# MIND MAP

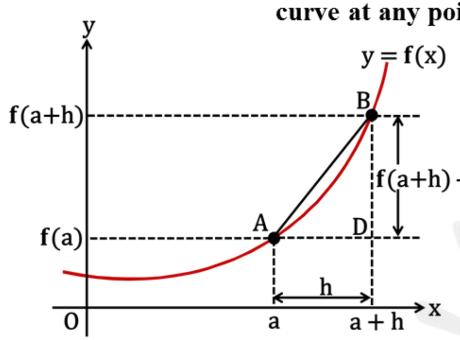
## Differentiability

## 1. Concept of Derivative

Two fold meaning of derivative

**Physical meaning** Instantaneous rate of change of function

**Geometrical meaning** Slope of the tangent drawn to the curve at any point (if exists)




At one instant, slope at A of both the tangents from both sides are equal, i.e.,

$\Rightarrow \lim\limits_{B\to A} m_{AB} = \lim\limits_{C\to A} m_{AC}$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

= Finite quantity

∴ This is called Derivative of f(x) at x = a.

## 2. Right Hand Derivative

The right hand derivative of f(x) at x = a

$$f'(a^+) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Provided the limit exists & is finite. (h > 0)

## 3. Left Hand Derivative

The left hand derivative of f(x) at x = a

$$f'(a^-) = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

Provided the limit exists & is finite. (h > 0)

Hence, f(x) is said to be derivable or differentiable at x = a, If

$\boxed{f'(a^+) = f'(a^-) = \text{Unique finite quantity}}$

Then derivate of f(x) at x = a is denoted by f ′(a). Where $f'(a) = f'(a^-) = f'(a^+)$ at x = a.

**Note :**

❖ All polynomial, rational, trigonometric, logarithmic and exponential function are continuous and differentiable in their domains.

❖ If the function y = f(x) is differentiable at x = a, Then a unique non vertical tangent can be drawn to the curve y = f(x) at the point P(a, f(a)) & f′(a) represent the slope of the tangent at point P.

❖ Constant function is differentiable.

❖ [x] is non-differentiable at integers.

❖ {x} is non-differentiable at integers.

❖ sgn (x) is non-differentiable at x = 0.

## 4. Derivability & Continuity

(a) If a function f(x) is derivable at x = a, then f(x) is continuous at x = a.

(b) Let $f'(a^+) = p$ & $f'(a^-) = q$

When p & q are finite (whether equal or not)

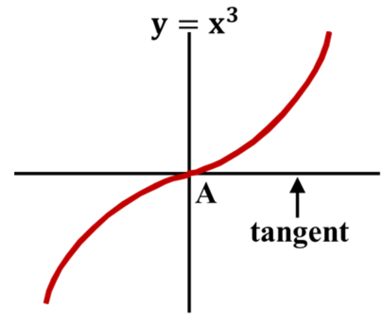
Then ‘f’ is continuous at x = a

But converse is NOT necessarily true.

(i) p = q ‘f’ is differentiable at x = a

$\Rightarrow$ ‘f’ is continuous at x = a

Here, x axis is tangent to the curve at x = 0.



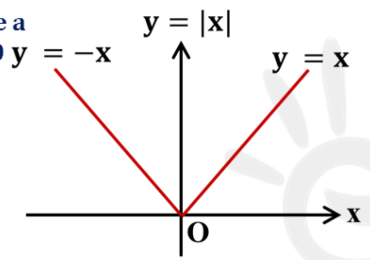
## 4. Derivability & Continuity

(ii) $p \neq q \Rightarrow$ 'f' is not differentiable at $x = a$, but 'f' is still continuous at $x = a$.

In this case, we have a sudden change in the direction of the graph of the function at $x = a$.

This point is called a corner point of the function. At this point, there is no tangent to the curve.

It does not have a tangent at $x = 0$




It is called a corner point.

## 4. Derivability & Continuity

(c) Let $f'(a^+) = p$ & $f'(a^-) = q$

When p or q or both may not be finite

In this case, 'f' is not differentiable at $x = a$

& nothing can be concluded about continuity of the function at $x = a$.

**Vertical Tangent :**

If $y = f(x)$ is continuous at $x = a$ &

# MIND MAP

# Differentiability

$\lim\limits_{x \to a} |f'(x)|$ approaches to $\infty$, then $y = f(x)$ has a vertical tangent at $x = a$.

If a function has vertical tangent at $x = a$, then it is non differentiable at $x = a$.

## 5. Derivability Over an Interval

(A) $f(x)$ is said to be differentiable over an open interval $(a, b)$; if it is differentiable at each & every point of $(a, b)$.

(B) $f(x)$ is said to be differentiable over the closed interval $[a, b]$ if :

(i) $f(x)$ is differentiable in $(a, b)$ &

(ii) For the points a and b, $f'(a^+)$ & $f'(b^-)$ exist.

## 6. Algebra of Differentiability

(i) If $f(x)$ & $g(x)$ are differentiable at $x = a$, then functions,

$f(x) + g(x)$; $f(x) - g(x)$; $f(x) . g(x)$;

$f(x)/g(x)$ if $g(a) \neq 0$

Will also be differentiable at $x = a$.

(ii) If $f(x)$ is differentiable at $x = a$ and $g(x)$ is not differentiable at $x = a$, then

(a) Sum or difference, i.e. $f(x) \pm g(x)$ must be non differentiable at $x = 0$,

(b) Product function $f(x) . g(x)$ may or may not be differentiable at $x = a$.

(iii) If $f(x)$ and $g(x)$ are both non differentiable at $x = a$, then

(a) Sum or difference, i.e. $f(x) \pm g(x)$ may or may not be differentiable at $x = a$.

(b) Product function $f(x).g(x)$ may or may not be differentiable.

## 7. Important Concepts

(a) Let $f(x)$ be a function whose derivative is $f'(x)$, then

$$\lim_{y \to x} \frac{f(x) - f(y)}{x - y} = f'(x)$$

(b) Derivative of continuous function need not be a continuous function.

i.e. if $f'(x)$ exists everywhere in its domain, then $f'(x)$ need not to be necessarily continuous everywhere.