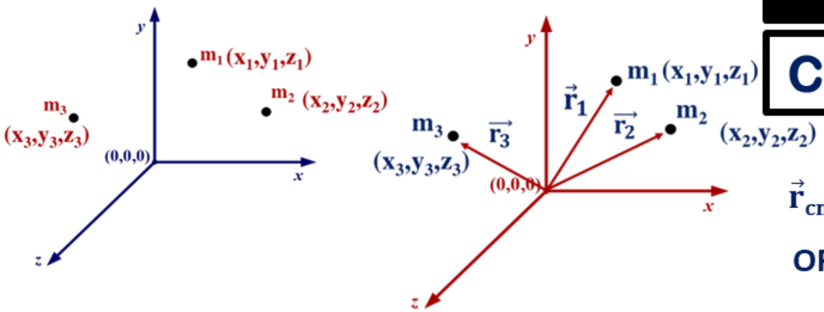
MIND MAP

Center of Mass

Centre of mass of discrete point masses



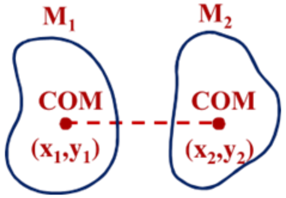
$$X_{cm} = \left(\frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}\right) = \frac{\sum m_i x_i}{\sum m_i}$$

$$Y_{cm} = \left(\frac{m_1y_1 + m_2y_2 + \ldots}{m_1 + m_2 + \ldots}\right) = \frac{\sum m_i y_i}{\sum m_i}$$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

OR $\vec{r}_{CM} = X_{CM}\hat{i} + Y_{CM}\hat{j}$

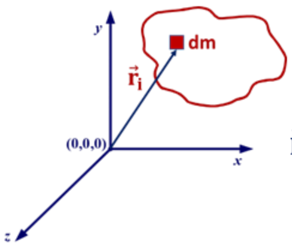
Com of several groups of particles



$$X_{CM} = \frac{M_1x_1 + M_2x_2}{M_1 + M_2}$$

$$Y_{CM} = \frac{M_1y_1 + M_2y_2}{M_1 + M_2}$$

Com of continuous bodies



$$\vec{r}_{cm} = \frac{\int \vec{r}_i \, dm}{\int dm}$$

Locate COM of m_1 and m_2

d

$\leftarrow d_2 \rightarrow$

$\leftarrow d_1 \longrightarrow$ Com

m_1 0, 0

m_2 d, 0

$$\frac{d_1}{d_2} = \frac{m_2}{m_1}$$

Com of two particles divides internally the line joining two particles in inverse ratio of their masses.

Linear mass density (λ)

Mass per unit length is called linear mass density λ $\lambda = \frac{M}{L}$

For uniform object, λ is same for every element

Surface mass density (σ)

Mass per unit area

For uniform objects $\sigma = \frac{M}{A}$

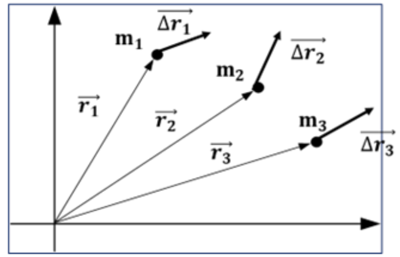
Volumetric mass density (ρ)

Mass per unit volume

If uniform then $\rho = \frac{M}{V}$

For uniform objects COM is at geometrical centre.

Displacement of COM due to displacement of particles of system



$$\Delta\vec{r}_{cm} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

Velocity and Acceleration of COM

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots}{m_1 + m_2 + \ldots}$$

MIND MAP

Center of Mass

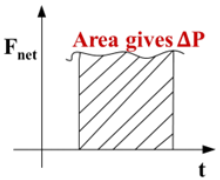
Linear Momentum

Linear Momentum is the product of Mass and Velocity $\vec{P} = m\vec{v}$

SI Unit is (kg m/s) or (N-s)

For a particle.

$\vec{F}_{net} = \frac{d\vec{P}}{dt}$ $\quad < \vec{F}_{avg} > = \frac{\Delta \vec{P}}{\Delta t}$



$\int \vec{F}_{net}\, dt = \Delta \vec{P}$

Kinetic Energy of a particle

KE of particle $= \frac{P^2}{2m}$

Linear Momentum for system of particles

It is vector sum of L.M. of all the particles in system.

$\vec{P}_{sys.} = \sum \vec{P}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots = M\vec{V}_{cm}$

M = Mass of System

$\vec{V}_{cm}$ = Velocity of COM of System

For calculation of linear momentum of a system we can assume whole mass to be concentrated at COM moving with $\vec{V}_{cm}$.

Kinetic Energy of System of particles

KE of system of particles is the algebraic summation of KE of all its constituent particles

$$KE = \sum \frac{1}{2} m_i v_i^2 \neq \frac{1}{2} M (V_{cm})^2$$

For calculation of Kinetic Energy of a system we CANNOT assume whole mass to be concentrated at COM moving with $\vec{V}_{cm}$.

Principle of conservation of L.M.

If net external force acting on a system is zero for a time interval then in that interval linear momentum of the system is conserved (i.e. remains constant)

Impulse-Momentum Theorem

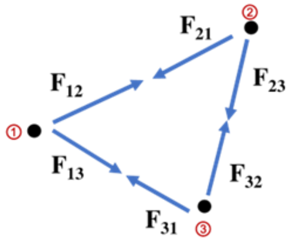
$$\boxed{\vec{I}_{net} = \vec{P}_f - \vec{P}_i}$$

Impulse is a vector quantity whose unit is same as that of momentum (kgm/s) or (N-s.)

(i) If linear momentum of a system is zero, then its KE may or may not be zero.
(ii) If KE of system is zero then its linear momentum must be zero.

Newton's Second Law for System of Particles

$$\boxed{\sum \vec{F}_{ext.} = M\vec{a}_{cm}}$$



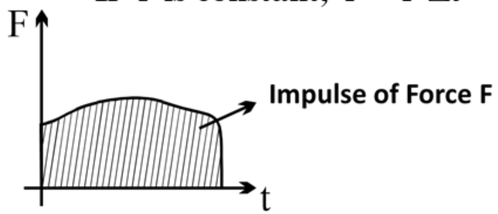
$M\vec{a}_{cm} = F_{1ext} + F_{1int}\ F_{2ext} + F_{2int} + \ldots\ldots$

Impulse : Impulse of a force for a time interval "t_1" to "t_2" is defined as

$$\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F} dt}$$

Area under F-t graph gives Impulse of Force F

If $\vec{F}$ is constant, $\vec{I} = \vec{F}\,\Delta t$



MIND MAP

Center of Mass

Impulsive Force

Force which acts for a very small time duration and whose magnitude is very large is called Impulsive Force.
In presence of impulsive forces, non-impulsive forces (like mg, spring force) can be neglected.

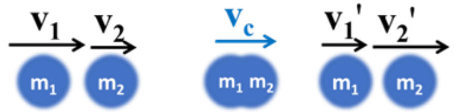
Collision

- **Head On Collision**: When during collision, velocities of both the objects are along the common normal of colliding objects

- **Oblique Collision**: If velocities of any of the colliding objects is not along the common normal

Elastic bodies are bodies which regain their original shape without any loss of energy.

Head-on Elastic Collision



KE max	KE min	KE max
PE min	PE max	PE min

(i) Total ME of system remains constant.
(ii) Initial KE & Final KE are equal but it is not constant throughout the process.
(iii) At the instant when velocities of both the bodies are same then PE is maximum & KE is minimum.

Coefficient of Restitution (e)

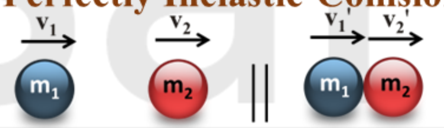
$$\boxed{e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}}$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$ For elastic collision e = 1

Inelastic Collision $0 < e < 1$

Inelastic

Perfectly Inelastic Collision $e = 0$



Perfectly Inelastic

Both bodies stick together and move with same velocity after collision

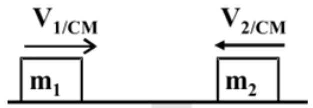
COM Frame

$\vec{P}_{sys} = M\vec{v}_{cm}$ $\quad \vec{P}_{sys/cm} = M\vec{v}_{cm/cm}$
$\vec{P}_{sys/cm} = 0$

In Centre of Mass frame, Linear Momentum of a System is Zero

Kinetic Energy of Two Particle System in CM Frame

In CM Frame



$$\frac{m_1 m_2}{m_1 + m_2} = \mu$$ (reduced mass)

$$\boxed{KE_{sys/CM} = \frac{1}{2}\mu (v_{rel})^2}$$

$$\boxed{KE_{sys} = KE_{sys/CM} + \frac{1}{2}(M)V_{cm}^2}$$

total mass of system

If $m_1 = m_2$

$v_2' = v_1 \qquad v_1' = v_2$

$\Rightarrow$ velocities are interchanged.

MIND MAP

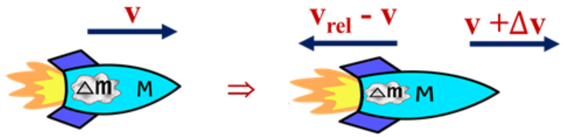
Center of Mass

KE loss in collision

$KE_{loss} = KE_i - KE_f$

$= \frac{1}{2}\mu v_{app.}^2 (1 - e^2)$ $(\because e = \frac{v_{sep}}{v_{app}})$

Variable Mass



$v_{rel.}$ is known as exhaust speed which is the speed of exhaust relative to the rocket

$M(\Delta v) = \Delta m\, v_{rel.}$

$F_{thrust} = M\frac{\Delta v}{\Delta t} = \frac{\Delta m}{\Delta t} v_{rel.}$

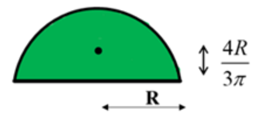
Recoil of Gun

$\boxed{F_{thrust} = \frac{m_{bullet} v_{rel.}}{\Delta t}}$

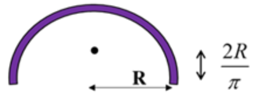
$v_{rel.}$ is known as muzzle velocity i.e. velocity of bullet w.r.t gun.

Center of Mass of some Shape

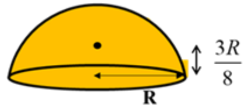
Semi circular plate



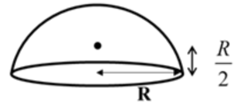
Semi circular ring



Solid hemisphere



Hollow hemisphere

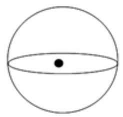


Disk

Ring

Solid Sphere

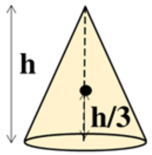
Hollow Sphere



Triangle

Centroid

Hollow cone



Solid cone