# MIND MAP

# Differential Equation

## 1. Order And Degree of Differential Equation

### Introduction

Consider the Differential Equation,

$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2} \ldots\ldots\ldots\right) = 0$$

❑ *ORDER:*

It is the order of the highest differential coefficient occurring in the eq.

❑ *DEGREE:*

It is the degree of the highest order derivative occurring in the eq. after it has been expressed in a form free from radicals & fractions.

### Note

1. If differential equation cannot be written as polynomial in derivatives, then degree is not defined.
2. If degree exists, then it is always a Natural Number.

## 2. Formation of a Differential Eq.

Let we have,

An equation in independent and dependent variables having some arbitrary constants

$$f(x, y, c_1, c_2, c_3, \ldots c_n) = 0$$

Then, a Differential Eq. is obtained as follows :

1. Differentiate the given eq. w. r. t. the independent variable (say x) as many times as the number of arbitrary constants in it.

### Note

No. of independent arbitrary constants decide the order of Differential Eq.

2. Eliminate the arbitrary constants.
3. The eliminant is the required Differential Equation.

### Note

1. A Differential Eq. represents a family of curves all satisfying some common properties.

This can be considered as the geometrical interpretation of the differential equation.

2. The arbitrary constants in a Differential Eq. are said to be independent, when it is impossible to deduce an equivalent relation containing fewer arbitrary constants.

## 3. Solution of Differential Equation

- ❖ Finding the unknown function is called Solving or Integrating the Differential Eq.
- ❖ It is a relation between variables involved which satisfies the Differential Eq.
- ❖ The solution of the Differential Eq. is also called its Primitive. Because it can be regarded as a relation derived from it.

## 4. General & Particular Solutions

### General Solution :

The solution of a Differential Eq. which contains :

No. of independent arbitrary constants = Order of the Differential Eq.

Then the sol. is called the General solution

OR

Complete Integral

OR

Complete Primitive

### ❑ Particular Solutions :

A solution obtainable from the general solution by giving particular values to the constants is called a Particular Solution.

**Note**

The general solution of a differential equation of nth order contains 'n' & only 'n' independent arbitrary constants.

## 5. Solution of D.E. by Variable Separable Form

If the differential equation can be expressed as :

$f(x)\,dx + g(y)\,dy = 0$

Then,

It is said to be Variable Separable Type of Differential Eq.

## 6. DE Reducible to variable separable

Type-2 : By Substitution Method

$$\frac{dy}{dx} = f(ax + by + c) \quad (b \neq 0)$$

(If $b = 0$ this is directly Variable Separable)

To solve this, Substitute : $t = ax + by + c$

Then the eq. reduces to separable type in the variable t and x which can be solved.

## 7. Homogeneous Differential Equations

A differential equation of the form

$$\frac{dy}{dx} = \frac{f(x, y)}{\phi(x, y)}$$

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# Differential Equation

Where, $f(x, y)$ & $\phi(x, y)$ are homogeneous functions of x & y, and of the same degree

Then, this type of eq. is called Homogeneous.

This eq. may also be reduced to the form

$$\frac{dy}{dx} = g\left(\frac{x}{y}\right) \quad \text{OR} \quad \frac{dy}{dx} = h\left(\frac{y}{x}\right)$$

It is solved by putting $y = vx$ ,

Such that y changes to another variable v and the D.E. is transformed to Variable Separable Type.

**Note**

1. The function $f(x, y)$ is said to be a homogeneous function of degree 'n', if $f(tx, ty) = t^n . f(x, y)$ for any real number $t(\neq 0)$,

## 8. Equations Reducible To The Homogeneous Form

Consider $\dfrac{dy}{dx} = \dfrac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

Case -1 $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ Parallel Lines

Substitute : $a_1x + b_1y = t$

Case -2 $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ Intersecting Lines

Substitute : $x = X + h$ ; $y = Y + k$

Where (h, k) is intersection point of lines

$a_1x + b_1y + c_1 = 0$

& $a_2x + b_2y + c_2 = 0$

Case -3 If $a_2 + b_1 = 0$

Method -1 Use method of intersecting lines.

Method -2 Cross multiply & use $xdy + ydx = d(xy)$

## 9. Linear Differential Equations

A differential equation is said to be linear if :

❖ Dependent variable & its differential coefficients occur in the first degree only.

❖ And they are never multiplied together.

The nth order Linear Differential Eq. is of the form :

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## Differential Equation

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots\ldots + a_n(x)\,.\,y = \phi(x)$$

Where, $a_0(x), a_1(x), \ldots\ldots a_n(x)$ are called the coefficients of the Differential Eq.

**Note :**

A Linear Differential Eq. is always of the first degree but every Differential Eq. of the first degree need not be linear. e.g. $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y^2 = 0$

is not linear, though its degree is 1.

### Type - 1 : Linear D. E. of First Order

The most general form of a Linear D. E. of first order is :

$$\frac{dy}{dx} + Py = Q \quad \text{OR} \quad \frac{dx}{dy} + Rx = S$$

Where, P & Q are functions of x

R & S are functions of y.

To solve such equations, multiply both sides by $e^{\int P\,dx}$ OR $e^{\int R\,dy}$

$$\frac{dy}{dx} + Py = Q, \quad \frac{dy}{dx}e^{\int Pdx} + Pye^{\int Pdx} = Qe^{\int Pdx}$$

$$\Rightarrow \frac{d}{dx}(ye^{\int Pdx}) = Q\,e^{\int Pdx}$$

$$\Rightarrow ye^{\int Pdx} = \int Q.\,e^{\int Pdx} + c$$

$$\Rightarrow y(\text{I. F.}) = \int Q(\text{I. F.}) + c$$

**Note**

1. The factor $e^{\int P\,dx}$ is called integrating factor of the Differential Eq. and is popularly abbreviated as I. F.
2. On multiplying by the I. F., the L. H. S. becomes the derivative of the product of y and the I.F.
3. Sometimes a given differential equation becomes linear if we take y as the independent variable and x as the dependent variable.

e.g. The eq. : $(x + 2y^3)\frac{dy}{dx} = y$

can be written as $\frac{dx}{dy} = \frac{x + 2y^3}{y} = \frac{1}{y}x + 2y^2$

Which is a Linear D. E.

### 10. Equations Reducible To Linear Form

#### Bernoulli's Equation

The equation : $\frac{dy}{dx} + Py = Q.\,y^n$

Where, P & Q are functions of x

Is reducible to the Linear Form by dividing it by $y^n$

Then, Substitute $y^{(-n+1)} = z$

Now, solve as normal Linear DE.

### 11. Exact Differential Equations

Sometimes just by inspection, a first order degree differential equation can be written in the exact differential form which can be easily integrated.

**Following exact differentials must be remembered**

1. $x\,dy + y\,dx = d(xy)$
2. $\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$
3. $\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$
4. $\frac{x\,dy + y\,dx}{xy} = d(\ell n\, xy)$
5. $\frac{dx + dy}{x + y} = d(\ell n(x + y))$
6. $\frac{y\,dx - x\,dy}{xy} = d\left(\ell n \frac{x}{y}\right)$
7. $\frac{x\,dy - y\,dx}{xy} = d\left(\ell n \frac{y}{x}\right)$
8. $\frac{x\,dy - y\,dx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$
9. $\frac{y\,dx - x\,dy}{x^2 + y^2} = d\left(\tan^{-1}\frac{x}{y}\right)$
10. $\frac{x\,dx + y\,dy}{x^2 + y^2} = d\left[\ell n\sqrt{x^2 + y^2}\right]$