MIND MAP

Application of Derivative

1. Tangent and Normal To The Curve at a Point

Tangent

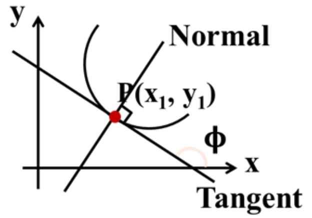
Slope of tangent at P : $m_T = \left(\frac{dy}{dx}\right)_P = \tan\Phi$

Where, P = point of contact

Φ = angle made by tangent with x−axis.

Eq. of the tangent at P (x_1, y_1) :

$$y - y_1 = \left[\frac{dy}{dx}\right]_{(x_1,y_1)}(x - x_1)$$



Normal

Slope of tangent at P : $m_T = \left(\frac{dy}{dx}\right)_P$

Slope of normal at P : $m_N = -\left(\frac{dx}{dy}\right)_P$

Eq. of normal at P (x_1, y_1)

$$= (y - y_1) = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x - x_1),$$

where, P = foot of normal

Important Points To Remember

(a) (i) If $\left[\frac{dy}{dx}\right]_{P(x_1,y_1)} = 0 \Rightarrow (m = 0)$

$\Rightarrow$ Tangent at P is parallel to x−axis & converse.

(ii) If $\left[\frac{dy}{dx}\right]_{P(x_1,y_1)} \to \infty$ $\Rightarrow$ Tangent at P is parallel to y−axis.

(b) (i) If tangent is || to $ax + by + c = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{a}{b} \Rightarrow m_1 = m_2$

(ii) If tangent is $\perp$ to $ax + by + c = 0$

$\Rightarrow \left[\frac{dy}{dx}\right]_{P(x_1,y_1)} \cdot \left(-\frac{a}{b}\right) = -1 \Rightarrow m_1 . m_2 = -1$

(c) If the tangent at P (x_1, y_1) on the curve is equally inclined to the coordinate axes then,

$\Rightarrow \left[\frac{dy}{dx}\right]_{P(x_1,y_1)} = \pm 1 \Rightarrow m = \pm 1$

(d) If the tangent at P (x_1, y_1) makes equal non zero intercepts on the coordinate axes .

Then, $\Rightarrow \left[\frac{dy}{dx}\right]_{P(x_1,y_1)} = -1 \Rightarrow m = -1$

(e) If the tangent at P (x_1, y_1) cuts equal distances on the coordinate axes from origin then,

$\Rightarrow \left[\frac{dy}{dx}\right]_{P(x_1,y_1)} = \pm 1 \Rightarrow m = \pm 1$

2. Tangent & Normal in Parametric Form

If the equation of the curve is represented parametrically i.e. $x = f(t)$ & $y = g(t)$

Where, $\frac{dx}{dt} = f'(t)$ & $\frac{dy}{dt} = g'(t)$, $\{f'(t) \neq 0\}$

Then, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$

Common Parametric Coordinates on a curve

(a) $x^2 + y^2 = r^2$ $\quad x = r\cos\theta, y = r\sin\theta$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad x = a\cos\theta, y = b\sin\theta$

(c) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\quad x = a\sec\theta, y = b\tan\theta$

(d) $y^2 = 4ax$ $\quad x = at^2, y = 2at$

(e) $x^{2/3} + y^{2/3} = a^{2/3}$ $\quad x = a\cos^3\theta, y = a\sin^3\theta$

(f) $\sqrt{x} + \sqrt{y} = \sqrt{a}$ $\quad x = a\cos^4\theta, y = a\sin^4\theta$

MIND MAP

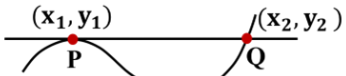
Application of Derivative

3. Eq. of Tangent/Normal in Some Special Cases

(a) If the tangent at P is meeting the curve $y = f(x)$ again at Q.

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(P)} = \frac{y_2 - y_1}{x_2 - x_1} \text{ ... (i) } \& \; y_1 = f(x_1) \text{ ...(ii)}$$

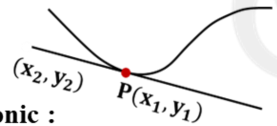
From (i) & (ii) We get the eq. of Tangent.



(b) If from an external point tangent is drawn to the curve. $y = f(x)$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(P)} = \frac{y_2 - y_1}{x_2 - x_1} \text{ ... (i) } \& \; y_1 = f(x_1) \text{ ...(ii)}$$

From (i) & (ii) We get the eq. of Tangent.



Note

(i) Equation of any conic :

$S = ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

Point of contact : $P(x_1, y_1)$

Then, eq. of tangent is obtained by replacing

$x^2 \to xx_1$, $y^2 \to yy_1$, $2x \to x + x_1$, $2y \to y + y_1$, $2xy \to xy_1 + x_1y$, $c \to c$

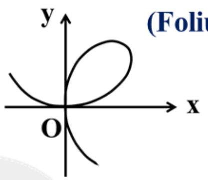
(ii) Curve : $\frac{x^n}{a^n} + \frac{y^n}{b^n} = 1$

Point of contact : $P(x_1, y_1)$

Eq. of Tangent : $\frac{x.x_1^{n-1}}{a^n} + \frac{y.y_1^{n-1}}{b^n} = 1$

(iii) Same line could be the tangent as well as normal to a given curve at a given point.

Example (Folium of Descartes)

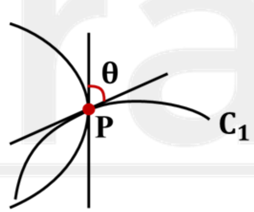


4. Angle of Intersection of Two Curves

Let, $C_1 = f(x)$ $C_2 = g(x)$

$\Rightarrow m_1 = f'(x) \Rightarrow m_2 = g'(x)$

Then, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$



Orthogonal Curves If the angle of intersection of two curves at every point of intersection is 90°, then curves are said to be orthogonal.

If the curves are orthogonal then,

$$\left(\frac{dy_1}{dx}\right)\left(\frac{dy_2}{dx}\right) = -1$$

(everywhere wherever they intersect)

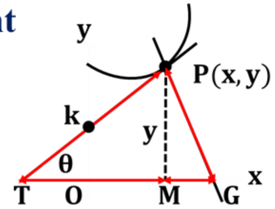
Note If the curves touch at P then $\theta = 0$

hence, $f'(x_1) = g'(x_1)$. $(m_1 = m_2)$

5. Length of Tangent, Normal, Subtangent, and Subnormal

Length of Tangent

$$\left|\frac{y\sqrt{1 + \left(dy/dx\right)^2}}{dy/dx}\right|$$



The Length of portion of tangent intercepted between the point of contact & x-axis.

Length of Subtangent The projection of length of tangent on x-axis $\left|\frac{y}{dy/dx}\right|$

Length of Normal The length of portion of Normal intercepted between foot of normal & x- axis. $y\sqrt{1 + \tan^2\theta}$

Length of Subnormal The projection of length of Normal on x-axis is known as Length of Sub Normal. $\left|y\left(\frac{dy}{dx}\right)\right|$

5. Approximation

$\Delta y \to$ Approximate change in value of y corresponding to Δx change in value of x .

$\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + f'(x)\,\Delta x$

MIND MAP

Application of Derivative

1. Increasing & Decreasing Nature of A Function at a Point

Increasing at x = a

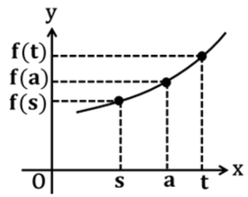
If, $f(s) \le f(a) \le f(t)$ whenever $s < a < t$,

Where,

$s, t \in (a - h, a + h) \cap D_f$

for some $h > 0$,

Then, f is said to be increasing at x = a.

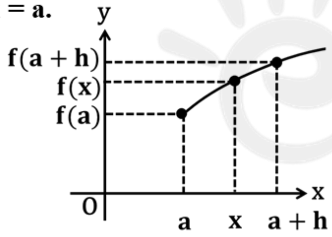


(1) When 'a' be left end of the interval

$f(a) \le f(x)\ \forall\ x \in (a, a + h) \cap D_f$

for some $h > 0$

$\Rightarrow$ f is increasing at x = a.

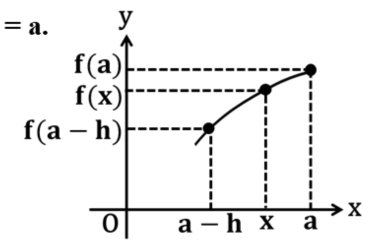


(2) When 'a' be right end of the interval

$f(x) \le f(a)\ \forall\ x \in (a - h, a) \cap D_f$

for some $h > 0$

$\Rightarrow$ f is increasing at x = a.



2. Strictly Increasing at x = a

If, $f(s) < f(a) < f(t)$

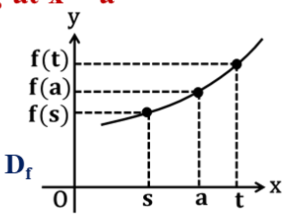
whenever $s < a < t$,

Where,

$s, t \in (a - h, a + h) \cap D_f$

for some $h > 0$,

Then, f is strictly increasing at x = a.



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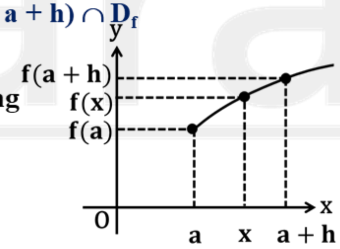
$\Rightarrow$ f is strictly increasing at x = a.

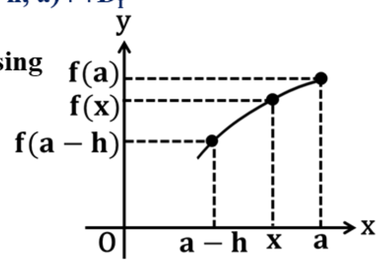


(2) When 'a' be right end of the interval

$f(x) < f(a)\ \forall\ x \in (a - h, a) \cap D_f$

for some $h > 0$

$\Rightarrow$ f is strictly increasing at x = a.



3. Decreasing at x = a

If, $f(s) \ge f(a) \ge f(t)$

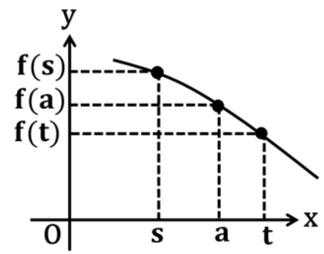
whenever $s < a < t$,

Where,

$s, t \in (a - h, a + h) \cap D_f$

for some $h > 0$,

Then, f is said to be decreasing at x = a.

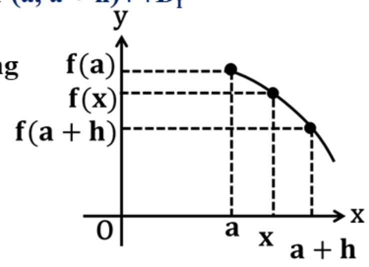


(1) When 'a' be left end of the interval

$f(a) \ge f(x)\ \forall\ x \in (a, a + h) \cap D_f$

for some $h > 0$

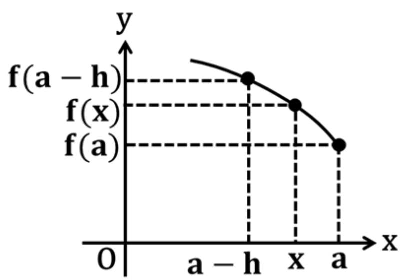
$\Rightarrow$ f is decreasing at x = a.



(2) When 'a' be right end of the interval

$f(x) \ge f(a)\ \forall\ x \in (a - h, a) \cap D_f$

for some $h > 0$ $\Rightarrow$ f is decreasing at x = a.



MIND MAP

Application of Derivative

4. Strictly Decreasing at x = a

If, $f(s) > f(a) > f(t)$

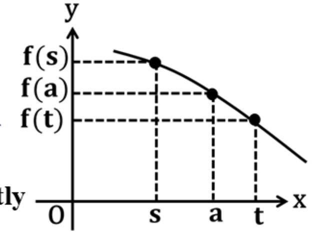
whenever $s < a < t$,

Where,

$s, t \in (a - h, a + h) \cap D_f$

for some $h > 0$,

Then, f is said to be strictly decreasing at x = a.

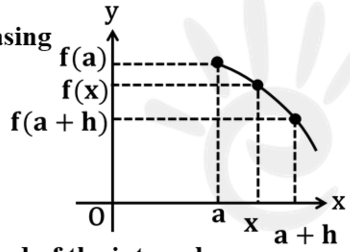


(1) When 'a' be left end of the interval

$f(a) > f(x)\ \forall\ x \in (a, a + h) \cap D_f$

for some $h > 0$

$\Rightarrow$ f is strictly decreasing at x = a.

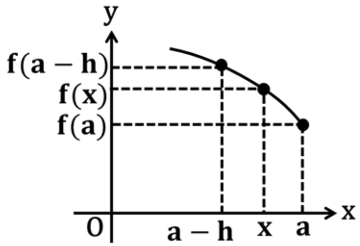


(2) When 'a' be right end of the interval

$f(x) > f(a)\ \forall\ x \in (a - h, a) \cap D_f$

for some $h > 0$

$\Rightarrow$ f is strictly decreasing at x = a.



5. Increasing & Decreasing Nature of a Function over an Interval

Consider an Interval $I \subseteq D_f$

(A) Increasing Over an Interval I :

If, $\forall\ x_1, x_2 \in I \quad x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$

Then f is increasing over the interval I.

(B) Decreasing Over an Interval I :

If, $\forall\ x_1, x_2 \in I \quad x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$

Then f is decreasing over the interval I.

(C) Strictly Increasing Over an Interval I :

If, $\forall\ x_1, x_2 \in I \quad x_1 < x_2 \Leftrightarrow f(x_1) < f(x_2)$

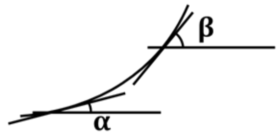
Then f is strictly increasing over the interval I.

(D) Strictly Decreasing Over an Interval I :

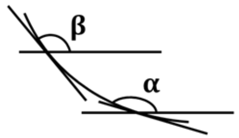
If, $\forall\ x_1, x_2 \in I \quad x_1 < x_2 \Leftrightarrow f(x_1) > f(x_2)$

Then f is strictly decreasing over the interval I.

6. Monotonicity For Differentiable Functions:



i.e . $\alpha, \beta \in$ acute angle
slope of tangents > 0
$f'(x) > 0$



i.e . $\alpha, \beta \in$ obtuse angle
slope of tangents < 0
$f'(x) < 0$

Consider an interval $I\ (\subseteq Df)$ That can be, [a, b] or (a, b) or [a, b) or (a, b]

(1) $f'(x) > 0\ \forall\ x \in I$
$\Rightarrow$ f is strictly increasing function over the interval I.

(2) $f'(x) \ge 0\ \forall\ x \in I$
$\Rightarrow$ f is increasing function over the interval I.

(3) $f'(x) \ge 0\ \forall\ x \in I$
& $f'(x) = 0$ at discrete points.
(Do not form any interval)
$\Rightarrow$ f is strictly increasing function over the interval I.

(4) $f'(x) < 0\ \forall\ x \in I$
$\Rightarrow$ f is strictly decreasing function over the interval I.

(5) $f'(x) \le 0\ \forall\ x \in I$
$\Rightarrow$ f is decreasing function over the interval I.

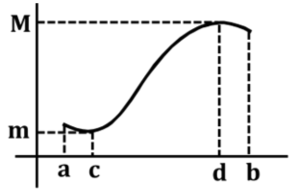
(6) $f'(x) \le 0\ \forall\ x \in I$
& $f'(x) = 0$ at discrete points.
$\Rightarrow$ f is strictly decreasing function over the interval I.

7. Greatest and Least Value of a Function

(a) Extreme value Theorem

If f is continuous on [a, b] then 'f' takes a

least value m
and
a greatest value M
on this interval.



MIND MAP

Application of Derivative

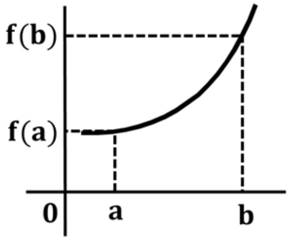
7. Greatest and Least Value of a Function

(b) If a continuous function $y = f(x)$ is increasing in the closed interval $[a, b]$, then,

$f(a)$ is the least value of $f(x)$

&

$f(b)$ is the greatest value of $f(x)$ in $[a,b]$.

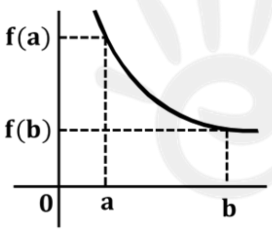


(c) If a continuous function $y = f(x)$ is decreasing in the closed interval $[a, b]$, then,

$f(b)$ is the least value of $f(x)$

&

$f(a)$ is the greatest value of $f(x)$ in $[a,b]$.



(d) If function is continuous & non-monotonic in $[a, b]$, then greatest & least value of function exist where (i) $f'(x)$ is zero.

(ii) $f'(x)$ does not exist.

(iii) At the end points of the interval.

(e) If a continuous function $y = f(x)$ has least value m and greatest value M over the interval $[a, b]$ then Range is $[m, M]$

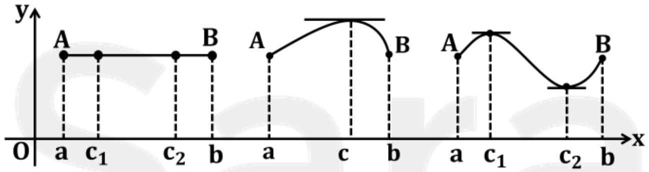
(f) In an open interval, a continuous function need not have either a maximum or a minimum value.

8. Rolle's Theorem

Let $f(x)$ be a function of x subject to the following conditions :

(i) $f(x)$ is a continuous function of x in $[a, b]$

(ii) $f(x)$ is derivable in (a, b) (iii) $f(a) = f(b)$



Then there exists at least one point $x = c$,

Such that, $a < c < b$ where $f'(c) = 0$

Converse of Rolle's theorem is NOT TRUE.

Note

- If $f(x)$ is differentiable function then between any two roots of the equation $f(x) = 0$ there will be at least one root of the equation $f'(x) = 0$.
- If f is a differentiable function in (a, b) and continuous in $[a, b]$ such that it has exactly n roots in $[a, b]$, then f' will have $(n - 1)$ roots $\in (a, b)$.
- If $f'(x)$ has exactly m roots in (a, b), then $f(x)$ will have at most $(m + 1)$ roots in $[a, b]$

9. Lagrange's Mean Value Theorem

LMVT Theorem

Function : $f(x)$

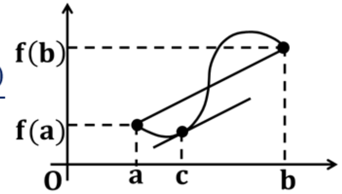
Continuous in $[a, b]$ — Derivable in (a, b)

Then, there exists at least one point $x = c$

Where,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$(a < c < b)$



Geometrical Interpretation

Slope of Secant joining the curve at $x = a$ & $x = b$ is equal to the

Slope of Tangent drawn to the curve at $x = c$

10. Significance of The Sign of 2nd Order Derivative

The sign of the 2nd order derivative determines the concavity of the curve.

i) If $f''(x) \geq 0 \ \forall \ x \in (a, b)$

Graph of $f(x)$ is concave upward in (a, b)

Tangent lies below the graph

ii) If $f''(x) \leq 0 \ \forall \ x \in (a, b)$

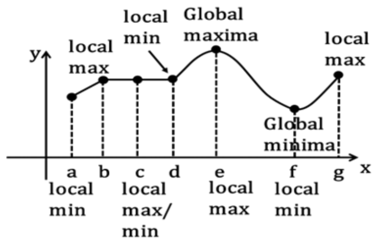
Graph of $f(x)$ is concave downward in (a, b)

Tangent lies above the graph

MIND MAP

Application of Derivative

11. Maxima & Minima



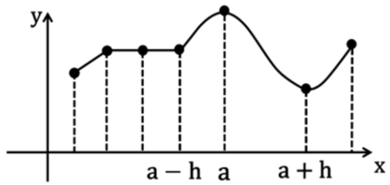
12. Local Maxima/Relative Maxima

A function f(x) is said to have a local maxima at x = a if :

$f(a) \geq f(x) \ \ \forall, x \in (a-h, a+h) \cap D_{f(x)}$

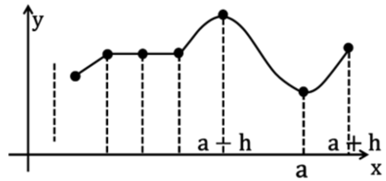
Where,

h is some very small (+ve) real number.



A function f(x) is said to have a local minima at x = a if :

$f(a) \leq f(x) \forall, x \in (a-h, a+h) \cap D_{f(x)}$

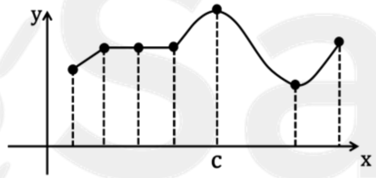


13. Absolute Maxima (Global Maxima)

A function " f " has an absolute maxima (or global maxima) at c if :

$f(c) \geq f(x) \quad \forall, x \in D_{f(x)}$

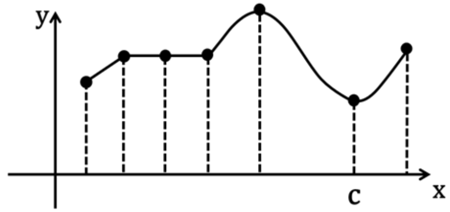
The number f (c) is called the maximum value of (f) in D.



A function " f " has an absolute minima (or global minima) at c if :

$f(c) \leq f(x) \forall, x \in D_{f(x)}$

The number f (c) is called the minimum value of (f) in D.

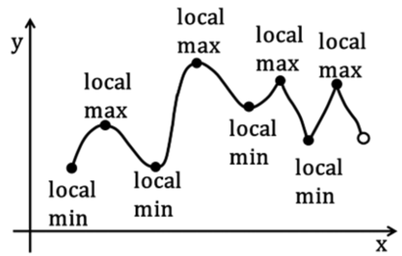


Note :

(i) The term 'extrema' is used both for maxima or minima.

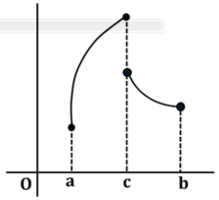
(ii) A local maximum (minimum) value of a function may not be the greatest (least) value in a finite interval.

(iii) A function can have several extreme values such that local min. value may be > than a local max. value.



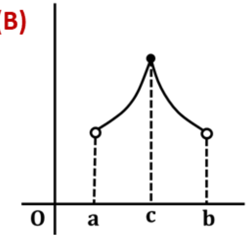
Analyse the various graphs for points of extrema:

(A)



1) Absolute maxima at x = c
2) Absolute minima at x = a
3) Local maxima at x = c
4) Local minima at x = a

(B)



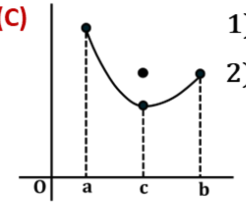
1) Absolute maxima at x = c
2) Absolute minima at x = No absolute minima
3) Local maxima at x = c

MIND MAP

Application of Derivative

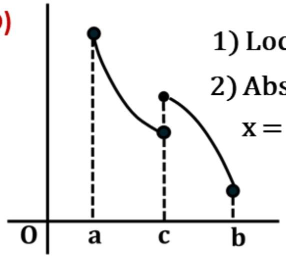
Analyse the various graphs for points of extrema:

(C)



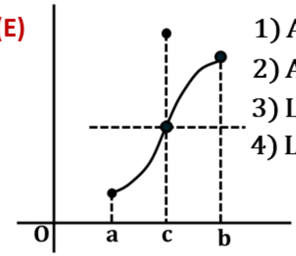
1) Local maxima at $x = c$
2) Absolute max. / min. at
$x =$ No absolute max. / min.

(D)



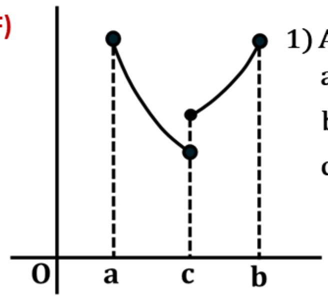
1) Local maxima at $x = c$
2) Absolute max. / min. at
$x =$ No absolute max. / min.

(E)



1) Absolute maxima at $x = c$
2) Absolute minima at $x = a$
3) Local maxima at $x = c$
4) Local minima at $x = a$

(F)



1) At $x = c$:
a) Maxima
b) Minima
c) None of these ✓

14. Derivative Test For Ascertaining Maxima /Minima

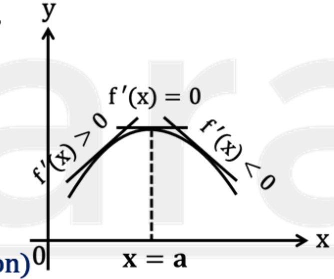
First Derivative Test

(For differentiable function)

The point (say $x = a$) where $f'(x) = 0$

- If $f'(x)$ changes sign from $(+) \rightarrow (-)$ in the neighborhood of $x = a$,

then,
$x = a$ is said to be a point of local maxima.



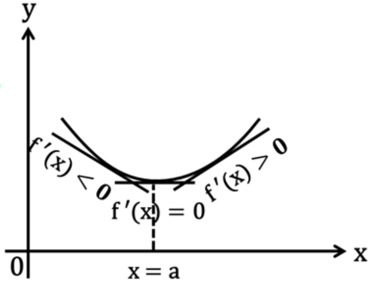
15. First Derivative Test

(For differentiable function)

The point (say $x = a$) where $f'(x) = 0$

- If $f'(x)$ changes sign from $(-) \rightarrow (+)$ in the neighborhood of $x = a$,

then,
$x = a$ is said to be a point of local minima



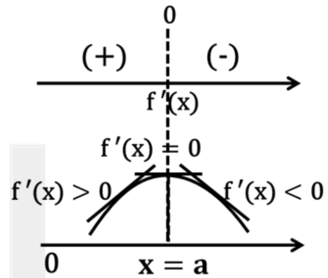
16. Second Derivative Test

If $f(x)$ is continuous & differentiable at $x = a$

where, $f'(a) = 0$ and $f''(a)$ also exists

Then, for ascertaining maxima/minima at $x = a$

(i) If $f''(a) < 0$ $\Rightarrow x = a$ is a point of local maxima

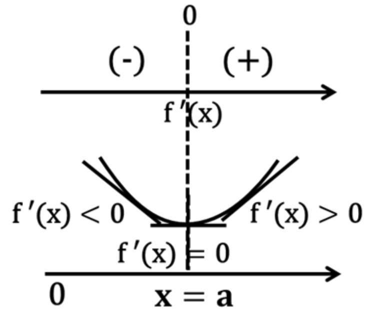


If $f(x)$ is continuous & differentiable at $x = a$

where, $f'(a) = 0$ and $f''(a)$ also exists

Then, for ascertaining maxima/minima at $x = a$

(ii) If $f''(a) > 0$ $\Rightarrow x = a$ is a point of local minima



16. Second Derivative Test

If $f(x)$ is continuous & differentiable at $x = a$

where, $f'(a) = 0$ and $f''(a)$ also exists

Then, for ascertaining maxima/minima at $x = a$

(iii) If $f''(a) = 0$ $\Rightarrow$ Second derivative test fails

To identify maxima/minima at this point either first derivative test or higher derivative test can be used.

17. n[th] Derivative Test

Let $f(x)$ be a function such that :

$f'(a) = f''(a) = f'''(a) = \ldots\ldots = f^{n-1}(a) = 0$

& $f^n(a) \neq 0$

Then, ❑ If n is even :

$$\Rightarrow \begin{cases} f^n(a) > 0 \Rightarrow \text{Minima} \\ f^n(a) < 0 \Rightarrow \text{Maxima} \end{cases}$$

Then, ❑ If n is odd :

Then neither maxima nor minima is obtained at $x = a$.

18. Special Point on A Given Curve

Stationary Points:

The stationary points are the points of domain where $f'(x) = 0$.

MIND MAP

Application of Derivative

Critical Points:

There are three kinds of critical points :

(i) The point at which $f'(x) = 0$

(ii) The point at which $f'(x)$ does not exist.

(iii) The end points of interval. (if included)

These points belongs to domain of the function.

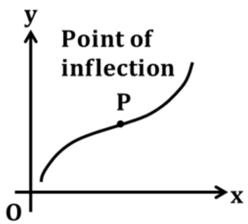
Note :

Local maxima & local minima occurs at critical points only.

But not all critical points will correspond to local maxima/local minima.

Point of Inflection:

- A point where the graph of a function has a tangent line and where the concavity changes is called a point of inflection.



- For finding point of inflection of any function, compute the solutions of

$\frac{d^2y}{dx^2} = 0$ or does not exist.

If $\frac{d^2y}{dx^2} = 0$ at $x = a$,

And sign of $\frac{d^2y}{dx^2}$ changes about this point then $x = a$ is called point of inflection.

- For finding point of inflection of any function, compute the solutions of

$\frac{d^2y}{dx^2} = 0$ or does not exist.

If $\frac{d^2y}{dx^2}$ does not exist at $x = a$,

And sign of $\frac{d^2y}{dx^2}$ changes about this point and tangent exist then $x = a$ is called point of inflection.

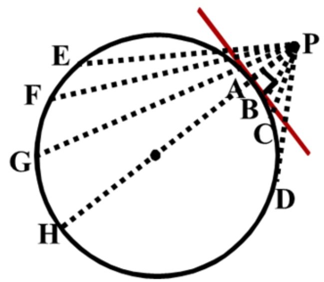
19. Optimization Problems

Maximum & Minimum Distance of a Fixed Point From a Given Curve

Therefore, Min. distance = PA

Max. distance = PH

PA & PH are Normal to the circle which passes through the centre of the circle.



MIND MAP

Application of Derivative

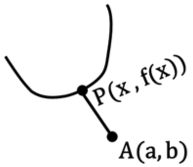
Maximum & Minimum Distance of a Fixed Point From a Given Curve

Given a curve : $y = f(x)$

A Fixed point : $A(a, b)$

Moving point on the curve : $P(x, f(x))$

AP will be maximum / minimum if it is normal to the curve at P.



Proof

$F(x) = (x-a)^2 + (f(x)-b)^2$

$\Rightarrow F'(x) = 2(x-a) + 2(f(x)-b) \cdot f'(x) = 0$

$\therefore f'(x) = -\left(\dfrac{x-a}{f(x)-b}\right).$

Also , $m_{AP} = \dfrac{f(x)-b}{x-a}$

Hence, $f'(x) \cdot m_{AP} = -1.$

Note

Greatest/ Least distance between two curves always occur along common normal.

20. Nature of Roots of Cubic Equation

$f(x) = ax^3 + bx^2 + cx + d$

Exactly 1 real root $f(\alpha) f(\beta) > 0$

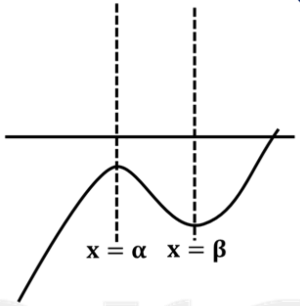
2 repeated & 1 distinct real roots $f(\alpha) f(\beta) = 0$

3 real & distinct roots $f(\alpha) f(\beta) < 0$

2 repeated & 1 distinct real roots $f(\alpha) f(\beta) = 0$

Exactly 1 real root $f(\alpha) f(\beta) > 0$

21. Real & Distinct Roots



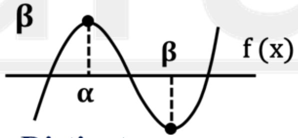
$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$D > 0$ → α, β

$f(\alpha) f(\beta) < 0$

$+ \quad \alpha \quad - \quad \beta \quad +$ $f'(x)$



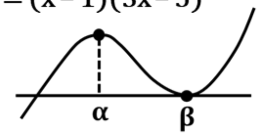
Two Repeated Root & One Distinct

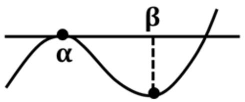
$f'(x) = 0 \qquad D > 0$

$f(\alpha) f(\beta) = 0$

$f(x) = (x-1)^2(x-2)$

$\Rightarrow f'(x) = (x-1)(3x-5)$





Three Repeated Roots

$f(x) = 0 \rightarrow$ cubic $\qquad \Rightarrow f'(x) = 0 \rightarrow$ quadratic

$D = 0 \rightarrow$ has root α. $\qquad \Rightarrow f(\alpha) = f'(\alpha) = f''(\alpha) = 0$

One Real & Two Imaginary Roots

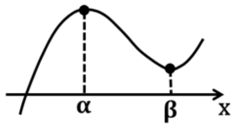
(i) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f(\alpha) f(\beta) > 0$

$D > 0$ → α, β

$+ \quad \alpha \quad - \quad \beta \quad +$



(ii) $f(x) = 0 \rightarrow$ cubic $\qquad \Rightarrow f'(x) = 0 \rightarrow$ quadratic

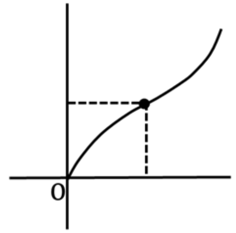
$D < 0 \qquad \therefore 1-$real $\qquad 2-$imaginary roots

$f(x) = x^3 - x^2 + x$

$\Rightarrow f'(x) = 3x^2 - 2x + 1$

$D < 0$

$f''(x) = 6x - 2$



(iii) $f(x) = 0 \rightarrow$ cubic

$\Rightarrow f'(x) = 0 \rightarrow$ quadratic $\qquad D = 0 \rightarrow$ has root α.

$f'(\alpha) = f''(\alpha) = 0$ but $f(\alpha) \neq 0$

Note

Graph of every cubic polynomial must have exactly one point of inflection.