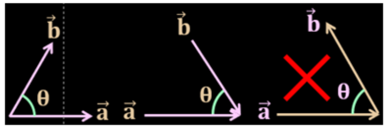
MIND MAP

Vectors

Angle between Vectors



$\boxed{\hat{\vec a\,\vec b} = \theta}$

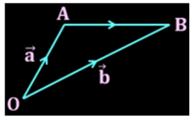
Important Points

Unit Vector $\hat a = \dfrac{\vec a}{|\vec a|}$

Negative of a Vector

$\overrightarrow{AB} = \vec a \Rightarrow \overrightarrow{BA} = -\vec a$

Position Vector



$\overrightarrow{AB} = \vec b - \vec a$

Multiplication of Vectors by Scalars

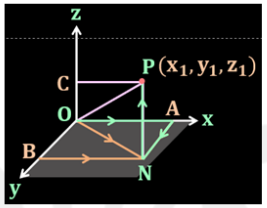
$\vec a \to$ vector $\quad \xrightarrow{\vec a}$

$m \to$ scalar $\quad \xrightarrow{\;m\vec a\;}$

$\vec a, \vec b \to$ vector $\quad m, n \to$ scalar

1. $m(\vec a) = (\vec a)m = m\vec a$
2. $m(n\vec a) = n(m\vec a) = (mn)\vec a$
3. $(m+n)\vec a = m\vec a + n\vec a$
4. $m(\vec a + \vec b) = m\vec a + m\vec b$
5. $\hat a = \dfrac{\vec a}{|\vec a|}$

Representation of Vectors in Space



in terms of 3 Orthonormal Triad of Unit Vectors

$A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are two points in space

$$\overrightarrow{AB} = (x_2 - x_1)\hat i + (y_2 - y_1)\hat j + (z_2 - z_1)\hat k$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Linear Combination of Vectors

$$\vec r = x\vec a + y\vec b + z\vec c + \ldots$$

Linearly Dependent

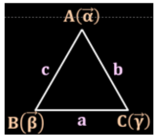
$x_1\vec a_1 + x_2\vec a_2 + \ldots + x_n\vec a_n = 0$

'n' scalars $x_1, x_2, x_3, \ldots, x_n$, not all zero

Linearly Independent

$x_1 = x_2 = x_3 = \ldots = x_n = 0$

Centres of Triangle



(a) Position vector of centroid $= \dfrac{\vec\alpha + \vec\beta + \vec\gamma}{3}$

(b) Incentre $= \dfrac{a\vec\alpha + b\vec\beta + c\vec\gamma}{a+b+c}$

(c) Excentres $\dfrac{-a\vec\alpha + b\vec\beta + c\vec\gamma}{-a+b+c}$; $\dfrac{a\vec\alpha - b\vec\beta + c\vec\gamma}{a-b+c}$ & $\dfrac{a\vec\alpha + b\vec\beta - c\vec\gamma}{a+b-c}$

(d) Circumcentre

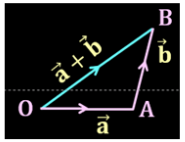
$$\frac{\vec\alpha\sin 2A + \vec\beta\sin 2B + \vec\gamma\sin 2C}{\sum \sin 2A}$$

(e) Orthocentre

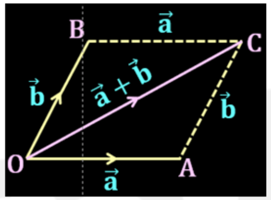
$$\frac{\vec\alpha\tan A + \vec\beta\tan B + \vec\gamma\tan C}{\sum \tan A}$$

Vector Addition

a) Triangle Law of Vectors



b) Parallelogram Law of Vectors



$\overrightarrow{OA}$ & $\overrightarrow{OB}$ vectors $\vec a$ & $\vec b$

$\overrightarrow{OC}$ sum $\vec a + \vec b$

c) Properties

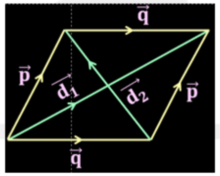
1. $\vec a + \vec b = \vec b + \vec a$
2. $(\vec a + \vec b) + \vec c = \vec a + (\vec b + \vec c)$
3. $\vec a + \vec 0 = \vec a = \vec 0 + \vec a$
4. $\vec a + (-\vec a) = \vec 0 = (-\vec a) + \vec a$

Note:

(i) $|\vec a + \vec b| \neq |\vec a| + |\vec b|$

(ii) $|\vec a + \vec b| \leq |\vec a| + |\vec b|$

(iii) $|\vec a + \vec b| \geq \big||\vec a| - |\vec b|\big|$

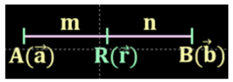


$\vec d_1 = \vec p + \vec q \quad \vec d_2 = \vec p - \vec q$

$\vec p = \left|\dfrac{\vec d_1 + \vec d_2}{2}\right| \quad \vec q = \left|\dfrac{\vec d_1 - \vec d_2}{2}\right|$

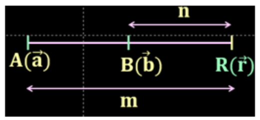
Section Formula

a) Internal Division in ratio m : n



$\vec r = \dfrac{n\vec a + m\vec b}{m+n}$

b) External Division in ratio m : n



$\vec r = \dfrac{m\vec b - n\vec a}{m-n}$

Test of Collinearity of 3 points

3 points A, B, C will be collinear, if

$\overrightarrow{AB} = \lambda\,\overrightarrow{BC}$

Necessary and sufficient condition:

If & only if there exist scalars x, y, z; not all zero simultaneously such that;

$x\vec a + y\vec b + z\vec c = 0$, where $x + y + z = 0$

MIND MAP

Vectors

Scalar Product (Dot Product)

$\boxed{\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta, \text{ where } \theta \in [0, \pi]}$

For two non zero vectors $\vec{a}$ & $\vec{b}$ & $\vec{a} \neq \lambda\vec{b}$ if

(i) $\vec{a}.\vec{b} > 0$ — θ is acute
(ii) $\vec{a}.\vec{b} = 0$ — $\theta = \pi/2$
(iii) $\vec{a}.\vec{b} < 0$ — θ is obtuse

General expression

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\boxed{\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3}$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\,.\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Properties

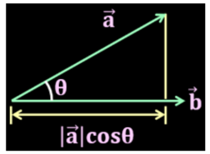
1. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
2. $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$
3. $\vec{a}.\vec{a} = |\vec{a}|^2 = \vec{a}^2$
4. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
5. $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

Important Identities

1. $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = \vec{a}^2 - \vec{b}^2$
2. $(\vec{a} + \vec{b})^2 = \vec{a}^2 + 2\vec{a}.\vec{b} + \vec{b}^2$
$= (\vec{a} + \vec{b}).(\vec{a} + \vec{b})$
$= (\vec{a})^2 + (\vec{b})^2 + 2|\vec{a}||\vec{b}|\cos\theta$
3. $(\vec{a} - \vec{b})^2 = \vec{a}^2 - 2\vec{a}.\vec{b} + \vec{b}^2$
$= (\vec{a})^2 + (\vec{b})^2 - 2|\vec{a}||\vec{b}|\cos\theta$
4. $(\vec{a} + \vec{b} + \vec{c})^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\sum(\vec{a}.\vec{b})$
5. $a_1b_1 + a_2b_2 + a_3b_3 \leq \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\sum(\vec{a}.\vec{b})$

Geometrical Interpretation

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} = \vec{a}.\hat{b}$
(+ve, –ve or 0)

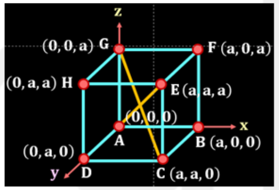


Vector component of $\vec{a}$ along $\vec{b}$

$= (\vec{a}.\hat{b})\hat{b} = \left(\dfrac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$

Vector component of $\vec{a} \perp \vec{b} = \vec{a} - \left(\dfrac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$

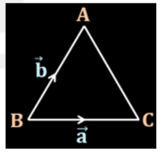
Angle between the diagonals of a cube



$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

Vector Area of a Plane Triangle

a) Area of the Δ (in terms of sides)



$= \dfrac{|\vec{a} \times \vec{b}|}{2} = \dfrac{1}{2}|\vec{BA} \times \vec{BC}|$

$= \dfrac{1}{2}|\vec{AB} \times \vec{AC}| = \dfrac{1}{2}|\vec{CA} \times \vec{CB}|$

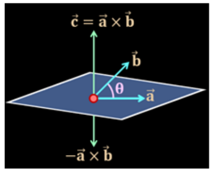
b) Vector Area of the Δ
(in terms of position vectors $\vec{a}, \vec{b}, \vec{c}$)

$\vec{\Delta} = \dfrac{1}{2}[(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})]$

c) Condition of collinearity of 3 points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

Vector Product (Cross Product)



$\boxed{\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}}$

$\sin\theta = \pm\dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

Unit vector $\perp$ to the plane of $\vec{a}$ & $\vec{b}$ is

$\pm\dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

Vector of mag. 'r' $\perp$ to the plane of $\vec{a}$ & $\vec{b}$

$= \pm\dfrac{r(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$

Langrange's Identity

$|\vec{a} \times \vec{b}|^2 = \vec{a}^2\vec{b}^2 - (\vec{a}.\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$

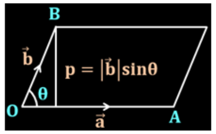
Properties

1. $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$
2. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
3. $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$
4. $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
$[\hat{i} \times \hat{j} = \hat{k}]\,[\hat{j} \times \hat{k} = \hat{i}]\,[\hat{k} \times \hat{i} = \hat{j}]$
5. $(\vec{a} \times \vec{b}).\vec{a} = 0$
6. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Geometrical Interpretation

Area of parallelogram

$= |\vec{a}||\vec{b}|\sin\theta$



If diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$

Area $= \dfrac{1}{2}|\vec{d}_1 \times \vec{d}_2|$