MIND MAP

Limit

1. Meaning of $x \to a$

We denote it by $x \to a$ (x is not equal to a)

If x approaches to a from right side

then we denote it by $x \to a^+$.

If x approaches to a from left side

then we denote it by $x \to a^-$.

2. Limit

❖ Let a function f(x), at any value of $x = a$, is either not defined or is indeterminate

❖ But is defined and finite at left and/or right neighbourhood points very-very close to $x = a$

❖ Then the value of the function at the neighbourhood points is called Limiting value of function.

❖ Or the function tends to approach its limit as x tends to a.

$f(x) = \dfrac{x^2 - a^2}{x - a}$ is indeterminate at $x = a \left(\dfrac{0}{0}\right)$

Then, limiting value is :

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \ell$$

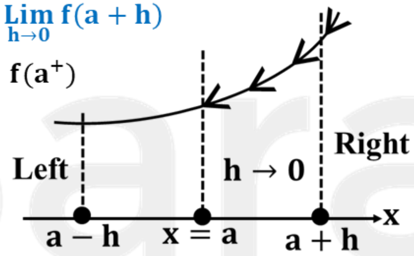
ℓ is called limit of f(x) when $x \to a$ (x tends to a or x approaches a)

3. Right Hand Limit (R.H.L.)

When $x \to a$ from the values of $x > a$ or from right side of a, then corresponding limit is called R.H.L. & is written as

$\lim\limits_{x \to a^+} f(x)$ or $\lim\limits_{h \to 0} f(a + h)$

$f(a + 0)$ or $f(a^+)$

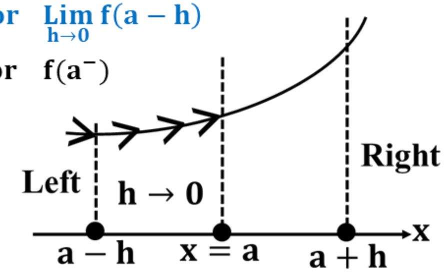


4. Left Hand Limit (L.H.L.)

When $x \to a$ from the values of $x < a$ or from left side of a, then corresponding limit is called L.H.L. & is written as

$\lim\limits_{x \to a^-} f(x)$ or $\lim\limits_{h \to 0} f(a - h)$

$f(a - 0)$ or $f(a^-)$

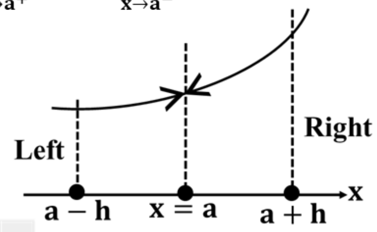


5. Existence of Limit

If R.H.L. = L.H.L. = Finite quantity

i.e. both limits exist, equal and finite.

$\Rightarrow \lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x) =$ Finite quantity



6. Fundamental Theorems of Limits

1. Constant Multiple Rule : $\lim\limits_{x \to c} K.f(x) = KL$
2. Sum Rule : $\lim\limits_{x \to c} (f(x) + g(x)) = L + M$
3. Difference Rule : $\lim\limits_{x \to c} (f(x) - g(x)) = L - M$
4. Product Rule : $\lim\limits_{x \to c} (f(x) \,.\, g(x)) = L \,.\, M$
5. Quotient Rule : $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$ & $M \neq 0$

Note

1. $\lim\limits_{x \to a} (\log f(x)) = \ell og\left(\lim\limits_{x \to a} f(x)\right) = \ell og(L)$ provided $L > 0$
2. $\lim\limits_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to 0} f(x)} = \sqrt[n]{L}$
3. $\lim\limits_{x \to a} f(x)$ where, $f(x) = C \Rightarrow \lim\limits_{x \to a} f(x) = C$

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Limit

6. Fundamental Theorems of Limits

Note

4. $\lim\limits_{x\to a}|f(x)| = |\lim\limits_{x\to a} f(x)| = |L|$

5. $\lim\limits_{x\to a}(f(x))^{g(x)} = \left(\lim\limits_{x\to a} f(x)\right)^{\lim\limits_{x\to a} g(x)} = L^M$

6. $\lim\limits_{x\to a} f(g(x)) = f\left(\lim\limits_{x\to a} g(x)\right) = f(M)$

if f is continuous at $g(x) = M$

7. Indeterminate Forms

1. $\frac{0}{0}$ 2. $\frac{\infty}{\infty}$ 3. $\infty - \infty$

4. $0 \times \infty$ 5. 1^{∞} 6. 0^0 7. ∞^0

8. Methods to Evaluate Limit

(i) Factorization Method :

Consider $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$

If by substituting $x = a$, $\frac{f(x)}{g(x)}$ reduces to the form $\frac{0}{0}$ then $(x-a)$ is a factor of both f(x) and g(x).

So, we first factorize f(x) and g(x) and then cancel out the common factor to evaluate the limit.

Standardization

Result $\lim\limits_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) Rationalization Method :

This is particularly used when either the numerator or the denominator or both involve expression consists of squares roots and on substituting the value of x the rational expression takes the form $\frac{0}{0}$

Algebraic limit Using Some Standard limits :

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots$$

where $|x| < 1$

9. Sandwich / Squeeze Play Theorem

Let f(x), g(x) & h(x) be 3 functions such that :

$f(x) \le g(x) \le h(x)$

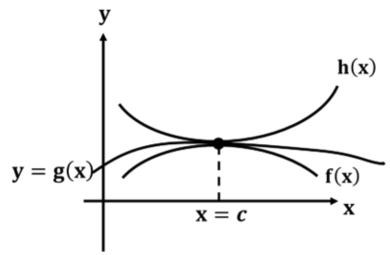
For all values of x in some neighborhood of $x = c$

Then, $\lim\limits_{x\to c} f(x) \le \lim\limits_{x\to c} g(x) \le \lim\limits_{x\to c} h(x)$

And if, $\lim\limits_{x\to c} f(x) = \lim\limits_{x\to c} h(x) = L$

Then,

$\lim\limits_{x\to c} g(x) = L$



10. Limits of Trigonometric Functions

1. $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

2. $\lim\limits_{x\to a}\frac{\sin(x-a)}{(x-a)} = 1$

3. $\lim\limits_{x\to a}\frac{\tan(x-a)}{(x-a)} = 1$

4. $\lim\limits_{x\to 0}\frac{\sin^{-1}x}{x} = 1$

5. $\lim\limits_{x\to 0}\frac{\tan^{-1}x}{x} = 1$

6. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$

Note:

Let[.] denotes greatest integer function

$\lim\limits_{x\to 0}\left[\frac{\sin x}{x}\right] = 0$ $\left[\lim\limits_{x\to 0}\frac{\sin x}{x}\right] = 1$

$\lim\limits_{x\to 0}\left[\frac{\tan x}{x}\right] = 1$ $\lim\limits_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right] = 1$

$\left[\lim\limits_{x\to 0}\frac{\sin^{-1}x}{x}\right] = 1$

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11. Limit of Exponential/ Logarithmic Functions

1. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$
2. $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ell n\,a \quad (a>0)$
3. $\lim\limits_{x\to 0}\dfrac{\ell n(1+x)}{x}=1$

12. 0^0 & ∞^0 Forms

Limits of the form $\lim\limits_{x\to a}\{f(x)\}^{g(x)}$:

Let $L=\lim\limits_{x\to a}(f(x))^{g(x)}$

$\Rightarrow \log_c L=\log_c\left[\lim\limits_{x\to a}(f(x))^{g(x)}\right]$

$=\lim\limits_{x\to a} g(x)\log_c[f(x)]$

13. $(1)^{\infty}$ Form

Result 1. $\lim\limits_{x\to 0}(1+x)^{1/x}=e$

2. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$

Generalized Formula for $(1)^{\infty}$

$$\lim_{x\to a}\{f(x)\}^{g(x)}=e^{\lim\limits_{x\to a} g(x)[f(x)-1]}$$

14. Limit Using Series Expansion

1. $e^x=1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dots\dots.$

   $e^{-x}=1-\dfrac{x}{1!}+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+\dots\dots.$

2. $a^x=e^{x\log_e a}=e^{x\ell na}$

   $a^x=1+\dfrac{x\ell na}{1!}+\dfrac{x^2\ell n^2 a}{2!}+\dfrac{x^3\ell n^3 a}{3!}+\dots a>0$

3. $\ell n(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\dots\dots.$

   for $-1<x\le 1$

   $\ell n(1-x)=-x-\dfrac{x^2}{2}-\dfrac{x^3}{3}-\dfrac{x^4}{4}+\dots\dots.$

4. $\sin x=x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\dots\dots.$

5. $\cos x=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\dots\dots.$

6. $\tan x=x+\dfrac{x^3}{3}+\dfrac{2x^5}{15}+\dots\dots.$

7. $\tan^{-1}x=x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\dots\dots.$

8. $\sin^{-1}x=x+\dfrac{1^2}{3!}x^3+\dfrac{1^2.3^2}{5!}x^5+\dfrac{1^2.3^2.5^2}{7!}x^7+\dots$

   $=x+\dfrac{x^3}{3!}+\dfrac{9x^5}{5!}+\dots\dots.$

9. $\sec^{-1}x=1+\dfrac{x^2}{2!}+\dfrac{5x^4}{4!}-\dfrac{61x^6}{6!}+\dots\dots.$

10. $(1+x)^n=1+nx+\dfrac{n(n-1)x^2}{2!}+\dots\dots n\in Q$

15. L'Hospital Rule

If $\lim\limits_{x\to a} f(x)=0$ or (∞) & $\lim\limits_{x\to a} g(x)=0$ or (∞)

Where, f(x) & g(x) both are differentiable functions at $x\to a$.

Then, $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ are of $\left(\dfrac{0}{0}\right)$ or $\left(\dfrac{\infty}{\infty}\right)$ Forms can be solved by differentiating both N^r & D^r separately.

Thus, $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\left(\dfrac{0}{0}\right)$ or $\left(\dfrac{\infty}{\infty}\right)$ Forms

$=\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}=\lim\limits_{x\to a}\dfrac{f''(x)}{g''(x)}=\dots..$