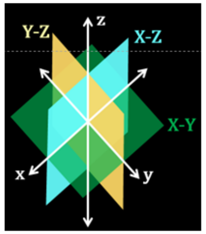
MIND MAP

3 D Geometry

1. Coordinates of a Point in Space

Octants & sign of the coordinates



Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

2. Distance Formula

Dis. b/w 2 pts. $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$

$$\boxed{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

3. Section Formula

❑ Internal Division

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

❑ External Division

$$\left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}, \frac{m_1z_2-m_2z_1}{m_1-m_2}\right)$$

❑ Mid Point

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

3. Centres of a Triangle

ΔABC

$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$

❑ Centroid :

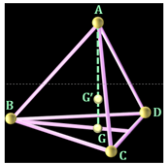
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

❑ Incentre :

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$

❑ Centroid of n sided polygon :

$$(x,y,z) = \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n}, \frac{\sum_{i=1}^{n} z_i}{n}\right)$$



❑ Centroid/Mean Centre of Gravity : Tetrahedron

$$\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$

4. Direction Cosines of Vector/Line

$$\vec{a} = a_1 i + a_2 j + a_3 k$$

$$\cos\alpha = \frac{a_1}{|\vec{a}|} = \ell \quad \cos\beta = \frac{a_2}{|\vec{a}|} = m \quad \cos\gamma = \frac{a_3}{|\vec{a}|} = n$$

$$\hat{a} = \ell\hat{i} + m\hat{j} + n\hat{k}$$

$$\ell^2 + m^2 + n^2 = 1$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

5. Dir. cosines of the line joining 2 given pts.

$$\frac{x_2-x_1}{\ell} = \frac{y_2-y_1}{m} = \frac{z_2-z_1}{n} = r$$

if ℓ, m & n are dir. cosines of line, then $-\ell, -m$ & $-n$ are also dir. cosines of the same line.

6. Direction Ratios of a Line

Relationship b/w DCs & DRs

$$\ell = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}; m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}; n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Angle b/w 2 lines with dir. cosines ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 is

$$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2 = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \times \sqrt{a_2^2+b_2^2+c_2^2}}$$

7. The Plane

Eq. of Plane in 3D :

$ax + by + cz + d = 0$

(i) Eq. of y-z plane : $x = 0$

(ii) Eq. of z-x plane : $y = 0$

(iii) Eq. of x-y plane : $z = 0$

8. Eq. of a plane passing through a fixed pt.

Vector Form: $(\vec{r} - \vec{a}).\vec{n} = 0$

Cartesian Form: $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

9. Eq. of a plane passing through 3 given pts.

$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$A(\vec{a})$, $B(\vec{b})$, $C(\vec{c})$

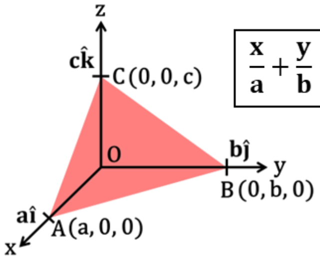
$$[\vec{r}-\vec{a} \quad \vec{r}-\vec{b} \quad \vec{r}-\vec{c}] = 0$$

$$[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}] = 0$$

MIND MAP

3 D Geometry

10. Intercept Form of the Plane



$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

11. Normal Form of the Plane

Vector Form: $\boxed{\vec{r}.\hat{n}=d}$ $(d>0)$

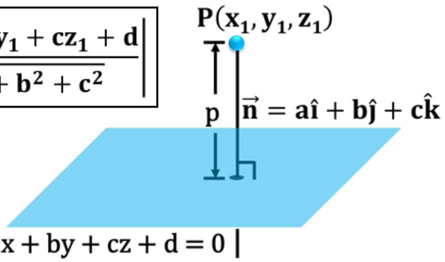
Cartesian Form: $\boxed{\ell x+my+nz=d}$

12. Parametric Equation of Plane

$\vec{r}=\vec{a}+\lambda\vec{p}+\mu\vec{q}$ $\quad [\vec{r}-\vec{a}\quad \vec{p}\quad \vec{q}]=0$

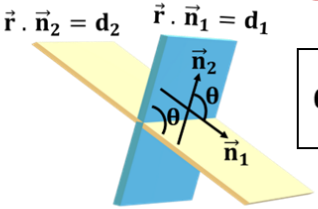
13. Perpendicular Dis. of a pt. from a Plane

$$\boxed{\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|}$$



If plane is $\vec{r}.\vec{n}=d$ & point is $\vec{a}$, then $\boxed{\left|\frac{\vec{a}.\vec{n}-d}{|\vec{n}|}\right|}$

14. Angle Between Two Planes



$$\boxed{\theta=\cos^{-1}\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}}$$

If planes are $a_1x+b_1y+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

(i) Parallel & distinct if, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\neq\frac{d_1}{d_2}$

(ii) Perpendicular if, $a_1a_2+b_1b_2+c_1c_2=0$

(iii) Identical if, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$

15. Eq. of a Plane Parallel to a given plane

Cartesian Form: Plane : $ax+by+cz+d=0$

Parallel Plane : $ax+by+cz+K=0$

Vector Form: Plane : $\vec{r}\cdot\vec{n}_1=d_1$

Parallel Plane : $\vec{r}\cdot\vec{n}_1=d_2$

16. Distance between two Parallel Planes

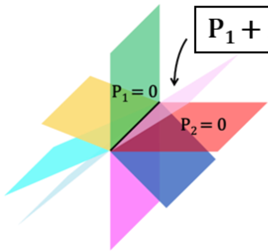
$P_1 : a_1x+b_1y+c_1z+d_1=0$
& $P_2 : a_2x+b_2y+c_2z+d_2=0$

$$\boxed{\text{Distance}=\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|}$$

17. Eq. of the Bisector Planes b/w the Planes

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}=\pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

18. Family of Planes



$\boxed{P_1+\lambda P_2=0}$ $\forall\lambda\in R$

3 D Geometry

19. Two Sides of a Plane

Points : $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$

Plane : $ax+by+cz+d=0$

$ax_1+by_1+cz_1+d$
& $ax_2+by_2+cz_2+d$

Same signs	Opposite signs
Pts. lie on the same side	Pts. lie on the opposite sides

1. Equation of the Straight Line

Symmetrical form/Vector form:

Point $A(\vec{a})$; Parallel to vector $\vec{b}$

$$\vec{r} - \vec{a} = \lambda\vec{b}$$

$$\vec{r} = \vec{a} + \lambda\vec{b}, \quad \lambda \in R$$

Point $A(x_1, y_1, z_1)$; Dir. Ratios a, b, c

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$$

$$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = \lambda$$

2. Eq. of a Line passing through 2 pts.

Cartesian form:

Points : $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$

Dir. Ratios : $x_2 - x_1$, $y_2 - y_1$ & $z_2 - z_1$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

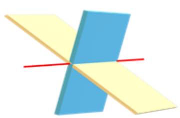
Vector form:

Points : $A(\vec{a})$ & $B(\vec{b})$

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}), \quad \lambda \in R$$

3. General Form

A line is obtained by the intersection of 2 planes.

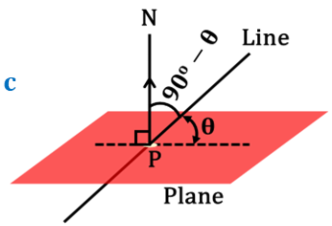


4. Angle b/w a Line & a Plane

Cartesian Form:

Plane : $ax + by + cz + d = 0$

Dir. Cosines : ℓ, m, n



$$\cos(90^\circ - \theta) = \frac{a\ell + bm + cn}{\sqrt{a^2 + b^2 + c^2}}$$

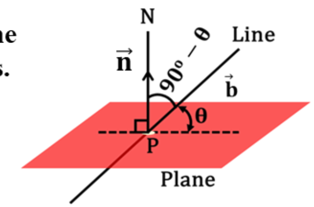
Conditions for :

1) Line || to plane:

$$\ell a + mb + nc = 0$$

2) Line $\perp$ to plane: $\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$

5. Angle b/w a Line and a Plane



Vector Form:

Line: $\vec{r} = \vec{a} + \vec{b}t$

Plane: $\vec{r} \cdot \vec{n} = q$,

$$\theta = \sin^{-1}\left(\frac{\vec{n} \cdot \vec{b}}{|\vec{n}||\vec{b}|}\right)$$

6. Condition for a Line in a Plane

Line: $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$

Plane : $ax + by + cz + d = 0$

Conditions : $a\ell + bm + cn = 0$

$$ax_1 + by_1 + cz_1 + d = 0$$

7. Gen. Eq. of Plane containing a Line

Plane : $ax + by + cz + d = 0$

Line Contained : $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

8. Eq. of Plane containing a given line & el to another given line

Containing $\frac{x - x_1}{\ell_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ $\quad \vec{r} = \vec{a} + \lambda\vec{b}$

ll to $\frac{x - x_2}{\ell_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ $\quad \vec{r} = \vec{c} + \mu\vec{d}$

Eqn. of plane: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$

$$[\vec{r} - \vec{a} \;\; \vec{b} \;\; \vec{d}] = 0$$

9. Condition for two lines to be intersecting OR to be coplanar

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$[\vec{c} - \vec{a} \;\; \vec{b} \;\; \vec{d}] = 0$$

10. Shortest Dis. b/w Two Lines

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} \div \sqrt{\sum (m_1 n_2 - m_2 n_1)^2}$$

11. Unsymmetrical Form of St. Line

$$a_1 x + b_1 y + c_1 z + d_1 = 0$$

$$\& \; a_2 x + b_2 y + c_2 z + d_2 = 0$$

together represents the line

12. Line of Greatest Slope in a Plane