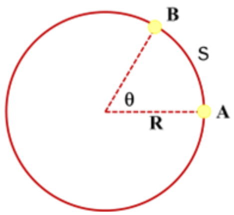
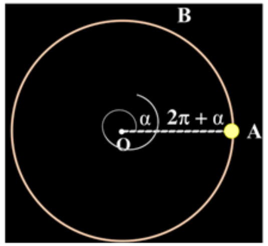
MIND MAP

Circular Motion



Angle $\theta = \frac{S}{R} = \frac{\textit{arc length}}{\textit{radius}}$ radian



Angular Displacement
$= 2\pi + \alpha$

Angular Velocity (ω)

Angular Velocity is defined as the rate of change of angular position w.r.t time.

Angular Velocity
- Average: $<\omega> = \frac{\Delta\theta}{\Delta t}$
- Instantaneous: $\omega = \frac{d\theta}{dt}$

SI unit is rad/s
Dimension is $[T^{-1}]$

$1\ \text{RPM} = \frac{\pi}{30}\ \text{rad/sec}$

Angular Acceleration (α)

Angular Acceleration is defined as rate of change of angular velocity w.r.t. time.

Angular Acceleration
- Average: $<\alpha> = \frac{\Delta\omega}{\Delta t}$
- Instantaneous: $\alpha = \frac{d\omega}{dt}$

SI unit is rad/s^2
Dimension is $[T^{-2}]$

If α = constant, then

$\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right) t$

$\theta = \omega_f t - \frac{1}{2}\alpha t^2$

Time Period (T)

The time taken by an object to make one revolution is known as its Time Period

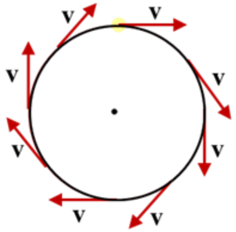
$T = \frac{2\pi}{\omega}$

Frequency (f)

The number of revolutions made in one second is known as Frequency.

$f = \frac{1}{T} = \frac{\omega}{2\pi}$

Uniform Circular Motion



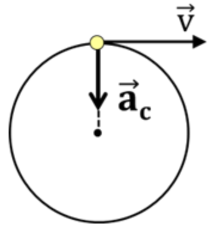
If a particle is moving in a circle with constant speed then its motion is called Uniform Circular Motion (UCM).

$|\vec{a}| = v\omega$

$|\vec{a}| = \frac{v^2}{R} \left(\because \omega = \frac{v}{R}\right)$

$|\vec{a}| = \omega^2 R$

$a_c = \frac{v^2}{R}$



This acceleration acts towards the centre, so it is called Centripetal Acceleration (a_c)

- Centripetal acceleration is perpendicular to the velocity and is responsible for changing the direction of the velocity.
- In U.C.M, $\vec{a}_c$ is not constant as its magnitude is constant but direction is changing.

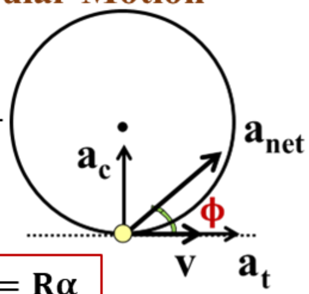
Non–Uniform Circular Motion

If a particle is moving in a circle with variable speed then its motion is called Non-Uniform Circular Motion.

$\tan\phi = \frac{a_c}{a_t}$

$\boxed{a_{net} = \sqrt{(a_c)^2 + (a_t)^2}}$

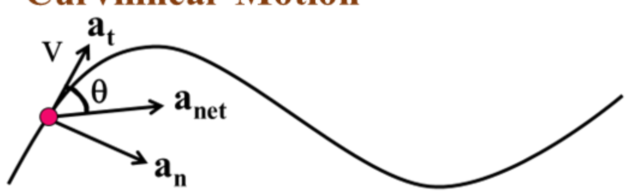
$\boxed{a_t = R\alpha}$



MIND MAP

Circular Motion

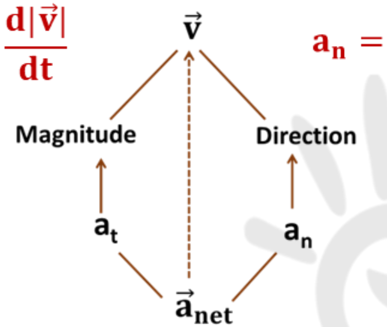
Curvilinear Motion



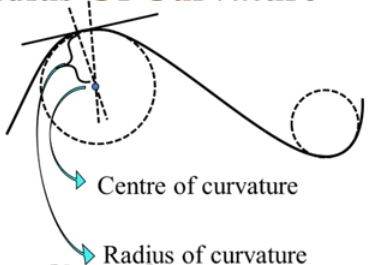
a_t is responsible for changing the magnitude of velocity i.e. speed of particle.

a_n is responsible for changing the direction of velocity. Its direction is towards the CONCAVE side.

$a_t = \frac{d|\vec{v}|}{dt}$ $\qquad$ $a_n = \frac{v^2}{R}$



Radius Of Curvature



Radius of curvature $R = \frac{v^2}{a_n}$

Radius of curvature is property of curve & not of motion of particle.

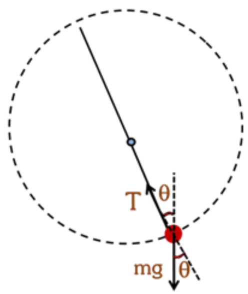
Dynamics of Circular Motion

The net resultant force providing the centripetal acceleration is called Centripetal Force.

$$\boxed{\sum F_c = m\,a_c = \frac{mv^2}{R} = m\omega^2 R}$$

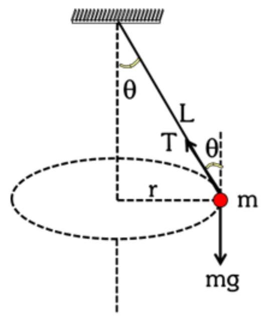
It should not be included in FBD drawn in inertial frame.

Vertical Circular Motion



$a_t = g\sin\theta$

Conical Pendulum



Time period of pendulum 'T'

$$T = \frac{2\pi r}{v}$$

$$= 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

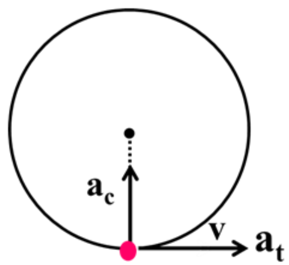
Dynamics of Non-UCM

Along radial

$$\sum F_C = ma_C = \frac{mv^2}{R} = m\omega^2 R$$

Along tangential $\sum F_t = ma_t$

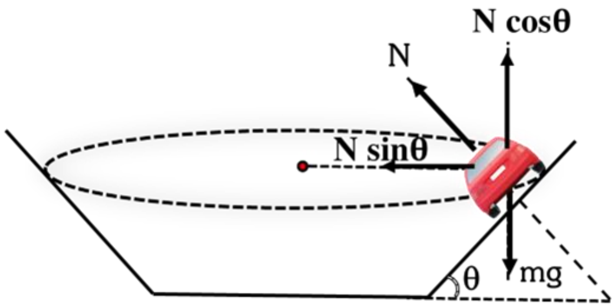
$$F_{net} = \sqrt{\left(\sum F_C\right)^2 + \left(\sum F_t\right)^2}$$



MIND MAP

Circular Motion

Banking of Roads



No friction

$\tan\theta = \dfrac{v^2}{Rg}$ $\qquad v_0 = \sqrt{Rg\tan\theta}$

Friction is present

$v < v_0$

For minimum speed, $f = \mu N$

$v_{min} = \sqrt{Rg\left(\dfrac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$

$v > v_0$

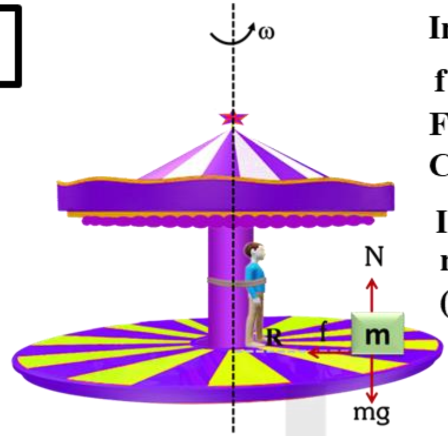
For maximum speed, $f = \mu N$

$v_{max} = \sqrt{Rg\left(\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

vehicle can successfully turn on a banked road in a circle of radius R for

$v_{min} \le v \le v_{max}$

Centrifugal Force



In ground frame

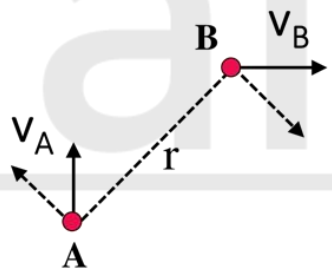
$f = m\omega^2 R$

Friction is the Centripetal Force.

In man's frame

$m\omega^2 R$ Pseudo Force (Centrifugal Force)

Angular Velocity in General



$$\omega = \frac{(v_\perp)_{rel}}{r} = \frac{\text{relative velocity} \perp \text{ to line joining two particles}}{\text{separation between two particles}}$$