Wave on String

Power Transmission in Travelling Wave on String

$P = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$ valid for all waves on string

$$\langle P \rangle = \frac{1}{2}\underbrace{\sqrt{T\mu}}_{\text{Property of medium}}\ \overbrace{A^2\omega^2}^{\text{Property of Source}}$$

General Equation of Standing Wave

$y = 2A\sin(kx + \phi_1)\sin(\omega t + \phi_2)$

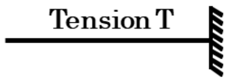
Amplitude of component wave = A

Coefficient of x = k

Wavelength of component waves $= \frac{2\pi}{k}$

Loop length $= \frac{\lambda}{2} = \frac{\pi}{k}$

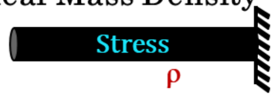
Velocity of Wave on String



$v_w = \sqrt{\frac{T}{\mu}}$

$\underbrace{\mu}$ = mass per unit length

Linear Mass Density



$v_w = \sqrt{\frac{T}{\mu}}$

Wave on String

WAVES

- Mechanical Waves
  - Medium is required
  - Waves on String, Sound Wave
  - Transverse Wave
    - Constituents of the medium oscillates perpendicular to the direction of wave propagation.
    - Waves on String
  - Longitudinal Wave
    - Constituents of the medium oscillates along the direction of wave propagation.
    - Sound Wave
- Non-Mechanical Waves
  - Medium is not required
  - Light Wave

Wave Function $y = f(x,t)$

Velocity of wave: $v_w = -\frac{\text{Coefficient of t}}{\text{Coefficient of x}}$

$y = A\sin(kx - \omega t)$

$v_w = -\frac{-\omega}{k} = \frac{\omega}{k}$

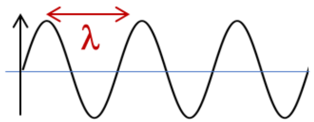
Wave travelling in + dir.

$y = A\sin(kx + \omega t)$

$v_w = -\frac{\omega}{k} = -\frac{\omega}{k}$

Wave travelling in – dir.

Sinusoidal Wave Equation



$y = A\sin(\omega t - kx + \phi)$

$\lambda = \frac{2\pi}{k}$ $\quad T = \frac{2\pi}{\omega}$ $\quad$ Angular wave number, $k = \frac{2\pi}{\lambda}$

Velocity & Acceleration of Particle

$v_P = \frac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$

$y = A\sin(\omega t - kx)$ → y: Displacement

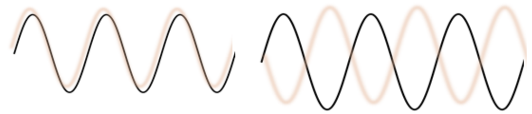
$a_P = \frac{\partial v_P}{\partial t} = -\omega^2 y$

Relation Between Particle Velocity & Wave Velocity

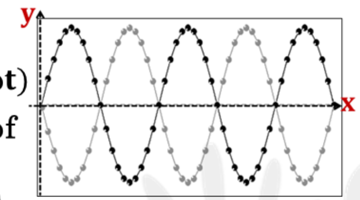
$v_P = -v_w\left(\frac{\partial y}{\partial x}\right)$ → $\frac{\partial y}{\partial x}$: Slope of the string at point x.

Wave on String

Interference of Waves



Standing Waves



$y_{res} = 2A\sin(kx)\cos(\omega t)$

For $x = x_0$ Equation of Motion

$y = \underbrace{2A\sin(kx_0)}_{A'}\cos(\omega t)$

$y = A'\cos(\omega t)$ → SHM

$y_{res} = 2A\sin(kx)\cos(\omega t)$

$v_P = \dfrac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$

$a_p = \dfrac{\partial^2 y}{\partial t^2} = -\omega^2 y$

For Nodes - Amplitude = 0

$\sin(kx + \phi_1) = 0$

$kx + \phi_1 = n\pi$

For Antinodes - Amplitude = max

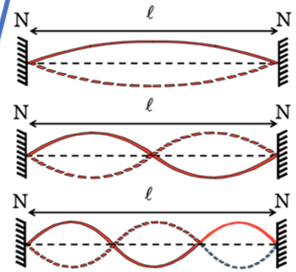
$\sin(kx + \phi_1) = \pm 1$

$(kx + \phi_1) = \left(n + \dfrac{1}{2}\right)\pi$

Energy in Standing Wave

For Node	For Anitnode
$KE = 0$	$PE = 0$
$\dfrac{\partial y}{\partial t} = 0$	$\dfrac{\partial y}{\partial x} = 0$
$P_{Node} = 0$	$P_{Antinode} = 0$

Normal Modes String Fixed at Both Ends



$\dfrac{\lambda}{2} = \ell$ $f_0 = \dfrac{v}{2\ell}$ Fundamental Frequency

$\dfrac{2\lambda}{2} = \ell$ $f = \dfrac{2v}{2\ell}$ 2nd Harmonic 1st Overtone

$\dfrac{3\lambda}{2} = \ell$ $f = \dfrac{3v}{2\ell}$ 3rd Harmonic 2nd Overtone

$\dfrac{n\lambda}{2} = \ell$ $f = \dfrac{nv}{2\ell}$ n^{th} Harmonic $(n-1)^{th}$ Overtone

(1) $\dfrac{n\lambda}{2} = \ell$

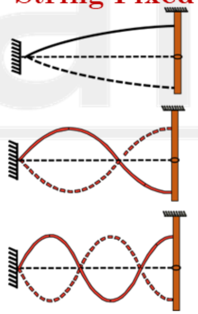
(2) $f = \dfrac{nv}{2\ell}$

(3) $f = \dfrac{n}{2\ell}\sqrt{\dfrac{T}{\mu}}$

(4) $f = n f_0$

For fundamental frequency, n = 1

String Fixed at One End & Free at Other



$\dfrac{\lambda}{4} = \ell$ $f = \dfrac{v}{4\ell}$ Fundamental Frequency

$\dfrac{3\lambda}{4} = \ell$ $f = \dfrac{3v}{4\ell}$ 3rd Harmonic 1st Overtone

$\dfrac{5\lambda}{4} = \ell$ $f = \dfrac{5v}{4\ell}$ 5th Harmonic 2nd Overtone

$\dfrac{(2n+1)\lambda}{4} = \ell$ $f = \dfrac{(2n+1)v}{4\ell}$ $(2n+1)$ Harmonic n^{th} Overtone

Power transmission through node & antinode is zero. So, energy of one loop (even half loop) remains conserved.