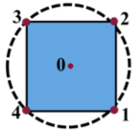
MIND MAP

Rotational Motion

CRTM



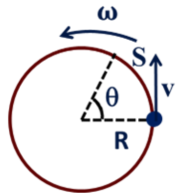
1. Since distance between two particles of a rigid body remains constant, so the relative motion of one particle w.r.t. other particle is circular motion.
2. Angular velocity of all the particles about a given point of a rigid body is same
$\omega_{1/0} = \omega_{2/0} = \omega_{3/0} = \omega_{4/0} = \omega_0$
3. This angular velocity of a rigid body about all the point of rigid body is same.

$\omega_{0/1} = \omega_{2/1} = \omega_{3/1} = \omega_{4/1}$
$\quad ||$
$\omega_{1/0} = \omega_{2/0} = \omega_{3/0} = \omega_{4/0}$ $= \omega_0$ — 'ω' of rigid body

Kinematics of Pure rotation motion

$\boxed{S = R\theta}$

$\boxed{|v| = R\omega}$



If α = constant

$\omega_f = \omega_i + \alpha t$

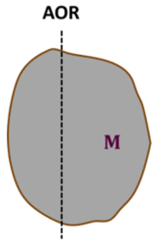
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

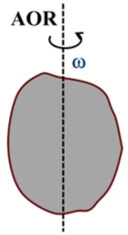
$\theta = \omega_f t - \frac{1}{2}\alpha t^2$

Radius of Gyration



$K = \sqrt{\frac{I}{M}}$ → RoG about an axis

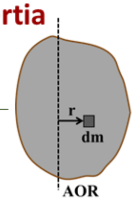
$I = MK^2$

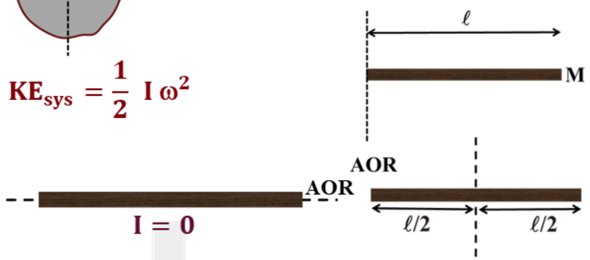


$KE_{sys} = \frac{1}{2} I\omega^2$

Moment of Inertia

$I = \int r^2\, dm$



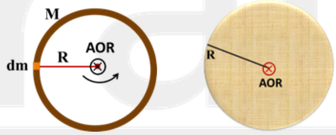


$I = \frac{M\ell^3}{3}$

$I = 0$

$I = \frac{M\ell^2}{12}$

Uniform Ring / Disk

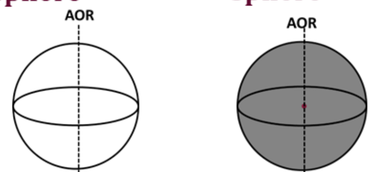


Uniform Ring: $I = MR^2$

Disk: $I = \frac{MR^2}{2}$

Moving mass parallel to AOR does not change moment of inertia.

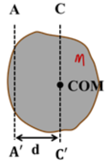
Uniform Hollow Sphere / Uniform Solid Sphere



Uniform Hollow Sphere: $I = \frac{2}{3}MR^2$

Uniform Solid Sphere: $I = \frac{2}{5}MR^2$

Parallel – axis theorem:

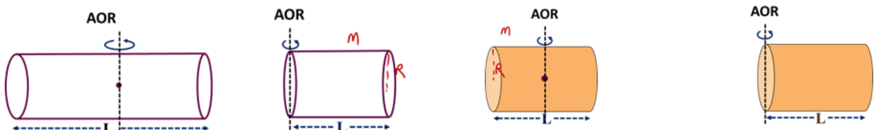


Among the given parallel axis MOI will be minimum about the axis passing through COM.

MOI about the parallel axis having same distance from COM is equal.

$I_{AA'} = I_{CC'} + Md^2$

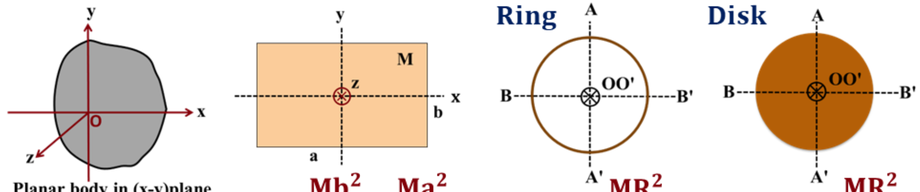
Hollow Cylinder / Solid Cylinder



Hollow Cylinder: $I = \frac{MR^2}{2} + \frac{ML^2}{12}$; $I = \frac{MR^2}{2} + \frac{ML^2}{3}$

Solid Cylinder: $I = \frac{MR^2}{4} + \frac{ML^2}{12}$; $I = \frac{MR^2}{4} + \frac{ML^2}{3}$

Perpendicular axis theorem:



$I_z = I_x + I_y$

$I_z = \frac{Mb^2}{12} + \frac{Ma^2}{12}$

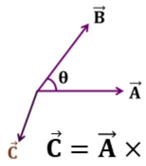
Ring: $I_{AA'} = \frac{MR^2}{2}$

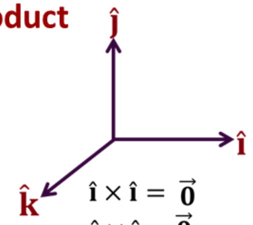
Disk: $I_{AA'} = \frac{MR^2}{4}$

MIND MAP

Rotational Motion

Cross Product





$\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta$

θ is angle b/w $\vec{A}$ & $\vec{B}$

Properties

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

$\vec{A} \times \vec{A} = \vec{0}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

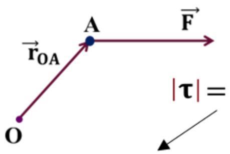
$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$

Torque

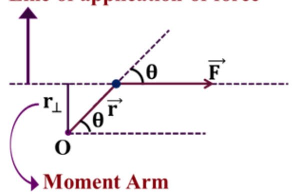


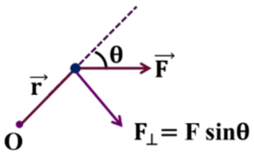
$\vec{\tau}_O = \vec{r}_{OA} \times \vec{F}$

$|\tau| = rF\sin\theta$

(r sinθ)F → $\boxed{|\tau| = r_\perp F}$

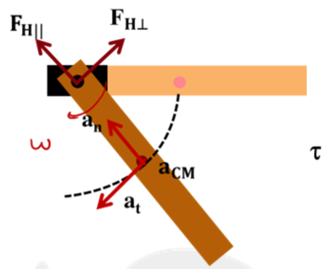
r(F sinθ) → $\boxed{|\tau| = rF_\perp}$





$F_\perp$ is perpendicular component of $\vec{F}$

Calculation of Hinge Force



$$F_H = \sqrt{F_{H||}^2 + F_{H\perp}^2}$$

($F_{H\perp}$) Perpendicular to rod

$$\sum F_\perp = m(a_{CM})_t = m\alpha\frac{L}{2}$$

$\tau = I\alpha \longrightarrow \alpha \longrightarrow (a_{CM})_t \longrightarrow F_{H\perp}$

($F_{H||}$) Parallel i.e. along the rod

$$\sum F_{||} = m(a_{CM})_n = m\omega^2\frac{L}{2}$$

W-E Theorem $\longrightarrow \omega \longrightarrow (a_{CM})_n \longrightarrow F_{H||}$

Inertial Pulley

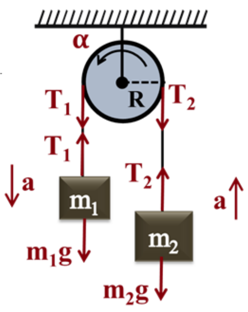
{No slipping of rope on disk}

$T_1R - T_2R = I\alpha$ ....(1)

$m_1g - T_1 = m_1a$ ....(2)

$T_2 - m_2g = m_2a$ ....(3)

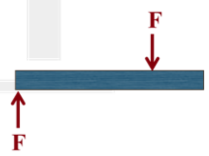
$\alpha R = a$ ....(4)



Work done in terms of Torque

$$W = \int \tau d\theta \qquad P = \tau\,\omega$$

Force Couple



A pair of equal & opposite forces is called force couple.

Torque of force couple is same about any point of the space.

Condition of Equilibrium

Translation Equilibrium

1) $\sum \vec{F} = 0$ → $\sum \vec{F}_x = 0$, $\sum \vec{F}_y = 0$, $\sum \vec{F}_z = 0$

Rotational Equilibrium

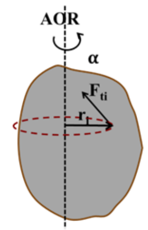
2) $\sum \vec{\tau} = 0$

If $\vec{F}_{net} = 0$ on a rigid body then $\vec{\tau}_{net}$ is same about every point of space.

$\sum \tau_{int} = 0$

$$\left(\sum \tau_{ext}\right)_{AOR} = (I)_{AOR}\alpha$$

Valid only in inertial frame.



MIND MAP

Rotational Motion

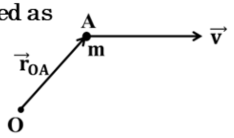
Angular Momentum of Particle

Angular Momentum of Particle about point O is defined as

$\vec{L}_O = \vec{r}_{OA} \times \vec{P}$

$= \vec{r}_{OA} \times (m\vec{v})$

$= m\,\vec{r}_{OA} \times \vec{v}$



where $\vec{P}$ is linear momentum of the particle present at A.

$\vec{L}_O$ is angular momentum about O.

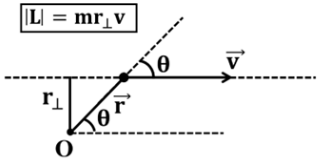
$$\vec{L} = \vec{r} \times \vec{P}$$

AM is also known as moment of LM.

$|L| = mrv\sin\theta$

$m(r\sin\theta)v \Rightarrow \vec{L} = \text{Constant}$

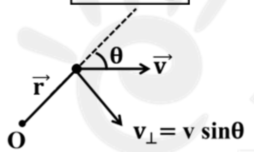
$\boxed{|L| = mr_{\perp}v}$



$r_{\perp}$ is the shortest distance from O to line of velocity.

$mr(v\sin\theta)$

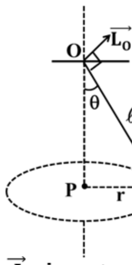
$\boxed{|L| = mrv_{\perp}}$



$v_{\perp} = v\sin\theta$

$v_{\perp}$ is perpendicular to $\vec{r}$

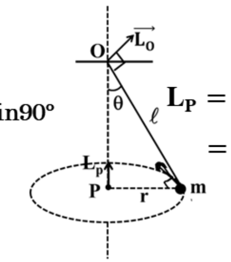
Conical Pendulum



$L_O = m\,l\,v\sin 90^{\circ}$

$= mvl$

$\vec{L}_O$ is not constant but its magnitude is constant.



$L_P = m\,\ell\sin\theta\,v\sin 90^{\circ}$

$= mv\ell\sin\theta$

$\vec{L}_P$ is constant

Angular Momentum Conservation Principle

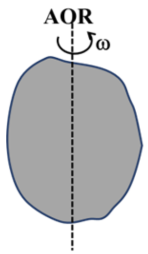
$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$ If $\vec{\tau}_{net} = 0 \Rightarrow \vec{L} = \text{Constant}$

If $\vec{\tau}_{net}$ is zero then its A.M. is conserved.

Angular Momentum of System of Particles

$$\vec{L}_{sys} = \sum \vec{L}_i$$

A.M. of Rigid Body Performing Pure Rotation About Fixed Axis



$$\left(L_{sys}\right)_{AOR} = I_{AOR}\,\omega$$

Relation Between Torque and Angular Momentum

$\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$ valid only in inertial frame.

$\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$

Particle

$\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$

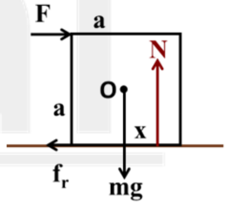
System

$\vec{\tau}_{Net} = \dfrac{d\vec{L}_{sys}}{dt}$

$\sum \tau_{ext} + \sum \tau_{int} = 0$

$(\vec{\tau}_{ext})_{Net} = \dfrac{d\vec{L}_{sys}}{dt}$

Condition of Toppling



$x > \dfrac{a}{2}$ Toppling will occur

$x = \dfrac{a}{2}$ On verge of toppling

$x < \dfrac{a}{2}$ No toppling

Angular Impulse

Linear Impulse $(\vec{I})$

$\vec{I} = \int \vec{F}\,dt$

Unit - Ns

$\vec{I}_{net} = \vec{P}_f - \vec{P}_i$

$\boxed{\vec{J} = \vec{r} \times \vec{I}}$

Angular Impulse $(\vec{J})$

$\vec{J} = \int \vec{\tau}\,dt$

Unit - Nms

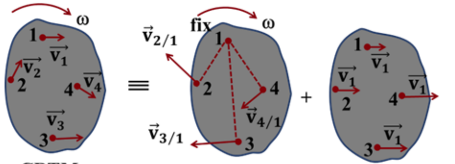
$\vec{J}_{net} = \vec{L}_f - \vec{L}_i$

MIND MAP

Rotational Motion

Combined Rotation + Translational Motion [CRTM]

Velocity Analysis



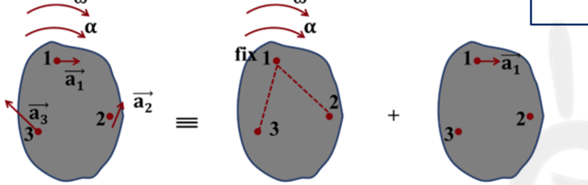
CRTM $\equiv$ Pure rotation + Pure translation

$\vec{v}_2 = \vec{v}_{2/1} + \vec{v}_1$

$\vec{v}_3 = \vec{v}_{3/1} + \vec{v}_1$

$\vec{v}_4 = \vec{v}_{4/1} + \vec{v}_1$

Acceleration Analysis



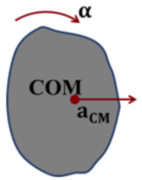
CRTM $\equiv$ Pure rotation + Pure translation

Dynamics of CRTM

For analysing its motion we apply two equation

$$\sum \vec{F}_{ext} = M\vec{a}_{CM}$$

$$\sum \vec{\tau}_{ext} = I\vec{\alpha}$$



Dynamics of CRTM

Second equation is valid in inertial frame.

1) $\sum \vec{F} = M\vec{a}_{CM}$ 2) $\sum \vec{\tau} = I\vec{\alpha}$

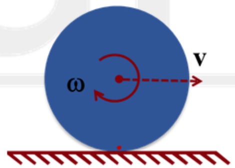
$\sum \vec{F}_{ext} = M\vec{a}_{CM}$ $\sum \vec{\tau}_{ext} = I\vec{\alpha}$

To apply second equation about non-inertial point, Pseudo force is applied at COM of body & τ of Pseudo force is also taken into account.

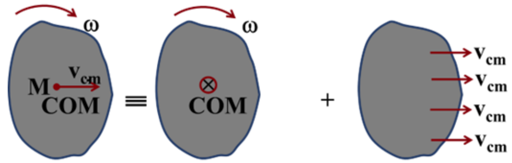
Rolling Motion

If during the motion, there is no relative slipping between the points of contact, then motion is called Rolling.

$$v = \omega R$$



Kinetic Energy of Rigid Body Performing CRTM

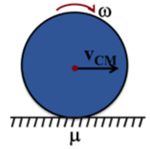


CRTM $\equiv$ Rotational KE + Translational KE

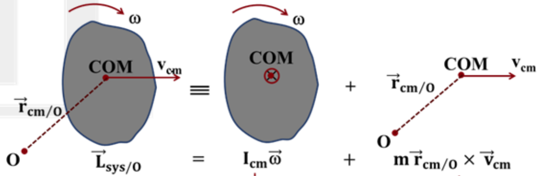
$$KE = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}Mv_{CM}^2$$

Keypoint

For a body performing rolling motion over a fixed surface, work done by friction force on the body will be zero as velocity of point of application of friction always zero.



Angular Momentum of R.B. Performing CRTM



$\vec{L}_{sys/O} = I_{cm}\vec{\omega} + m\vec{r}_{cm/O} \times \vec{v}_{cm}$

Rotational as rigid body; Pure rotation about COM

Translation as a particle

$$\vec{L}_{sys/O} = I_{cm}\vec{\omega} + m\vec{r}_{cm/O} \times \vec{v}_{cm}$$

Instantaneous Center of Rotation (I_{COR}) & Axis of Rotation (I_{AOR})

- Let at an instant of time velocity of point P is zero.
- To calculate velocity of other points of rigid body, rigid body can be assumed to perform pure rotation about an axis passing through point P at that instant. This point is called I_{COR} and axis passing through it is called I_{AOR}.