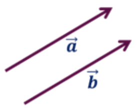
Vectors — MIND MAP

Two vectors are said to be equal if and only if they have same magnitude and the same direction.

$\vec{a} = \vec{b}$



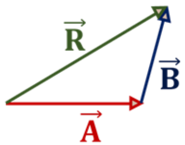
$0^\circ \le$ Angle between two vectors $\le 180^\circ$

Parallel Vectors	Anti-Parallel Vectors
Same direction and angle between vectors is 0°	Opposite direction and angle between vectors is 180°

Triangle rule of vector addition

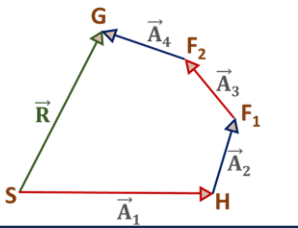
$\vec{A} + \vec{B} = \vec{R}$

$|\vec{R}| \neq |\vec{A}| + |\vec{B}|$



Polygon rule of vector addition

$\vec{R} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4$



Multiplication of vector with real number λ

Magnitude become $|\lambda|\,|\vec{A}|$, direction remains the same.

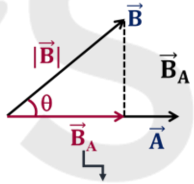
Scalar Product (Dot Product) of two Vectors

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\hat{i} \cdot \hat{i} = 1 \quad \hat{j} \cdot \hat{j} = 1 \quad \hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = 0 \quad \hat{i} \cdot \hat{k} = 0 \quad \hat{k} \cdot \hat{j} = 0$

Angle between two vectors $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$



$\vec{B}_A = |\vec{B}| \dfrac{(\vec{A} \cdot \vec{B})}{|\vec{A}|\,|\vec{B}|} \hat{A}$

component of B along A

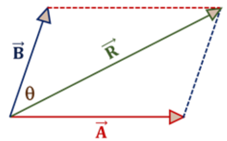
Parallelogram rule of vector addition

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

When $\theta = 0^\circ$ $\quad R_{max} = A + B$

When $\theta = 180^\circ$ $\quad R_{min} = |A - B|$

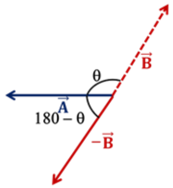
$R_{min} \le |\vec{R}| \le R_{max}$



Subtraction of vectors

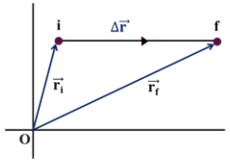
$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$|A - B| \le |\vec{A} - \vec{B}| \le A + B$



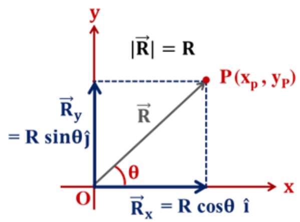
Displacement vector

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$



Resolution of a Vector into Components

Rectangular Components



$\vec{R} = \vec{R}_x + \vec{R}_y$

$\vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$

$|\vec{R}| \ge 0$

$\tan\theta = \dfrac{R_y}{R_x}$

$\hat{R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{R_x\hat{i} + R_y\hat{j}}{\sqrt{R_x^2 + R_y^2}}$

Position Vector $\vec{OP} = \vec{r} = x_p\hat{i} + y_p\hat{j}$

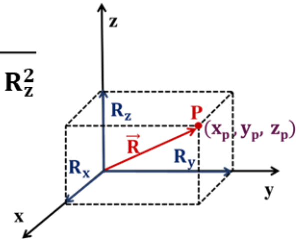
Representation of vector in 3D

$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

$R = |\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Position Vector

$\vec{r} = x_p\hat{i} + y_p\hat{j} + z_p\hat{k}$



Unit vector

$\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

magnitude = 1

purpose is to describe a direction in space

$|\hat{i}| = 1 = |-\hat{i}|$

$|\hat{j}| = 1 = |-\hat{j}|$

$|\hat{k}| = 1 = |-\hat{k}|$