MIND MAP

SIMPLE HARMONIC MOTION

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(1) Periodic
(2) Oscillatory
(3) $|F| \propto |x|$

$F_{net} = -kx$
$a = -\omega^2 x$
$\omega = \sqrt{\frac{k}{m}}$
$x = A\sin(\omega t + \phi)$

Time Period & Frequency $T = \frac{2\pi}{\omega}$ $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Velocity Analysis $x = A\sin(\omega t + \phi)$

$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$v = \omega\sqrt{(A^2 - x^2)}$

Max velocity $v_{max} = \omega A$ at $x = 0$ (Mean Position)

Min velocity $v_{min} = 0$ at $x = \pm A$ (Extreme Position)

Graph b/w 'v' & 'x' — Ellipse

$\frac{x^2}{A^2} + \frac{v^2}{(\omega A)^2} = 1$

Acceleration Analysis

$a = -\omega^2 x$

$x = 2 + 1\sin(\omega t)$

Position — eq^m position — Amplitude

Slope $= -\omega^2$

Physical Pendulum

$T = 2\pi\sqrt{\frac{I}{mgd}}$

I : MOI of system about hinge.
m : Mass of system
d : distance between COM and hinge.

Simple Pendulum

Ideal String, L, point mass

$T = 2\pi\sqrt{\frac{L}{g}}$

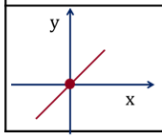
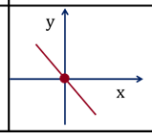
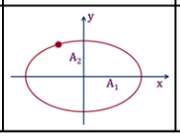
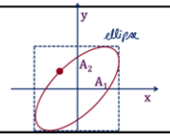
Composition of Two SHM's of same ω along mutually perpendicular directions

$x = 3\sin(\omega t) + 4\sin\left(\omega t + \frac{\pi}{2}\right)$

$x = 5\sin(\omega t + 53°)$

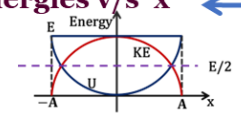
5, 4, 53°, 3, ωt

$\vec{x} = A_1\sin(\omega t)\hat{i}$ $\vec{y} = A_2\sin(\omega t + \phi)\hat{j}$

When $\phi = 0$	When $\phi = \pi$	When $\phi = \frac{\pi}{2}$	When $\phi \neq 0, \frac{\pi}{2}, \pi$
$y = \frac{A_2}{A_1}x$	$y = -\frac{A_2}{A_1}x$	$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$	
			

Energies v/s 'x'

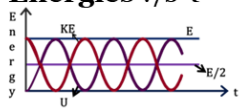
$U = \frac{1}{2}kx^2$
$KE = \frac{1}{2}k(A^2 - x^2)$
$KE + U = E = \frac{1}{2}kA^2$



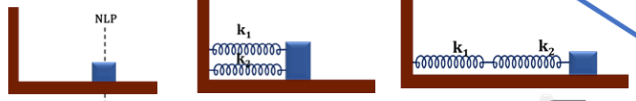
Energies v/s 't'

$U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

$KE = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$



Spring Block System



$T = 2\pi\sqrt{\frac{m}{k}}$

In a spring block system, if an additional constant force (along line of SHM) is applied on the block then time period remains same.

$k_{eq} = k_1 + k_2$

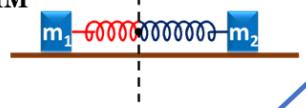
$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$= 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

Two Blocks SHM

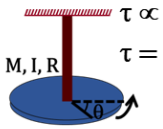


$\mu = \frac{m_1 m_2}{m_1 + m_2}$

Reduced Mass

$T_1 = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{k}}$

Torsional Pendulum



$\tau \propto -\theta$

$\tau = -C\theta$ (C = Torsional constant)

$T = 2\pi\sqrt{\frac{I}{C}}$

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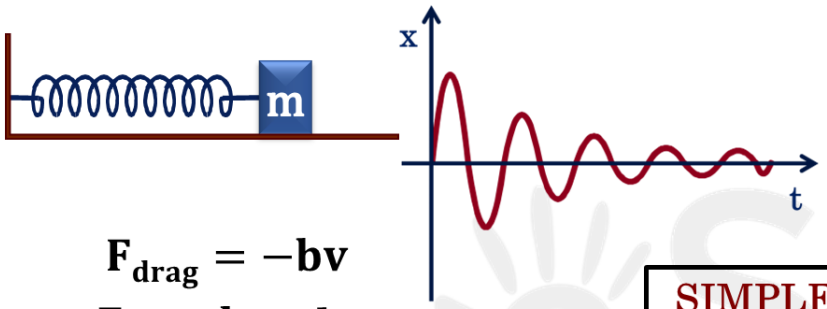
Damped SHM



$$F_{drag} = -bv$$

$$F = -kx - bv$$

$$x = Ae^{-bt/2m}\sin(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$E = \frac{1}{2}kA^2e^{-bt/m}$$

SIMPLE HARMONIC MOTION

Forced Oscillations and Resonance

$$F_{driving} = F_0 \sin(\omega_d t)$$

$$ma = -kx - bv + F_0 \sin(\omega_d t)$$

$$x = A\sin(\omega_d t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_d^2)^2 + (b\omega_d/m)^2}}$$

If small damping, (b very small)

$$A = \frac{F_0/m}{(\omega^2 - \omega_d{}^2)}$$

$$\omega_d \approx \omega \Rightarrow \text{RESONANCE}$$