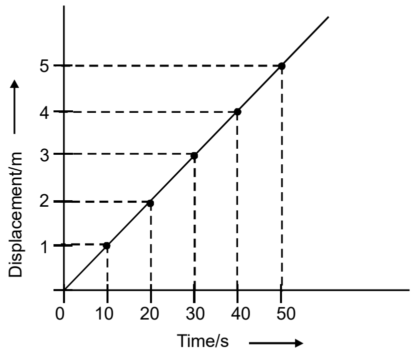
The position of a body at rest remains unchanged with time. Let us consider a body at a distance d from a reference point in a particular direction. Then from figure 4.1.



Graph shows that position of the body does not change w.r.t. time, so that body is said to be in rest.
Thus the velocity of a body at rest is zero.

◈ **Displacement – time graph of a body moving with uniform velocity :**

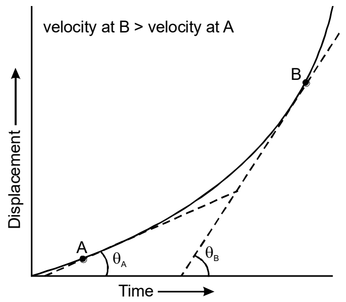
The displacement–time graph of a body moving with uniform (constant) velocity is a straight line inclined to the time–axis at certain angle.



The slope of the displacement–time graph for a body moving with uniform velocity is equal to the velocity of the body.

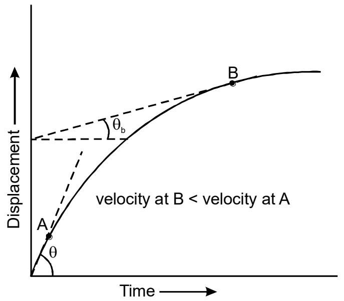
◈ **Displacement–time graph of a body moving with an increasing non–uniform velocity :**

The displacement–time graph of a body moving with an increasing non–uniform velocity is a curve (Figure). Here, the slope of the curve increases with time. So, the velocity of the body increases with time i.e., velocity at B > velocity at A



◈ **Displacement–time graph of a body moving with decreasing non–uniform velocity.**

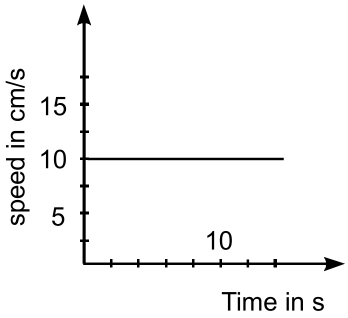
The displacement–time graph of a body moving with a decreasing non–uniform velocity is a curve (Figure). Here, the slope of the curve decreases with time. So, the velocity of the body decreases with time.
i.e., velocity at B < velocity at A



Can you think why ?
- If displacement $\propto t^2$ then what will be type of motion?

SPEED-TIME GRAPH

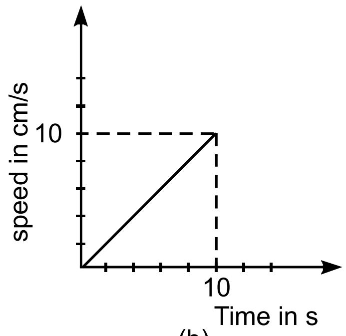
Some information about the motion of an object can also be obtained from its speed–time graph. Following figures give the speed–time graphs of four different objects in motion. Let us see what information can be obtained from these graphs.



(a)

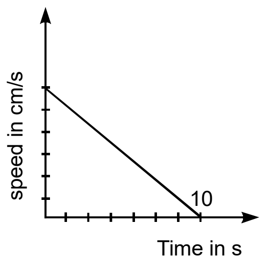
Figure (a) shows that for any time, the speed has the same value (10 cm/s). Thus it represents an object moving with a constant speed.

Whenever an object moves with a constant speed, its speed–time graph is a straight line, parallel to the time–axis.



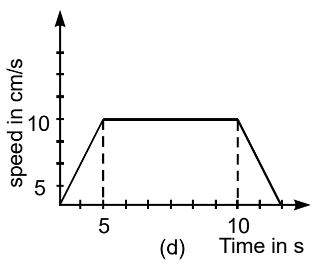
(b)

Figure (b) shows that the speed continuously increases with time. At time t = 0, the speed is 0. At t = 10 s, it becomes 10 cm/s. The straight–line nature of the graph indicates that the speed increases at a constant rate.



(c)

Figure (c) shows that the speed is 10 cm/s at t = 0 and gradually decreases as time passes. Thus it represents a decelerating object. Here also the speed changes at constant rate. At t = 10s, the speed becomes zero.



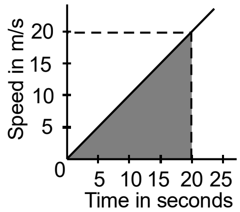
(d)

Figure (d) represents the motion of an object which speeds up from t = 0 to t = 5s, then moves at a constant speed from t = 5s to 10s and then decelerates to stop at t = 15 s.

- **NOTE :** distance covered by the body is shown by the area under speed–time graph.

ILLUSTRATIONS

16. Find the distance covered by a particle during the time interval t = 0 to t = 20s for which the speed–time graph is shown in figure 4.13.



Sol. The distance covered in the time interval 0 to 20s is equal to the area of the shaded triangle. It is

$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (20\text{s}) \times (20 \text{ m/s}) = 200 \text{ m}.$

VELOCITY-TIME GRAPH

If a graph is plotted taking the velocity of an object moving along a straight line on the vertical axis and time on the horizontal axis, we get a velocity–time graph.

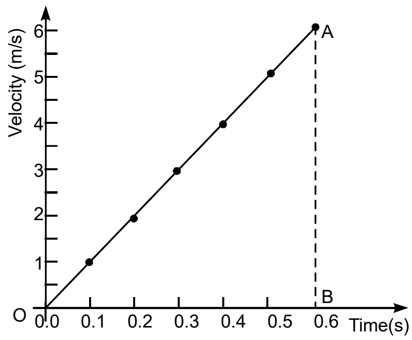
Suppose an object moves along a straight line in a fixed direction. That means the object does not turn around during its motion. Taking the direction of motion as the positive direction, the velocity of the object is given by the same value as its speed. Thus the speed–time graph for such an object is also its velocity–time graph.

The area under the speed–time graph gives the distance covered. But for a particle moving in a fixed direction, the distance covered in a time interval has the same value as its displacement in that time interval. So, **the area under the velocity–time graph of an object gives its displacement**.

Let us take an example. A ball is dropped from a height. We take the downward direction as positive. As the ball falls, its velocity increases. The velocity of the ball at different instants are given in Table-1. The velocity versus time graph is shown in figure.

Table-1 : Velocity of the falling ball at different instants:

Time in s	Velocity in m/s
0	0
0.1	1
0.2	2
0.3	3
0.4	4
0.5	5
0.6	6



What is the displacement of the ball in the time interval 0 to 0.6s ? It is equal to the area under the velocity–times graph from t = 0 to t = 0.6 s. This area is in the shape of a triangle. The area is

$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} (OB) \times (AB) = \frac{1}{2} \times (0.6\text{s}) \times (6 \text{ m/s}) = 1.8 \text{ m}.$

The ball has fallen through 1.8 m in 0.6 s.

Velocity–time graph of an object thrown upwards :

Suppose a ball is thrown upwards. We take the upward direction as the positive direction. The velocity decreases as the ball goes up. Table-2 gives the velocity of the ball at different instants.

Table-2 : Velocity of the rising ball at different instants :

Time (s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20

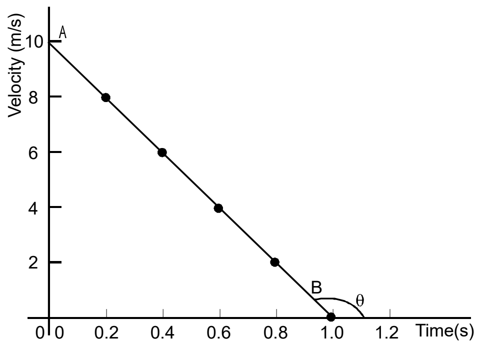
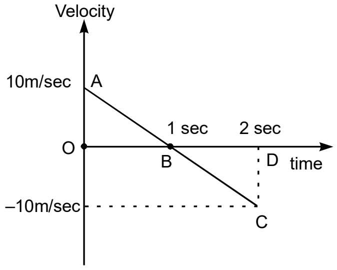


Figure shows the velocity–time graph. The plotted points fall on a straight line, AB. At t = 1.0 s, the velocity becomes zero. This means that the ball reaches the highest point at t = 1.0 s.

ILLUSTRATIONS

17. A ball is thrown vertically upwards with a velocity of 10m/sec. It strikes the ground after 2 sec. Its velocity–time graph is as shown in figure below. Find the displacement travelled by the ball in 2 second.

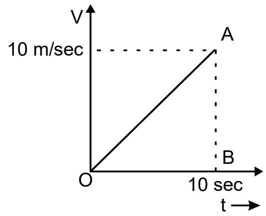


Sol. Displacement = area under velocity–time curve along time axis
= area of triangle AOB + area of triangle BDC

$= \frac{1}{2} \times OB \times AO + \frac{1}{2} \times BD \times CD = \frac{1}{2} \times 1\ sec \times 10m/sec + \frac{1}{2} \times 1 sec \times (10m/sec)$

= 5m – 5m = 0m

18. Velocity–time curve for a body moving with constant acceleration is shown in the figure. Calculate the displacement travelled by the body in 10 sec.



Sol. : Displacement = area under the velocity time curve along time axis = area AOB
Now AOB is a triangle with base = 10 sec and height = 10 m/sec.

So area = $\frac{1}{2}$ Base × height = $\frac{1}{2}$ × 10 sec × 10 m/sec = 5 × 10 = 50 m

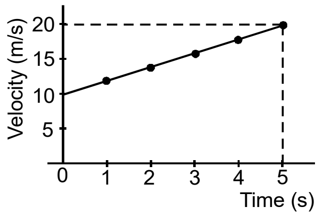
(a) Acceleration from velocity–time graph :

Suppose a particle moves with a uniform acceleration of 2 m/s^2 along a straight line. This means that the velocity increases by 2 m/s in one second. Also suppose its speed at t = 0 is 10 m/s.

Let us plot the velocity–time graph for this situation. We first find the values of the velocity at certain instants. At t = 0, the velocity is 10 m/s, at t = 1s it will become 10 m/s + 2m/s = 12 m/s, at t = 2s, it will become 14 m/s and so on. These values are given in Table-3 and the velocity–time graph is shown in Figure.

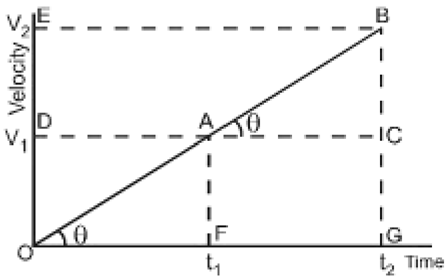
Table-3 :

Time (s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20



We see thzat the graph is a straight line. **Whenever the acceleration is uniform the velocity–time graph is a straight line.** We will now show that the slope of the velocity–time graph gives the acceleration.

Suppose the velocity–time graph of a particle moving along a straight line is as shown in figure. The graph is a straight line. At time t_1, the velocity is v_1, and at time t_2, it is v_2. These values are represented by the points A and B on the graph.



The acceleration of the object is

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{OE - OD}{OG - OF} = \frac{DE}{FG} = \frac{BC}{AC} = \tan\theta$$

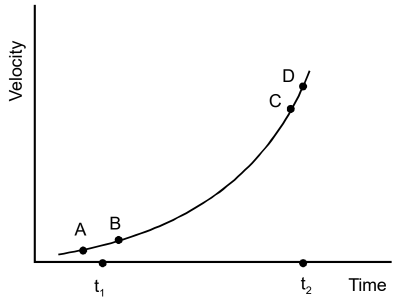
where θ is the angle made by the graph with the time–axis.

As defined earlier, the ratio $\frac{BC}{AC}$ = tan θ is called the slope of the line. Thus, we have the following:

The slope of the velocity–time graphs gives the acceleration for an object moving along a straight line.

◈ **Non–uniform acceleration :**

If the acceleration of an object moving along a straight line is not constant, the veolcity–time graph is not a straight line. Consider the velocity–time graph shown in figure. To find the acceleration at time t_1 we should treat the small part AB as a straight line and find its slope. Similarly, the slope of the small part CD gives the acceleration at time t_2. It is clear from the figure that the slope of CD is greater than that of AB. Thus, the acceleration at t_2 is greater than that at t_1. The graph in figure represents a motion in which acceleration increases with time.



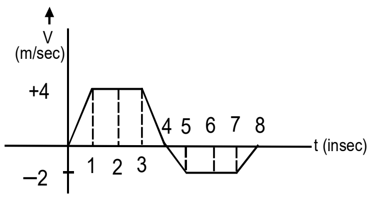
Displacement of particle from velocity time graph.

The area under the velocity–time curve along time axis gives the displacement of the particle

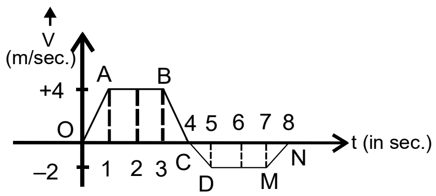
> **Can you think why ?**
> - Can 1-D motion be 2-D or 3-D?

ILLUSTRATION

19. The velocity versus time graph of a linear motion is shown in figure. Find the distance from the origin in 8 second.



Sol.



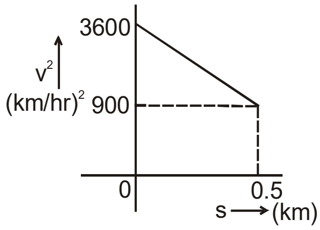
Distance in 8 second

S = Area of OABC + Area of CDMN

$$S = \frac{(2+4)\times 4}{2} + \frac{(2+4)\times 2}{2}$$

S = 18 m

20. A graph between the square of the velocity of a particle and the distance is moved by the particle is shown in the figure. Find the acceleration of the particle in km/h$^{2.}$

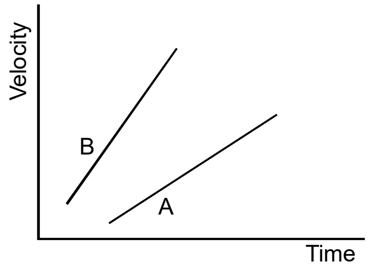


Sol. Given : $u^2 = 3600\ (\text{km/h})^2$, $v^2 = 900\ (\text{km/h})^2$, s = 0.6 km

From III equation of motion,

$$v^2 = u^2 + 2as \quad \Rightarrow a = \frac{v^2 - u^2}{2s} \quad \Rightarrow a = \frac{(900) - (3600)}{2\times 0.5} = \frac{-2700}{2\times 0.5} = -2700\ \text{km/h}^2$$

21. Figure shows the velocity–time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration ?



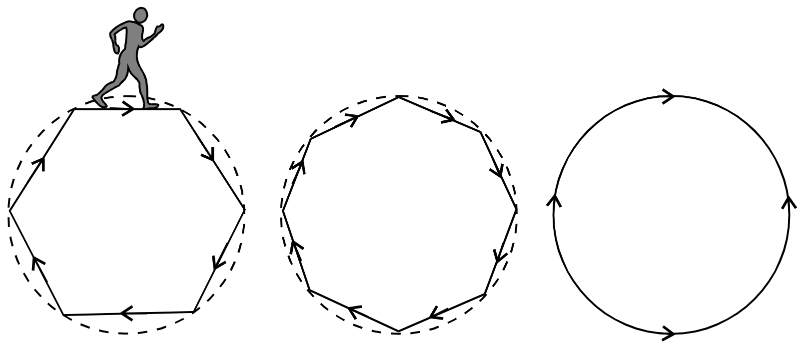
Sol. The slope of the velocity–time graph for B is greater than that for A. Thus, the acceleration of B is greater than that of A.

CIRCULAR MOTION

❖ **Definition :**
The motion of a body moving around a fixed point in a circular path is known as circular Motion.

- **Uniform Circular motion :** If the body covers equal distances along the circumference of the circle in equal intervals of time, the motion is said to be a uniform circular motion. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.
  Eg: Examples of uniform circular motion are:
  (i) Motion of moon around the earth.
  (ii) Motion of satellite around its planet.

❖ **Circular motion is known as accelerated motion :**
Explanation :
Consider a boy running along a regular hexagonal track (path) as shown in figure. As the boy runs along the side of the hexagon at a uniform speed, he has to take a turn at each corner changing direction but keeping the speed same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval will become smaller.



By increasing the number of sides of the regular polygon, we find that number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of a polygon with an infinite number of sides. On the circular track, the turning becomes a continuous process without any gap in between. The boy running along the sides of such a track will be performing a circular motion. Hence, circular motion is the motion of a body along the sides of a polygon of infinite number of sides with uniform speed, the direction changing continuously, it means the body moves with changing velocity in a circular path thus the uniform circular motion is known as accelerated motion.

DIFERENCE BETWEEN A UNIFORM LINEAR AND CIRCULAR MOITION

Uniform linear motion	Uniform circular motion
1. The direction of motion does not changes.	**1.** The direction of motion changes continuously.
2. The motion is non accelerated.	**2.** The motion is accelerated.

Boost your knowledge
☛ A body perform accelerated motion with uniform speed. The motion of the body is circular.

RADIAN

It is the unit of plane angle.

◈ **Definition :**

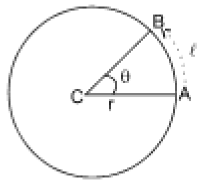
One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to its radius.

Eg. In figure , the arc AB of the circle has length ℓ and subtends an angle $\boldsymbol{\theta}$ at the centre **C**.

If $\angle ACB = \theta$ radians.

Then, $\theta = \frac{\ell}{r}$ radians.

[For $\ell = r$, $\theta = 1$ radian]



Angle subtended by the circumference at the centre,

$\theta = \frac{2\pi r}{r} = 2\pi$ radians $\{$or $2\pi^c\}$

[^{c}] is symbol for radian, just as (0) is symbol for degree.

Relation between radian and degree :

For complete circle at centre

$2\pi^c = 360^o$ or $1^c = \left|\frac{360}{2\pi}\right| = 57.3^o$

ANGULAR DISPLACEMENT AND ANGULAR VELOCITY

◈ **Definitions :**

(i) Angular displacement :

In a circular motion, the angular displacement of a body is the angle subtended by the body at the centre in a given interval of time. It is represented by the symbol θ (theta).

- The unit of angular displacement is radian.

(b) Angular velocity :

The angular displacement per unit time is called the angular velocity. It is represented by the symbol **ω** (omega).

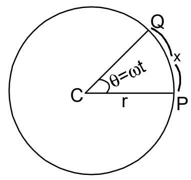
◈ **Expression for angular displacement and angular velocity :**

(i) Let a body move along a circle of radius r and perform a uniform circular motion. Let the body be at point P to start with and reach point Q after time t. Then, angular displacement = $\angle PCQ = \theta$

angular velocity = $\omega = \frac{\theta}{t}$ (i.e. $\theta = \omega t$)

(ii) In terms of time period and frequency: If the time period of the body is T(time taken in one complete round), the angular displacement = $2\pi^c$

Hence $\omega = \frac{2\pi}{T}$



But $\frac{1}{T} = n$ (frequency)

There $\omega = 2\pi n$

- The unit of angular velocity is rad/s

◈ **Angular acceleration :**

The rate of the change of angular velocity is called angular acceleration.

Let angular velocity at time t_1 is ω_1 and at time t_2 is ω_2 change in angular velocity in the time interval $t_2 - t_1$ is $\omega_2 - \omega_1$

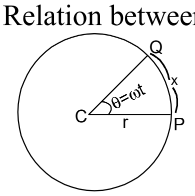
thus, rate of change of angular velocity = $\frac{\omega_2 - \omega_1}{t_2 - t_1}$

$\alpha = \frac{\Delta\omega}{\Delta t}$ (here α is average angular acceleration)

- The unit of angular acceleration is rad/s^2

◈ **Relation between Linear and Angular Quantities :**

(i) Relation between linear displacement and angular displacement.



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \frac{x}{r} \Rightarrow x = r\theta \quad ..... (i)$$

(ii) Relation between linear velocity and angular velocity.

From (i) $x = r\theta$

$$\frac{x}{t} = r\frac{\theta}{t} \Rightarrow v = r\omega \quad ..... (ii)$$

(iii) Relation between linear acceleration and angular acceleration.

From (ii) $v = r\omega$

$$\frac{v}{t} = r\frac{\omega}{t} \Rightarrow a = r\alpha \quad ..... (iii)$$

CENTRIPETAL FORCE

- Always acts towards centre.
- Centripetal force is required to move a particle in a circle.
- Because F_C is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

Note :

- Circular motion in horizontal plane is usually uniform circular motion.
- Remember that equations of motion are not applicable for circular motion.

CENTRIPETAL ACCELERATION

- In uniform circular motion the particle experiences an acceleration called the centripetal acceleration.
- $a_C = \frac{v^2}{r}$
- The direction of centripetal acceleration is along the radius towards the centre.

ILLUSTRATIONS

22. A fly wheel making 120 revolutions/minute. Find the angular speed of the wheel :

Sol. $\because$ 120 revolution/ minute = 2 rev/s

Angular speed = angle in one revolution × number of revolution/s = $2\pi \times 2 = 4\pi$ rad/s

23. A stone tied to the end of a string 80 cm is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec., what is the magnitude of the angular speed. $\left(\text{use}\,\pi = \frac{22}{7}\right)$

Sol. angular speed = angle × number of rev./s

$$\omega = 2\pi \times \frac{14}{25} = 3.52 \text{ rad/s}$$

24. Earth revolves around the Sun in 365 days. Calculate its angular speed.

Sol. T = 365 days = 365 × 24 × 60 × 60 s

$$\text{So, } \omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60} = 1.99 \times 10^{-7} \text{ rad/s}$$

25. A particle moves in a circle of radius 2 m and completes 5 revolutions in 10 seconds. Calculate the following :

(i) Angular velocity and

(ii) Linear velocity.

Sol. Since, it completes 5 revolutions in 10 seconds.

$$\therefore \text{ Time period} = \frac{10}{5} = 2\text{s}$$

(i)Now angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

(ii)Linear velocity is given by

$v = r\omega = 2\pi$

$\therefore$ $v = 2\pi$ m/s