MIND MAP

WPE

Work done by force $\vec{F}$ on an object is defined as

$$W_F = \int \vec{F}.d\vec{s}$$

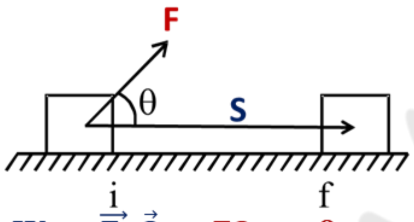
$\vec{F}$ is the force on object $d\vec{s}$ is displacement of point of application of force ($\vec{F}$)

SI unit of work is Joule (J).

Cgs unit is erg.

$\boxed{1 \text{ erg} = 10^{-7} \text{ joule}}$

If $\vec{F}$ is constant, then $W = \vec{F}.\vec{S}$



$W_F = \vec{F}.\vec{S} = FS\cos\theta$

θ is angle between $\vec{F}$ and $\vec{S}$

Work is a scalar quantity.

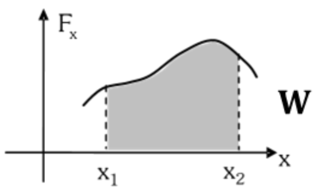
It can be –ve , zero or +ve.

$0° < \theta < 90° \Rightarrow W_F = +ve$

$\theta = 90° \Rightarrow W_F = 0$

$\theta > 90° \Rightarrow W_F = -ve$

Work as Area Under Curve



$$W = \int F_x dx$$

Work Done by Variable Force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable.

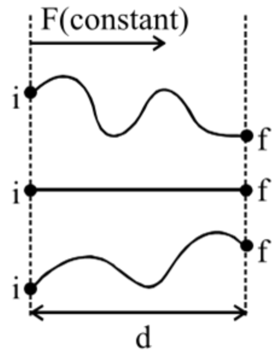
$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$W = \int F_x dx + Fydy + Fzdz$$

If particle goes from $(x_i\, y_i\, z_i)$ to $(x_f\, y_f\, z_f)$

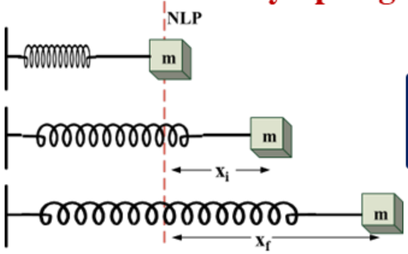
$$\boxed{W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz}$$



$W_1 = W_2 = W_2 = Fd$

Work done by constant force depends only on initial and final position and not on the path taken

Work Done by Spring Force



$$\boxed{W_{sp} = \frac{1}{2}k\left(x_i^2 - x_f^2\right)}$$

Where x_i and x_f are initial and final change in lengths from Natural Length of spring

MIND MAP

WPE

Work Energy Theorem

Work done by all the forces (external or internal) acting on a system is equal to the change in kinetic energy of the system.

$W_{net} = \Delta KE = KE_f - KE_i$

KE of a particle $= \frac{1}{2}mv^2 = \frac{p^2}{2m}$

where p is momentum of particle

$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

- Work energy theorem is valid in inertial frame.
- But we can use its equation in non-inerital frame by considering work of Psuedo Force.

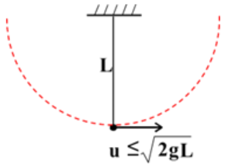
Work Energy Theorem For System of Particle

$$\boxed{(W_{net})_{sys} = (KE_f)_{sys} - (KE_i)_{sys}}$$

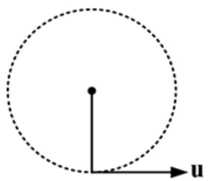
- $(W_{net})_{sys} = \sum W_F$ on system
- W_F on system = $\sum W_F$ on each part of system
- KE of system = $\sum$ KE of each particle of system.

- Work done by an Internal Force on the system is independent of the reference frame
- If distance between particles in a system along internal force remains unchanged always then work done by internal force on the system is 'zero'.

Vertical Circular Motion



$u \le \sqrt{2gl}$ (circular motion but not complete)



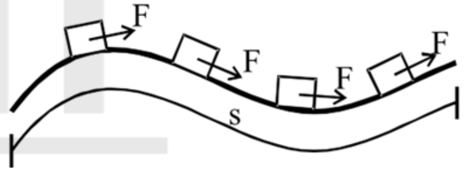
$u \ge \sqrt{5gl}$ (complete vertical circular motion)

Central Force

Force on an object whose magnitude only depends on the distance `r` of the object from a fixed point and is directed along the line joining object and the fixed point is referred as Central Force.

$$\boxed{\vec{F} = f(r)\hat{r}}$$

Work Done By Tangential Force



$W_F = \int \vec{F}.d\vec{s} = \int F\, ds \cos 0^\circ$

If $\vec{F}$ is constant in magnitude

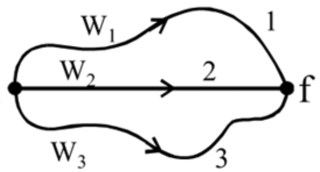
$W_F = F\int ds = Fs$ → Length of curve

Conservative Force

A Force is a Conservative Force when work done by it in any closed path (Loop) is zero.

$W_F = 0 \Rightarrow$ F is conservative

Work done by conservative force is independent of path. It depends only on initial & final position.

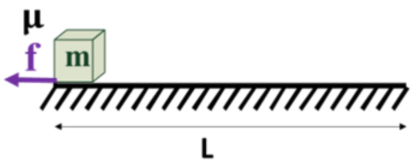


$W_1 = W_2 = W_3$ by F

MIND MAP

WPE

- Constant Forces are conservative.
- Central forces are conservative.



$W_{fric} = -\mu mgL$

Kinetic friction is an example of non-conservative force.

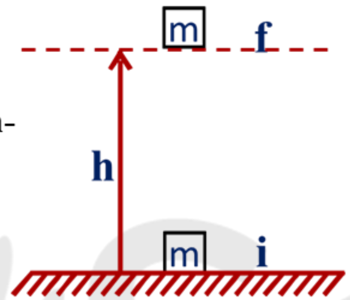
Potential Energy (U)

$\Delta U = -W_{int,C}$

$W_{int,C}$ is work done by internal conservative force

- In any system, change in potential energy is equal to the negative of work done by internal conservative force.
- PE is defined only for conservative forces.

Gravitational Potential Energy



$U_f = U_i + mgh$

If we take U at ground = 0 as reference value then

$U = mgh$

$W_g = -\Delta U = U_i - U_f$

$= mgh_i - mgh_f$

$= mg(h_i - h_f)$

Potential energy in terms of Force (conservative)

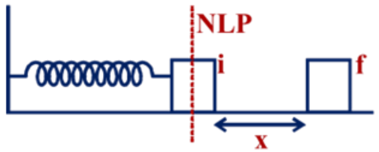
$$U_f - U_i = -\left(\int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz\right)$$

Mechanical Energy & its Conservation

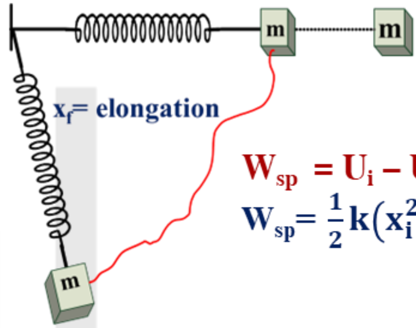
Mechanical Energy = Potential Energy + Kinetic energy

ME = U + KE

Potential energy associated with Spring Force



$U_f - U_i = \frac{1}{2} K x^2$



$W_{sp} = U_i - U_f$

$W_{sp} = \frac{1}{2} k\left(x_i^2 - x_f^2\right)$

Force in terms of potential energy

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

MIND MAP

WPE

A body is said to be in equilibrium when

$F_{net} = 0$

Stable Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position a force acts on particle which try to bring the particle back to equilibrium position.
- Such force is restoring and opposite to displacement.

Unstable Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position a force acts on particle which tries to take the particle away from the equilibrium position.

Neutral Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position the particle remains in equilibrium position

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = +ve$ Stable Equilibrium

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = -ve$ Unstable Equilibrium

Power

Power of a force is equal to rate of work done by that force.

$$P = \frac{dW}{dt}$$

$$P_{avg} = \langle P \rangle = \frac{\Delta W}{\Delta t}$$

S.I. unit of power is Watt (W)

Other unit is Horse Power (HP)

1 HP = 746 W.

$P = \frac{dW}{dt}$ Slope of W-t graph gives instantaneous Power

$W = \int dW = \int Pdt$

W is given by area under P-t graph