MIND MAP

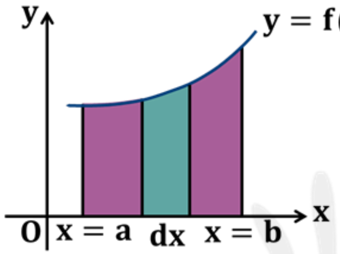
Area Under the Curve

1. Area Under the Curves

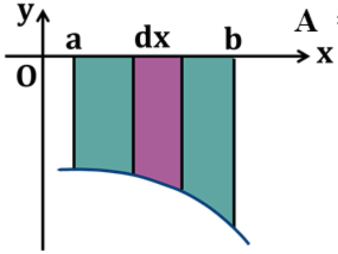
(1) Area bounded by the curve, the x – axis & the ordinates at (x = a, b) is given by :

$$A = \int_a^b y\,dx$$

Where, y = f (x) lies above the x–axis and (b > a)

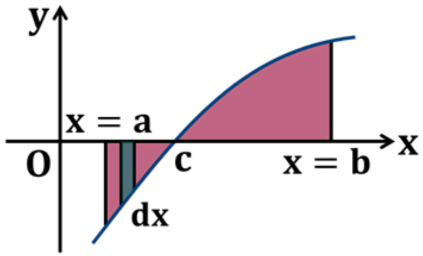


(2) If y = f (x) lies completely below the x–axis then,



$$A = \left| \int_a^b y\,dx \right|$$

(3) If curve crosses the x–axis at x = c then,



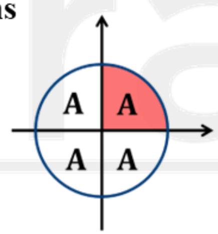
$$A = \left| \int_a^c y\,dx \right| + \int_c^b y\,dx$$

Note

If the curve be symmetric and suppose it has 'n' symmetric portions

Then,

Total Area = n (Area of symmetric portion)

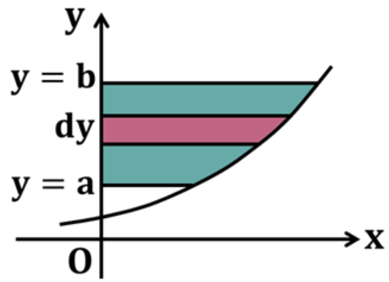


Total Area = 4A

(4) Area bounded by the curve, y–axis & the two abscissa at y = a & y = b is given by :

$$A = \int_a^b x\,dy \quad \text{(Integration w. r. t. 'y')}$$

$$= \int_a^b f(y)\,dy \quad \text{(dy : horizontal strip)}$$



2. Area Enclosed between two Curves

(1) Area bounded by two curves :

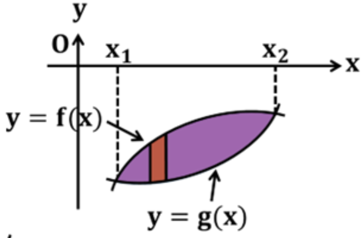
y = f (x) & y = g(x)

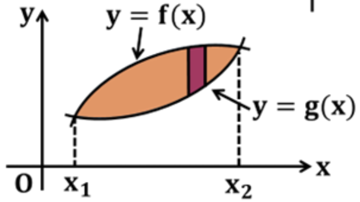
Such that, f (x) > g(x) is given by :

$$A = \int_{x_1}^{x_2} [f(x) - g(x)]\,dx$$

Where,

x_1 and x_2 are roots of equation f(x) = g(x)





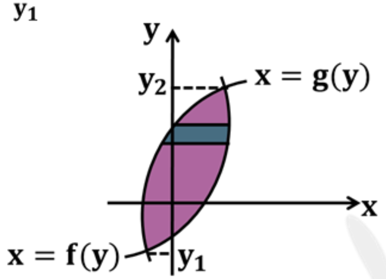
MIND MAP

Area Under the Curve

2. Area Enclosed between two Curves

(2) In case horizontal strip is taken
Then we have,

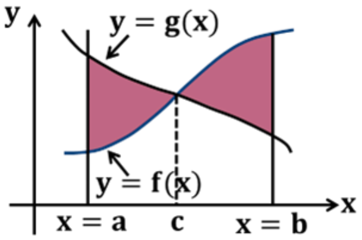
$$A = \int_{y_1}^{y_2} [f(y) - g(y)]dy$$



(3) If the curves y = f (x) and y = g(x) intersect at x = c, then required area is :

$$A = \int_a^c (g(x) - f(x))dx + \int_c^b (f(x) - g(x))dx$$

$$= \int_a^b |f(x) - g(x)|dx$$

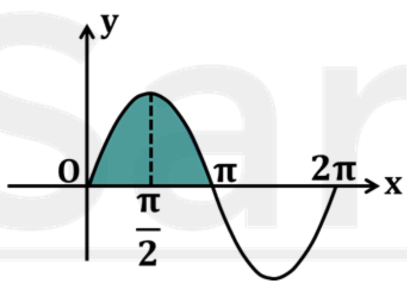


3. Standard Areas To Be Remembered

(1) Area bounded by :

y = sinx Where, $x \in [0, \pi]$

$$\therefore \text{Area} = \int_0^{\pi} \sin x \, dx = 2$$

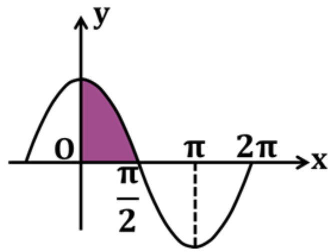


(2) Area bounded by :

y = cosx

Where, $x \in [0, \pi/2]$

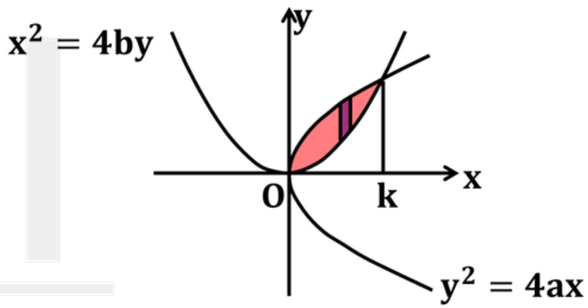
$$\therefore \text{Area} = \int_0^{\pi/2} \cos x \, dx = 1$$



(3) Area bounded by :

Parabolas $y^2 = 4ax$; $x^2 = 4by$

$(a > 0; b > 0)$ $A = \frac{16ab}{3}$

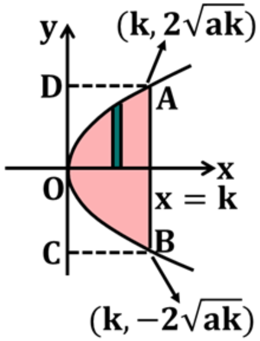


(4) Area enclosed by

Curve : $y^2 = 4ax$; its double ordinate at $x = k$

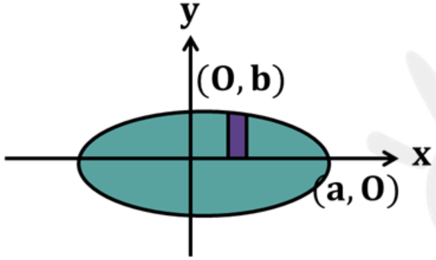
(chord perpendicular to the axis of symmetry)

Required Area $= \frac{2}{3}$(area of ABCD)



(5) whole area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Area $= \pi ab$



4. Finding Area by Shifting of Origin

Shifting of Origin

Since area remains invariant even if the coordinate axes are shifted.

Hence, shifting of origin in many cases prove to be very convenient in computing the areas.

MIND MAP

Area Under the Curve

area of $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

$\therefore$ Area $= \pi.(2).(3) = 6\pi$

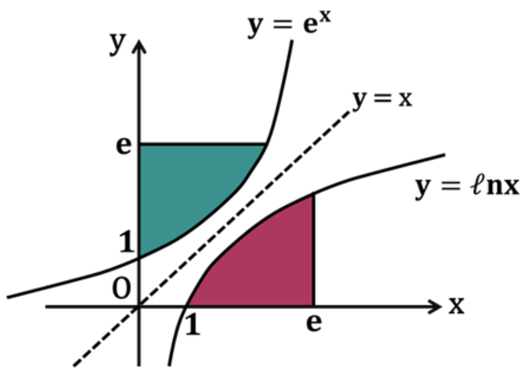
5. Area enclosed by inverse of a function

The area bounded by a curve (say y = f(x)) on x axis is equal to

The area bounded by the inverse of that curve $(f^{-1}(x))$ on y axis.

e.g.

A_1		A_2
Area bounded by $f(x) = \ell nx$ $(x = 1)$ & $(x = e)$ & x – axis	=	Area bounded by $f^{-1}(x) = e^x$ $(y = 1)$ & $(y = e)$ & y – axis



6. Variable Area Greatest & Least Value

Concept of Variable Area

(Greatest And Least Value)

$y = f(x) \longrightarrow$ A monotonic function in (a, b)

Then, the area bounded by :

ordinates at $x = a, x = b$

$y = f(x)$

$y = f(c)$, [where $c \in (a, b)$]

is minimum when $c = \frac{a+b}{2}$