MIND MAP

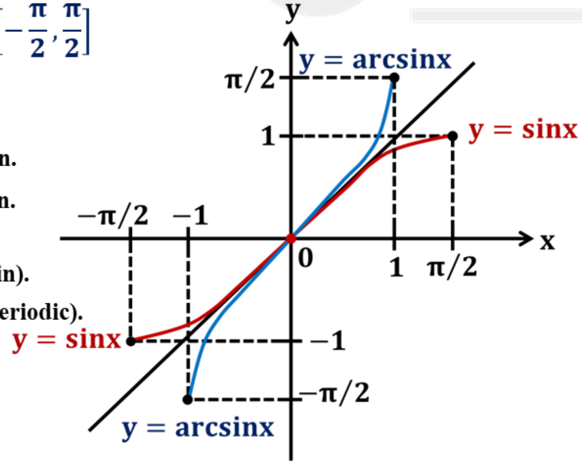
Inverse Trigonometric Function

1. Principal Values & Domains of Inverse Circular Functions

S. No.	f(x)	Domain	Range
(1)	$\sin^{-1} x$	$\lvert x\rvert \leq 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(2)	$\cos^{-1} x$	$\lvert x\rvert \leq 1$	$[0, \pi]$
(3)	$\tan^{-1} x$	$x \in R$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(4)	$\sec^{-1} x$	$\lvert x\rvert \geq 1$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ OR $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
(5)	$\operatorname{cosec}^{-1} x$	$\lvert x\rvert \geq 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
(6)	$\cot^{-1} x$	$x \in R$	$(0, \pi)$

2. Graphs of all 6 Inverse Circular Functions

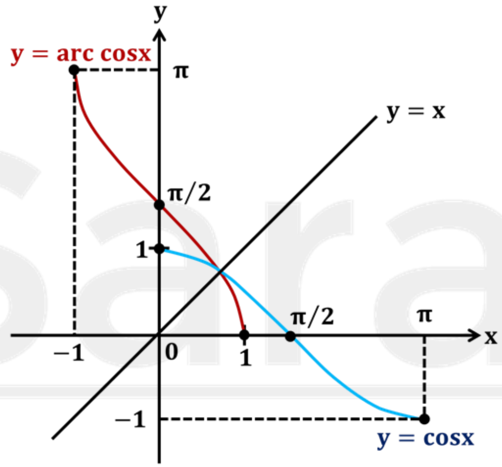
(1) $\sin^{-1}x, |x| \leq 1, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



More about arc sinx

- ✓ $\sin^{-1}x$ is monotonical increasing in its domain.
- ✓ It is a bounded function.
- ✓ It is an odd function (symmetric about origin).
- ✓ It is aperiodic (non – periodic).
- ✓ It is continuous.

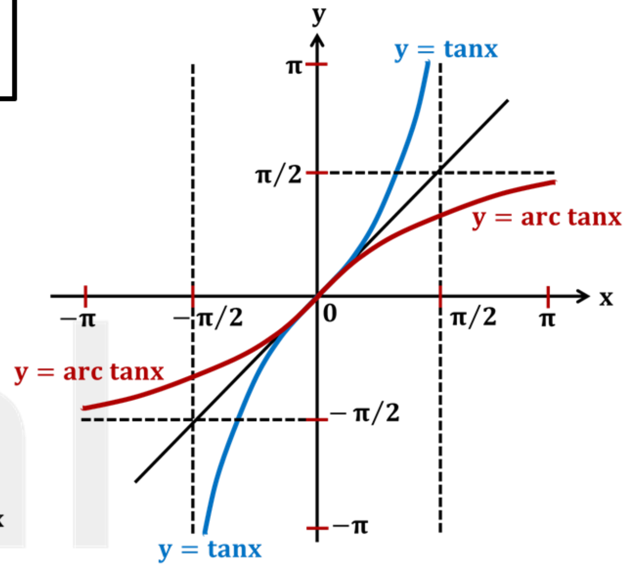
(2) $\cos^{-1}x, |x| \leq 1, [0, \pi]$



More about arc cosx

- ✓ $\cos^{-1}x$ is monotonic decreasing in its domain.
- ✓ It is a bounded function.
- ✓ It is neither even nor odd.
- ✓ It is aperiodic.
- ✓ It is continuous.

(3) $\tan^{-1}x, \quad x \in R, \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



More about arc tanx

- ✓ $\tan^{-1}x$ is monotonical increasing in its domain.
- ✓ It is a bounded function.
- ✓ It is an odd function.
- ✓ It is aperiodic.
- ✓ It is a continuous function.

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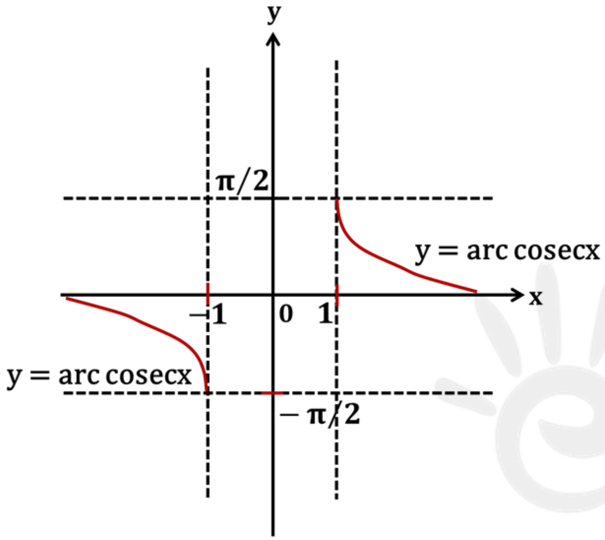
2. Graphs of all 6 Inverse Circular Functions

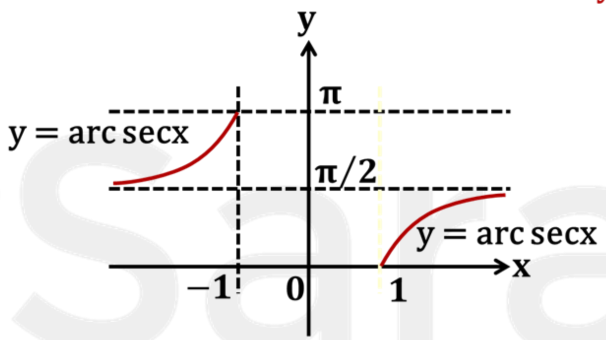
(4) $\operatorname{cosec}^{-1}x, |x| \geq 1, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$



More about arc cosecx

- It is monotonic decreasing function.
- It is aperiodic function.
- It is a bounded function.
- It is an odd function.
- It is continuous, wherever it is defined.

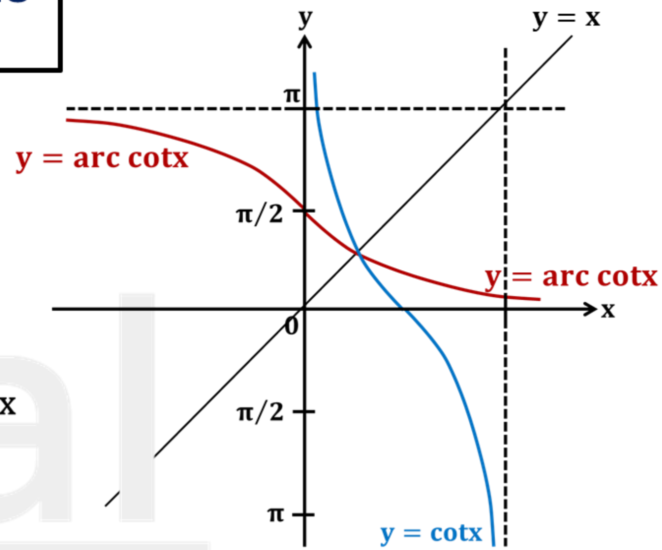
(5) $\sec^{-1}x, |x| \geq 1, \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



More about arc secx

- It is increasing wherever it is defined.
- It is aperiodic.
- It is neither even nor odd.
- It is continuous, wherever it is defined for $|x| \geq 1$.
- It is bounded function.

(6) $\cot^{-1}x, \quad x \in R, \quad (0, \pi)$



More about arc cotx

- It is monotonic decreasing function.
- It is bounded function.
- It is aperiodic.
- It is continuous everywhere.
- It is neither even nor odd.

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Inverse Trigonometric Function

3. Properties Of Inverse Trigonometric Function

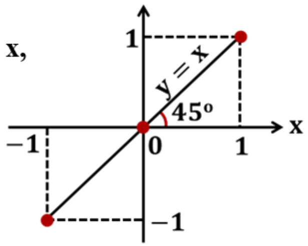
Property -1

(i) $y = \sin(\sin^{-1}x) = x$,

$x \in [-1, 1]$,

$y \in [-1, 1]$,

y is aperiodic.

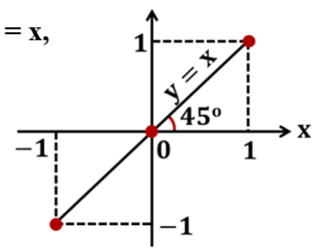


(ii) $y = \cos(\cos^{-1}x) = x$,

$x \in [-1, 1]$,

$y \in [-1, 1]$,

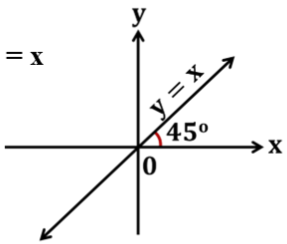
y is aperiodic.



(iii) $y = \tan(\tan^{-1}x) = x$

$x \in R, y \in R$

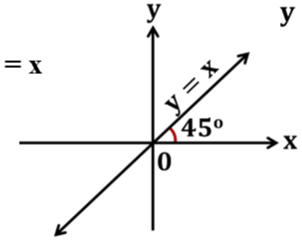
y is aperiodic.



(iv) $y = \cot(\cot^{-1}x) = x$

$x \in R, y \in R$

y is aperiodic.



(v) $y = \text{cosec}(\text{cosec}^{-1}x) = x$

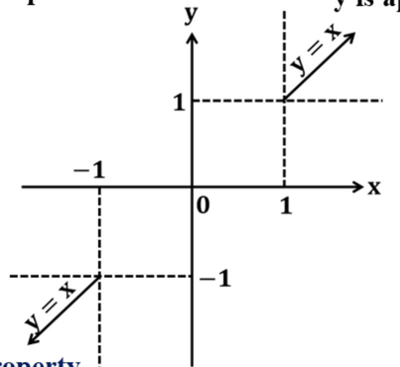
$|x| \geq 1, |y| \geq 1$

y is aperiodic.

(vi) $y = \sec(\sec^{-1}x) = x$

$|x| \geq 1, |y| \geq 1$

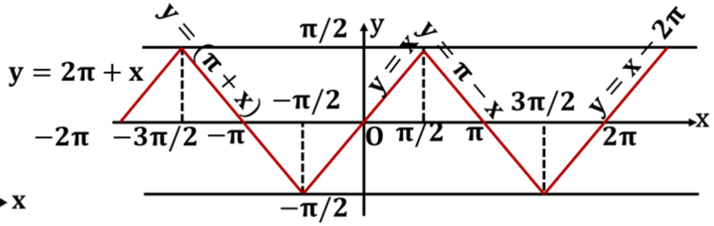
y is aperiodic.

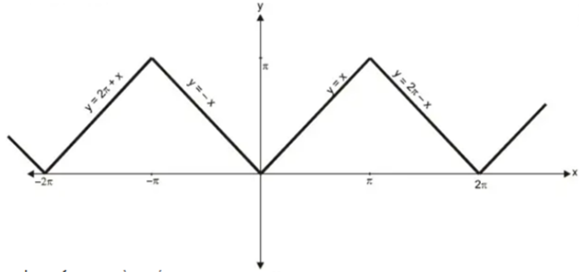


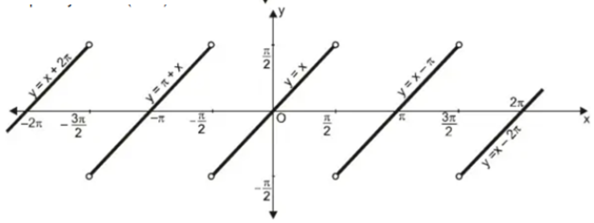
Property

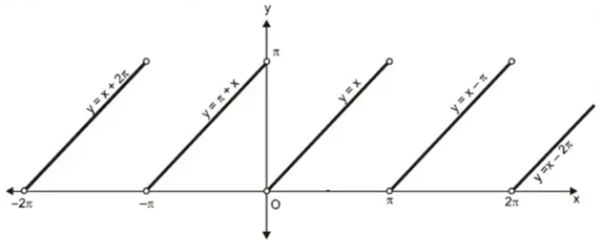
$y = (\sin^{-1}\sin x)$, $x \in R$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

periodic with period 2π









Property – 2

(i) $\sin^{-1}x = \text{cosec}^{-1}\frac{1}{x}$, where $x \in [-1, 1] - \{0\}$

(ii) $\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$, $|x| \geq 1$

(iii) $\cos^{-1}x = \sec^{-1}\frac{1}{x}$, where $x \in [-1, 1] - \{0\}$

(iv) $\sec^{-1}x = \cos^{-1}\frac{1}{x}$, $|x| \geq 1$

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(v) $\cot^{-1} x = \begin{cases} \tan^{-1}\frac{1}{x}\ ;\ x > 0 \\ \pi + \tan^{-1}\frac{1}{x}\ ;\ x < 0 \end{cases}$

Property -3

- $\sin^{-1}(-x) = -\sin^{-1}(x)\ ;\ |x| \le 1$
- $\tan^{-1}(-x) = -\tan^{-1}(x)\ ;\ x \in R$
- $\cos^{-1}(-x) = \pi - \cos^{-1}(x)\ ;\ |x| \le 1$
- $\cot^{-1}(-x) = \pi - \cot^{-1}(x)\ ;\ x \in R$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)\ ;\ |x| \ge 1$
- $\sec^{-1}(-x) = \pi - \sec^{-1}(x)\ ;\ |x| \ge 1$

Property – 4

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ |x| \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in R$

(iii) $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2},\ |x| \ge 1$

3. Properties of Inverse Trigonometric Function

Property – 5

If $x > 0, y > 0$, then

(i) $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x+y}{1-xy}\ ;\ \text{where } xy < 1 \text{ (acute angle)} \\ \pi + \tan^{-1}\frac{x+y}{1-xy}\ ;\ \text{where } xy > 1 \text{ (obtuse angle)} \end{cases}$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

Remember : (i) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

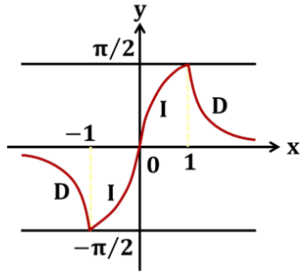
(ii) $\tan^{-1}(1) + \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$

(iii) $\frac{\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3}{\cot^{-1}1 + \cot^{-1}2 + \cot^{-1}3} = 2$

4. Simplification & Transformation of Inverse Functions by Elementary Substitution & Their Graphs

(1) $y = f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \begin{cases} -(\pi + 2\tan^{-1}x); & \text{if } x < -1 \\ 2\tan^{-1}x; & \text{if } |x| \le 1 \\ \pi - 2\tan^{-1}x; & \text{if } x > 1 \end{cases}$

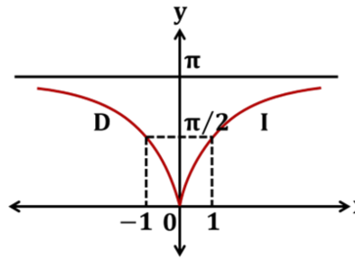


(where, I → increasing & D → decreasing)

(2) $y = f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Domain : R   Range : $[0, \pi)$

$= \begin{cases} -2\tan^{-1}x\ ;\ \text{if } x < 0 \\ 2\tan^{-1}x\ ;\ \text{if } x \ge 0 \end{cases}$



(3) $y = f(x) = \tan^{-1}\frac{2x}{1-x^2}$ Domain : $R - \{-1, 1\}$ Range : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \begin{cases} \pi + 2\tan^{-1}x\ ; & \text{if } x < -1 \\ 2\tan^{-1}x\ ; & \text{if } |x| < 1 \\ -(\pi - 2\tan^{-1}x)\ ; & \text{if } x > 1 \end{cases}$