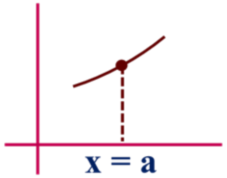
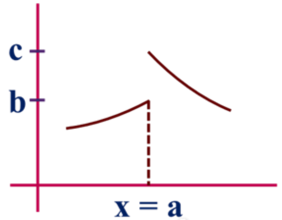
MIND MAP
Continuity

1. Concept of Continuity

The graph of a function is said to be continuous at $x = a$ if while travelling along the graph of the function through the point at $x = a$ either from left to right or from right to left, one does not have to lift his pen.





In case, if one has to lift his pen passing through point $x = a$, the graph of the function is said to have a break or discontinuous at $x = a$.

2. Formulative Definition of Continuity

A function $f(x)$ is said to be continuous at $x = a$,

If $\lim\limits_{x\to a} f(x)$ exists and is equal to $f(a)$.

If, $\lim\limits_{h\to 0} f(a-h) = \lim\limits_{h\to 0} f(a+h) = f(a)$

$= \text{finite quantity}$

$LHL|_{x=a} = RHL|_{x=a} = \text{value of } f(x)|_{x=a}$

$= \text{finite quantity}$

Continuity of a function is discussed

(i) At a point (ii) In its domain (iii) In an interval

Example $f(x) = \log x$

$f(x) = \log x$ is discontinuous at $x = 0$

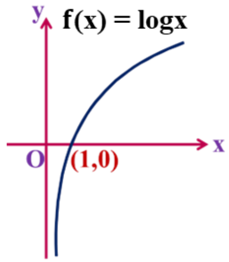
Continuous in the domain but not in $x \in R$.

Note: Continuity at $x = a$

$\Rightarrow$ Existence of limit at $x = a$

But, Existence of limit at $x = a$

$\not\Rightarrow$ Continuity at $x = a$



3. Reasons of Discontinuity

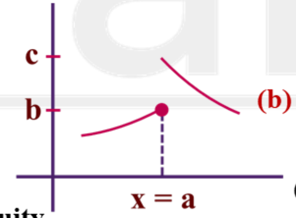
A function can be discontinuous due to the following reasons:

(i) $\lim\limits_{x\to a} f(x)$ does not exist

$\lim\limits_{h\to 0} f(a+h) \neq \lim\limits_{h\to 0} f(a-h)$

($f(a)$ may or may not be defined)

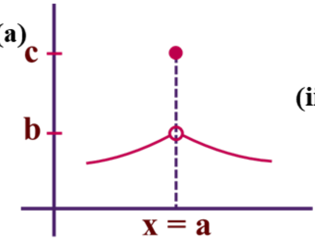
Called Non–removable discontinuity.



(ii) $\lim\limits_{x\to a} f(x)$ exist but is not equal to $f(a)$ (defined)

$\lim\limits_{h\to 0} f(a+h) = \lim\limits_{h\to 0} f(a-h) \neq f(a)$

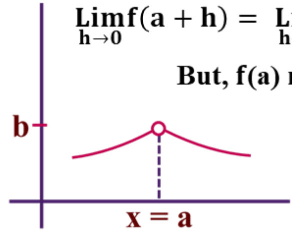
Called removable discontinuity (Isolated point).



(iii) Limit exist but $f(a)$ is not defined.

$\lim\limits_{h\to 0} f(a+h) = \lim\limits_{h\to 0} f(a-h)$

But, $f(a)$ not defined.



Removable discontinuity (missing point).

Note: (1) All polynomials, trigonometric, exponential & logarithmic functions are continuous in their domains.

4. Continuity In An Interval

(a) A function $f(x)$ is said to be continuous in (a, b) if $f(x)$ is continuous at each & every point $\in (a, b)$.

(b) A function $f(x)$ is said to be continuous in a closed interval $[a, b]$ if

(i) $f(x)$ is continuous in the open interval (a, b).

(ii) $f(x)$ is right continuous at 'a'

i.e. $\lim\limits_{x\to a^+} f(x) = f(a) = \text{finite quantity}$

(iii) $f(x)$ is left continuous at 'b'

i.e. $\lim\limits_{x\to b^-} f(x) = f(b) = \text{finite quantity}$

MIND MAP

Continuity

5. Continuity of Composite Functions

- ❖ If $f(x)$ is continuous at $x = a$ & $g(x)$ is continuous at $x = f(a)$, then the composite $g(f(x))$ is continuous at $x = a$.

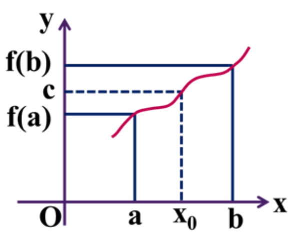
- ❖ Similarly, if $f(x)$ is continuous at $x \in R$ & $g(x)$ is continuous at $x = f(\forall x \in R)$, then the composite $g(f(x))$ is continuous at $x \in R$.

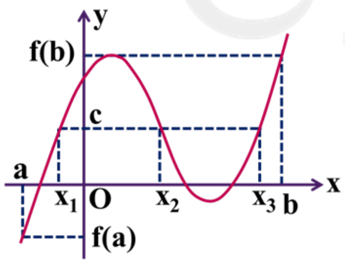
Note:

While checking points of discontinuity of gof(x), always include points of discontinuity of f(x).

6. Intermediate Value Theorem (IMVT)

If f is continuous on $[a, b]$ and $f(a) \neq f(b)$



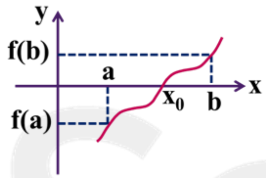


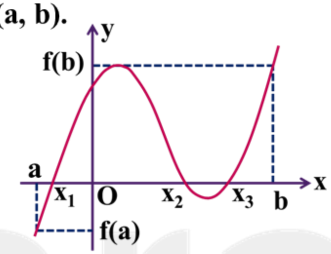
Then for every value c between f(a) and f(b) (including both), there is at least one number x_0 in $[a, b]$ for which $f(x_0) = c$.

(1) If $f(x)$ is continuous in $[a,b]$ $f(a).f(b) < 0$ and

{f(a) & f(b) posses opposite signs}

Then there exists at least one root of the equation $f(x) = 0$ in (a, b).





$f(x_0) = 0 \rightarrow x_0 \in (a, b)$

Note:

Continuity through the interval $[a, b]$ is essential for the validity of this IMVT theorem.

7. Important Points

(1) If $f(x)$ & $g(x)$ are continuous functions at $x = a$, then

$f(x) \pm g(x), f(x) . g(x)$ are continuous at $x = a$

Also, if $g(a) \neq 0$

$\frac{f(x)}{g(x)}$ is continuous at $x = a$

(2) If $f(x)$ is continuous at $x = a$ & $g(x)$ is discontinuous at $x = a$, then,

(a) $f(x) \pm g(x)$ must be discontinuous at $x = a$.

(b) $f(x) . g(x)$ or $\frac{f(x)}{g(x)}$; if $g(x) \neq 0$ will not necessarily be discontinuous at $x = a$.

(3) If $f(x)$ and $g(x)$ both are discontinuous at $x = a$ then,

$f(x) \pm g(x)$, $f(x) . g(x)$ or $\frac{f(x)}{g(x)}$; if $g(x) \neq 0$

will not necessarily be discontinuous at $x = a$

(4) Point functions are to be treated as continuous, i.e., if the domain of f(x) is a singleton, f(x) is a continuous function.

(5) (a) If a function $f(x)$ is continuous in a closed interval $[a, b]$, then it is bounded.

(b) A continuous function whose domain is some closed interval, must have its range also in closed interval.