MIND MAP

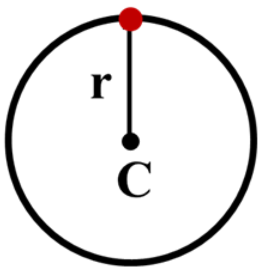
Circle

1. Circle

A circle is the locus of a point which moves in a plane in such a way that it's distance from a fixed point remains constant.

Fixed Point C: centre

Distance r: radius

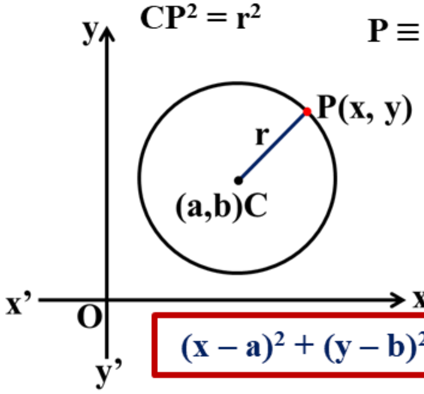


2. Eq. of the Circle In Various Forms

$C \equiv (a, b)$

$P \equiv (x, y)$

$CP^2 = r^2$

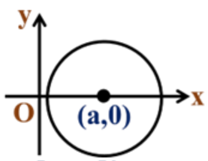


$\boxed{(x-a)^2 + (y-b)^2 = r^2}$

This is central form

If centre $C \equiv (0,0)$ & radius $\equiv r$

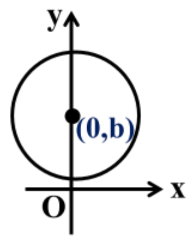
$\boxed{x^2 + y^2 = r^2}$



SPECIAL CASES :

(i) Centre lies on x-axis and radius r

$C \equiv (a, 0) \Rightarrow (x-a)^2 + y^2 = r^2$



(ii) Centre lies on y-axis and radius r

$C \equiv (0, b) \quad \Rightarrow x^2 + (y-b)^2 = r^2$

(iii) Point circle $(r = 0)$

$\Rightarrow (x-a)^2 + (y-b)^2 = 0$

3. General Eq. of Circle

The general eq. of the 2nd degree

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Represent a circle if, $\Delta \neq 0$ and

Coefficient of x^2 = Coefficient of y^2

and coefficient of $xy = 0$

$\boxed{x^2 + y^2 + 2gx + 2fy + c = 0}$

Centre is $(-g, -f)$

Radius $= \sqrt{g^2 + f^2 - c}$

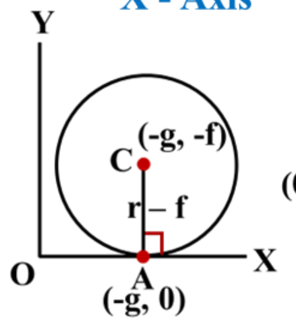
(i) If $g^2 + f^2 - c > 0$ equation represents a real circle

(ii) If $g^2 + f^2 - c = 0$ equation represents a point circle

(iii) If $g^2 + f^2 - c < 0$ equation represents an imaginary circle with real centre

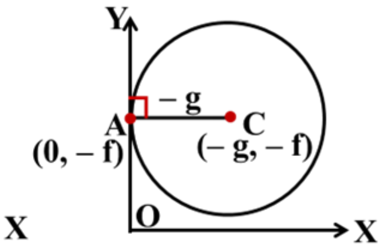
(iv) If c is negative, circle is always real because $g^2 + f^2 - c > 0$

(v) Circle Touching X - Axis



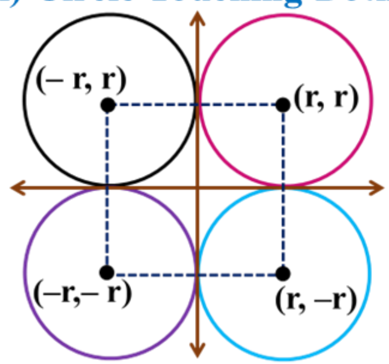
$(x+g)^2 + (y+f)^2 = f^2$

Circle Touching Y - Axis



$(x+g)^2 + (y+f)^2 = g^2$

(vi) Circle Touching Both Axes



Radius = r

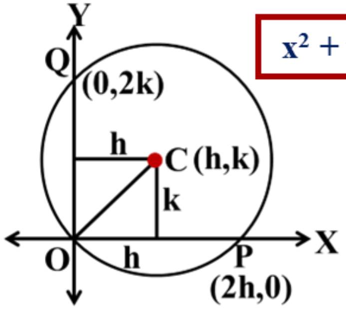
Centre $(\pm r, \pm r)$

$\boxed{(x \pm r)^2 + (y \pm r)^2 = r^2}$

MIND MAP

Circle

(vii) Circle Passing Through the Origin



$$\boxed{x^2 + y^2 - 2hx - 2ky = 0}$$

$C \equiv (h, k)$

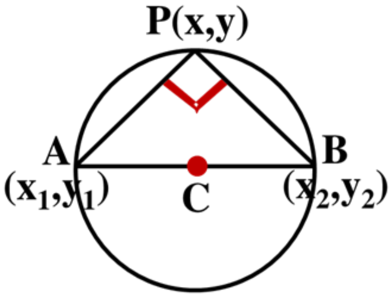
$r = \sqrt{h^2 + k^2}$

intercept cut on x-axis, OP = 2h

intercept cut on y-axis, OQ = 2k

4. Diameter Form

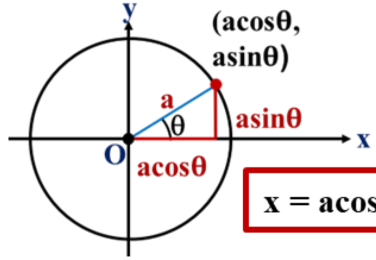
$$\boxed{(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0}$$



$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ is the circle of smallest radius passing through (x_1, y_1) and (x_2, y_2).

5. Parametric Form

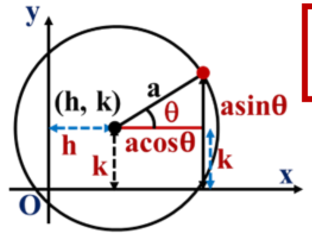
Circle: $x^2 + y^2 = a^2$



$$\boxed{x = a\cos\theta,\ y = a\sin\theta}$$

- Where, θ is a parameter $(0 \le \theta < 2\pi)$
- The point $(a\cos\theta, a\sin\theta)$ is also referred to as the point 'θ'.
- θ is the angle which the line joining the point to the centre of the circle makes with the x-axis.

Circle: $(x - h)^2 + (y - k)^2 = a^2$



$$\boxed{\begin{aligned} x &= h + a\cos\theta \\ y &= k + a\sin\theta \end{aligned}}$$

Circumcenter : (0,0)

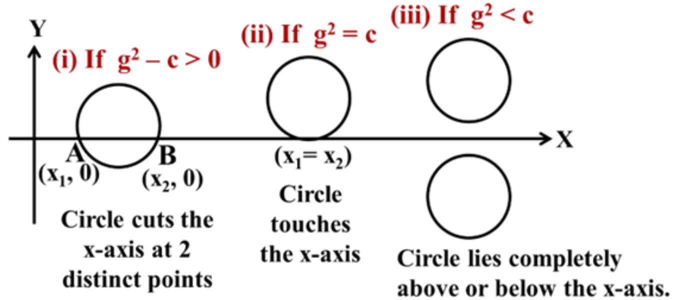
Centroid : $\dfrac{\Sigma\cos\theta_1}{3}, \dfrac{\Sigma\sin\theta_1}{3}$

Orthocenter : $(\Sigma\cos\theta_1 ; \Sigma\sin\theta_1)$

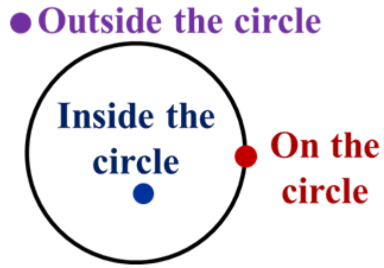
6. Intercept made by the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

on x-axis	on y-axis
$\boxed{2\sqrt{g^2 - c}}$	$\boxed{2\sqrt{f^2 - c}}$



7. Position of a Point w.r.t. a Circle



$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

P(h, k)

MIND MAP

Circle

7. Position of a Point w.r.t. a Circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0 \qquad P(h, k)$

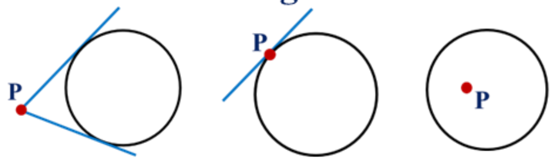
$S_1(P) = h^2 + k^2 + 2gh + 2fk + c$

(i) $S_1(P) = 0$ Point lies on the circle

(ii) $S_1(P) < 0$ Point lies inside the circle

(iii) $S_1(P) > 0$ Point lies outside the circle

Number of Tangents



2, 1 or No Tangents can be drawn from a point lying outside, on or inside the circle respectively.

8. Power of A Point w.r.t. a Circle

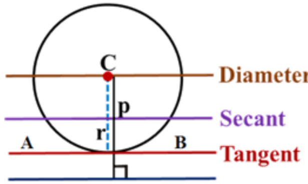
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Point: $P(x_1, y_1)$

$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

S_1 is called power of point $P(x_1, y_1)$ w.r.t. the circle S

9. Position of a Line w.r.t. a Circle



r: Radius

p :Length of the ⊥ from the centre on the line

(i) $p < r \Rightarrow$ The line intersects the circle in two distinct points

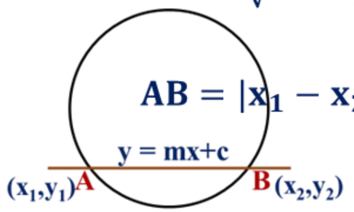
(ii) $p = r \Rightarrow$ The line touches the circle

(iii) $p > r \Rightarrow$ The line passes outside the circle

(iv) $p = 0 \Rightarrow$ the line passes through the centre

(v) Length of the chord = AB

$= 2AD = 2\sqrt{r^2 - p^2}$



$AB = |x_1 - x_2|\sqrt{1 + m^2}$

10. Equation of Tangent

Circle: $x^2 + y^2 = r^2$

Point form: (x_1, y_1)

$\boxed{xx_1 + yy_1 = r^2}$

Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

Point form: (x_1, y_1)

$\boxed{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0}$

Parametric form

Circle: $x^2 + y^2 = a^2$

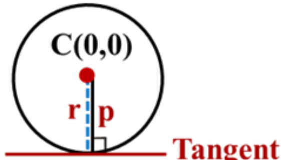
Point: $(a\cos\alpha, a\sin\alpha)$

$\boxed{x\cos\alpha + y\sin\alpha = a}$

Slope Form

Circle: $x^2 + y^2 = r^2$

Line: $y = mx + c$



Condition of tangency:

$\boxed{c = \pm r\sqrt{(1 + m^2)}}$

Eq. of Tangent : $\boxed{y = mx \pm r\sqrt{(1 + m^2)}}$

Note :

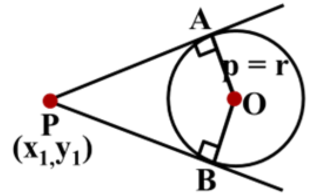
$D = 4(mc)^2 - 4(1 + m^2)(c^2 - r^2) = 0$

(i) $D > 0 \Rightarrow$ (Secant)

(ii) $D = 0 \Rightarrow$ (Tangent)

(iii) $D < 0 \Rightarrow$ line will not intersect the circle in any point.

11. Equations of Tangents From an External Point to a Circle



Step-1 : Assume equation of tangent as $y - y_1 = m(x - x_1)$

Step-2 : Equate $p = r$

Step-3 : Square both sides & solve quad. in 'm' whose roots are m_1 & m_2

Step-4 : Equations of tangents will be $y - y_1 = m_1(x - x_1)$ & $y - y_1 = m_2(x - x_1)$

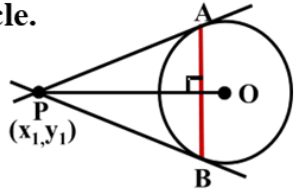
MIND MAP

Circle

12. Chord of contact

The chord AB joining the points of contact A and B of the tangents from P is called the Chord of Contact of $P(x_1, y_1)$ with respect to the circle.

It is $\perp$ to the line joining P to the centre of the circle.



Eq. of Chord of Contact:

$xx_1 + yy_1 = r^2$

Note

(i) The eq. has the same appearance as the eq. of the tangent for which the point of contact is (x_1, y_1), i.e. $T = 0$

(ii) For circle in general form $x^2 + y^2 + 2gx + 2fy + c = 0$

Chord of Contact will be

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(iii) If chord of contact is given & corresponding point is to be found then

1. Assume point (x_1, y_1)
2. Find chord of contact for this point
3. Now compare the coefficients of eq. with given chord equation.

13. Some Important Points

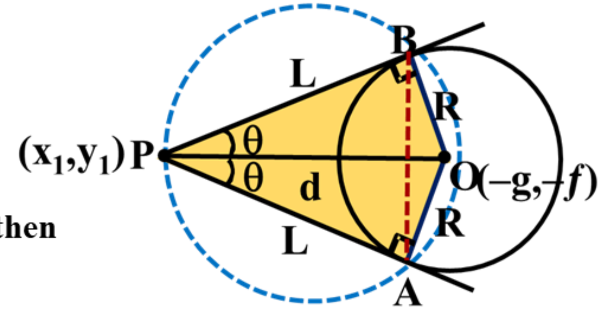
(i) Length of Tangent from an external point $P(x_1, y_1)$

$S = x^2 + y^2 + 2gx + 2fy + c = 0$

$$\boxed{L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}}$$

(a) Formula is applicable when coefficient of x^2 & y^2 is unity.

(b) The lengths of both tangents drawn from point P are equal, i.e. PA = PB



(ii) Area of Quad. PAOB (Formed by pair of tangents & pair of radii)

$= 2.\Delta POA$

$= 2 \cdot \frac{1}{2} R.L$

$$\boxed{= R.L = R\sqrt{S_1}}$$

(iii) Length of chord of contact AB

$$\boxed{AB = \frac{2RL}{\sqrt{R^2 + L^2}}}$$

(iv) Area of ΔPAB (Δ formed by pair of tangent & corresponding C.O.C.)

$$\boxed{\text{Area of } \Delta PAB = \frac{RL^3}{R^2 + L^2}}$$

(v) Angle between the pair of tangents

$$\boxed{2\theta = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)}$$

(vi) Eq. of the circle circumscribing the ΔPAB or quadrilateral PAOB (cyclic)

Circle described on OP as diameter

$$\boxed{(x - x_1)(x + g) + (y - y_1)(y + f) = 0}$$

MIND MAP

Circle

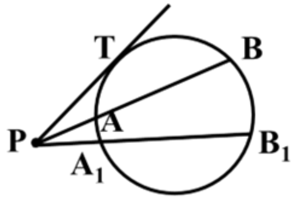
(vii) Power of a point

Square of the length of the tangent from the point P is called power of the point P w.r.t. a given circle

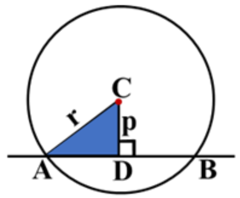
$PT^2 = S_1 = L^2$

$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

Power of a point remains constant w.r.t. a circle.



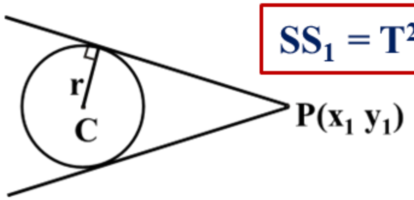
(viii) $PA.PB = PA_1 . PB_1 = PA_2 . PB_2 = ... = PT^2 = S_1$



14. Pair of Tangents

Combined equation of the pair of tangents drawn from an external point 'P' to a the circle $S = 0$ is given by

$\boxed{SS_1 = T^2}$



(i) $S \equiv x^2 + y^2 - a^2$ $\quad S_1 \equiv x_1^2 + y_1^2 - a^2$

$T = xx_1 + yy_1 - a^2$

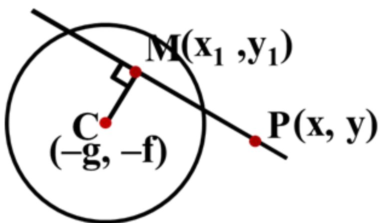
(ii) $S \equiv x^2 + y^2 + 2gx + 2fy + c$

$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$

15. Eq. of the Chord With Given Mid Point M (x_1, y_1)

$\boxed{T = S_1}$



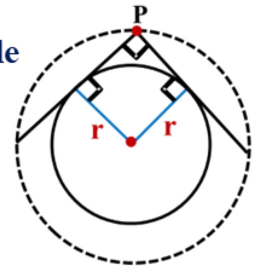
Note: Of all the chords which passes through a given point M(a, b) inside the circle, the shortest chord is one whose middle point is (a, b).

16. Director Circle

Locus of the point of intersection of 2 mutually $\perp$ tangents to a given curve is called the director circle of the given curve.

Eq. of Director Circle

$\boxed{x^2 + y^2 = 2r^2}$



17. Eq. of a Chord of a Circle in Parametric Form

Circle $x^2 + y^2 = a^2$

Joining Points: $(a\cos\alpha, a\sin\alpha)$; $(a\cos\beta, a\sin\beta)$

$$\boxed{x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}}$$

18. Point of intersection of tangents at the points $(a\cos\alpha, a\sin\alpha)$ & $(a\cos\beta, a\sin\beta)$

$$T = \left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{a\sin\frac{(\alpha+\beta)}{2}}{\cos\frac{(\alpha-\beta)}{2}}\right)$$

MIND MAP

Circle

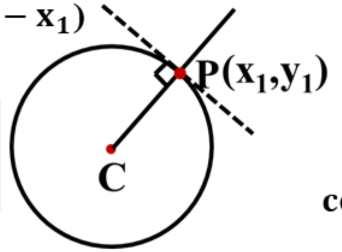
19. Normal to the Circle

Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

Pt. lying on the circle: $P(x_1, y_1)$

Eq. of the Normal at P:

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$



$$\boxed{\frac{y_1 + f}{x_1 + g} = \frac{y - y_1}{x - x_1}}$$

Every normal passes through the centre of the circle.

20. Angle of Intersection of 2 Circles

The angle b/w the tangents of 2 circles at the point of intersection of the 2 circles is called angle of intersection of 2 circles.

And, angle between tangents is equivalent to angle between radii (normals).

$$\boxed{\cos\theta = \left|\frac{2g_1g_2 + 2f_1f_2 - c_1 - c_2}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}\right|}$$

21. Orthogonality of 2 Circles

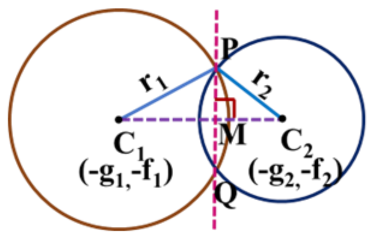
Two curves are said to be orthogonal if they intersect each other at 90° wherever they intersect.

Condition for Orthogonality of 2 Circles

$$\cos\theta = \left|\frac{2g_1g_2 + 2f_1f_2 - c_1 - c_2}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}\right| = 0$$

$$\boxed{2g_1g_2 + 2f_1f_2 = c_1 + c_2}$$

22. Common Chord of 2 Intersecting Circles



$$\boxed{S_1 - S_2 = 0}$$

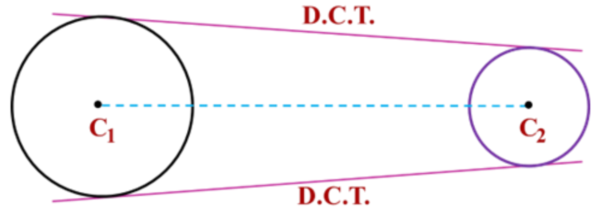
Provided that the coefficients of x^2 and y^2 are 1 in both S_1 and S_2.

Length of the common chord : PQ

$$PQ = 2PM = \sqrt{C_2P^2 - C_2M^2}$$

23. Direct Common Tangent (D.C.T)

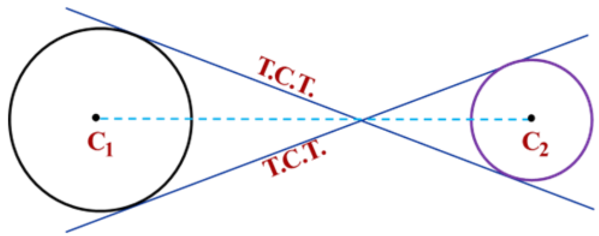
External common tangent



The centre of both the circles lie on the same side of the tangent line

24. Transverse Common Tangent (T.C.T)

Internal common tangent



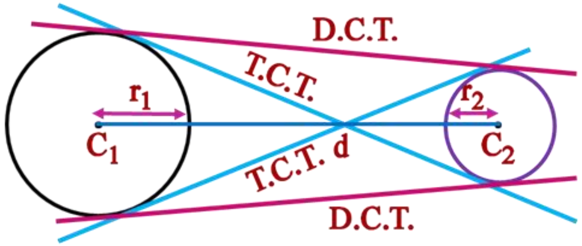
The centre of both the circles lie on the opposite side of the tangent line

MIND MAP

Circle

25. Position of The Circles & Number of Common Tangents

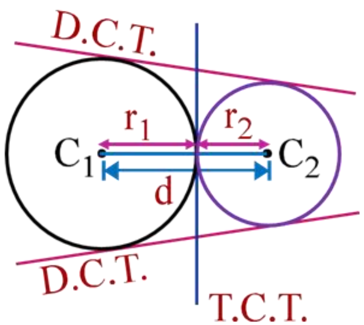
(1) If 2 circles are separated, then $d > r_1 + r_2$



4 common tangents → 2 D.C.T.
4 common tangents → 2 T.C.T.

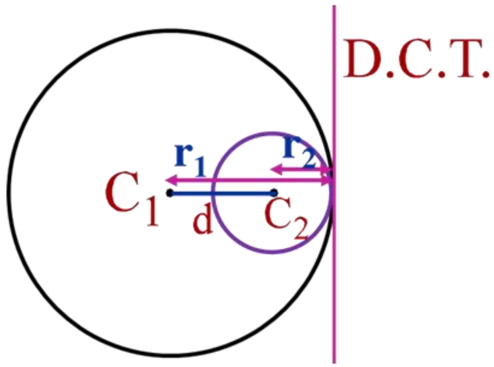
(2) If 2 circles touch externally, then $d = r_1 + r_2$

3 common tangents → 2 D.C.T.
3 common tangents → 1 T.C.T.
At their point of contact



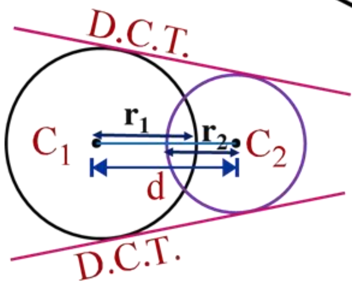
(3) If both the circles touch each other internally, then $d = |r_1 - r_2|$

1 common tangent ⟶ 1 D.C.T.
At their point of contact



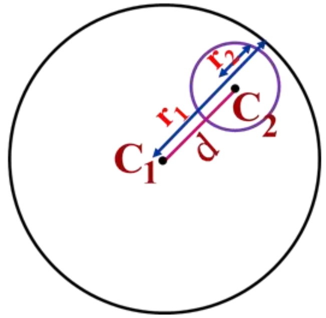
(4) If two circles intersect each other, then $|r_1 - r_2| < d < r_1 + r_2$

2 common tangents → 2 D.C.T.
2 common tangents → 0 T.C.T.

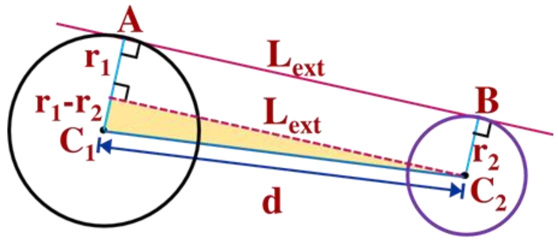


(5) If one circle is contained in other, then $d < |r_1 - r_2|$

$\therefore$ No common tangent

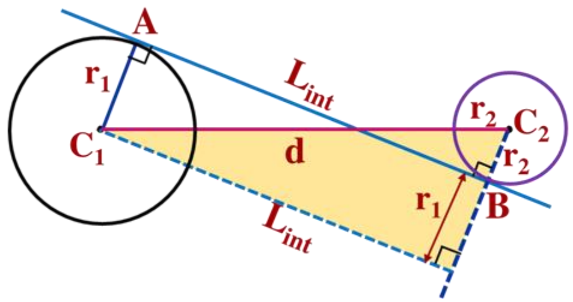


26. Length of common tangents



$$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$$

$$L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$$



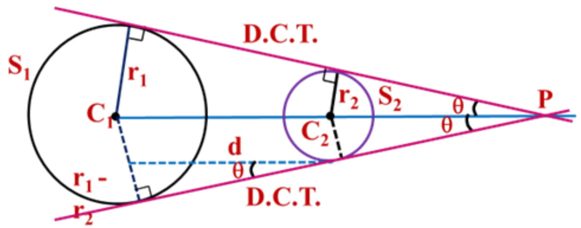
MIND MAP

Circle

Equation of D.C.T / T.C.T.

C_1, C_2, P lie in a same line

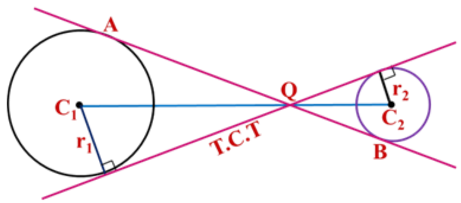
$$\frac{PC_1}{PC_2} = \frac{r_1}{r_2} \quad \sin\theta = \frac{r_1 - r_2}{c_1c_2} = \frac{r_1 - r_2}{d}$$



$$\frac{PC_1}{PC_2} = \frac{r_1}{r_2}$$

Direct Common Tangent (D.C.T.) meet at a point which divides the line joining the centre of circles externally in the ratio of their radii.

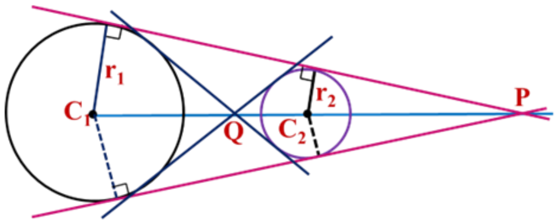
$$\frac{C_1Q}{C_2Q} = \frac{r_1}{r_2}$$



Transverse Common Tangent (T.C.T.) meets at a point which divides the line joining the centre of circles internally in the ratio of their radii.

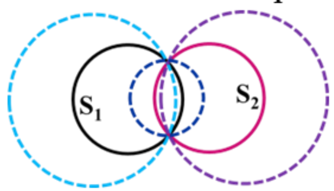
$$\frac{PC_1}{PC_2} = \frac{r_1}{r_2} \qquad \frac{C_1Q}{C_2Q} = \frac{r_1}{r_2} \qquad C_1, Q, C_2, P$$

constitute a harmonic range i.e. C_1Q, C_1C_2, C_1P are in H.P.



27. Family of Circles

Type-1 : The eq. of the family of circles passing through the points of intersection of 2 circles Circles: $S_1 = 0$ & $S_2 = 0$

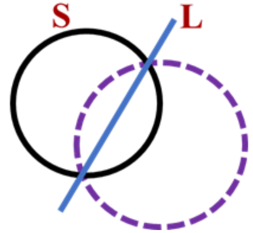


$$\boxed{\text{Family : } S_1 + \lambda S_2 = 0 \ ; \quad (\lambda \neq -1)}$$

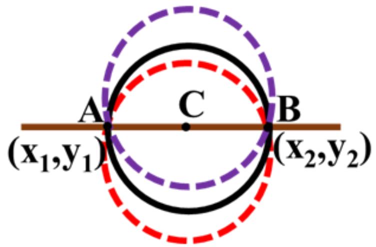
Type-2 : The equation of the family of circles passing through the point of intersection of a circle $S = 0$ & a line $L = 0$.

$$\boxed{S + \lambda.L = 0, \lambda \in R}$$

Converse is also true.



Type-3 : The equation of a family of circles passing through two given points $A(x_1, y_1)$ & $B(x_2, y_2)$



$$\boxed{(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda\left[\left((y - y_1) - \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right)\right] = 0}$$

Type-4 : The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point $P(x_1,y_1)$ lying on it

Treat P as point circle.

$$\boxed{(x - x_1)^2 + (y - y_1)^2 + \lambda[y - y_1 - m(x - x_1)] = 0}$$