

1. Definition

$$N = a^x \quad \dots (1)$$

$$\log_a N = x \quad \dots (2)$$

$$a^x = N \Leftrightarrow \log_a N = x$$

Where, $N > 0, a > 0, a \neq 1$

2. Three Fundamental Identities

$$(i) \log_a a = 1$$

$$(ii) \log_{(1/N)} N = -1$$

$$(iii) \log_a 1 = 0$$

(Basic constraints on number & base must be observed.)

Note :

$a^{\log_a N} = N$ is an identity for all $N > 0$ and $a > 0, a \neq 1$

3. Principal Properties of Logarithm

If m, n are arbitrary positive real numbers where $a > 0 ; a \neq 1$

$$(1) \log_a m + \log_a n = \log_a mn$$

$$(2) \log_a m - \log_a n = \log_a \frac{m}{n}$$

$$(3) \log_a m^x = x \log_a m$$

MIND MAP

Logarithm

4. Base changing theorem (BCT)

$$\frac{\log_c a}{\log_c b} = \log_b a$$

Important Results of BCT

$$1. \log_b a = \frac{1}{\log_a b}$$

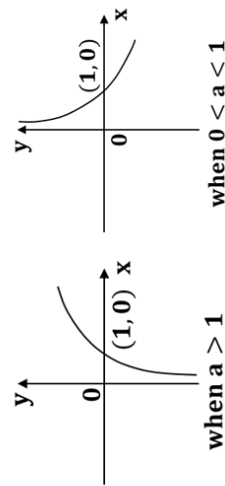
$$2. (\log_b a) \cdot (\log_c b) \cdot (\log_d c) = \log_d a$$

$$3. a^{\log_b(c)} = c^{\log_b a}$$

$$4. \log_a x^m = \frac{1}{x} \log_a m$$

5. Graph of Logarithmic Function

Graph of $y = \log_a x$



6. Antilogarithm

$$\log_a n = m \Leftrightarrow n = \text{antilog}_a m$$

7. Characteristic and Mantissa

$$\log_a N = \text{Integer} + \text{Fraction (+ve)}$$

Integer part $\rightarrow$ Characteristic (n)

Fractional part $\rightarrow$ Mantissa (m)

Important Notes :

(i) The mantissa part of log of a number is always positive ($0 \leq m < 1$)

(ii) If the characteristic of $\log_{10} N$ be n , then the number of digits in N is $(n + 1)$

(iii) If the characteristic of $\log_{10} N$ be $(-n)$, then there will be $(n - 1)$ zeros after decimal and before first significant digit in N .

Important Results from Graphs

$$\log_a x < \log_a y \Leftrightarrow \begin{matrix} x < y & \text{if } a > 1 \\ x > y & \text{if } 0 < a < 1 \end{matrix} \quad \begin{matrix} x > 0, y > 0 \\ x > 0, y > 0 \end{matrix}$$