MIND MAP

Methods of Differentiation

1. Derivative By First Principle

Derivative by first principle/ by delta method/ by ab-initio method :

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Finding the value of the limit is called finding the derivative by first principle.

2. Derivative of standard functions

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^x	e^x
a^x	$a^x \ell na, a>0$
$\ell n\, x, \log_e x$	$1/x$
$\log_a x$	$(1/x)\log_a e,\ a>0, a\neq 1$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
constant	0

Derivative of Inverse Trigonometric Functions

$f(x)$	$f'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}},\ -1<x<1$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}},\ -1<x<1$
$\tan^{-1}x$	$\frac{1}{1+x^2},\ x\in R$
$\operatorname{cosec}^{-1}x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert>1$
$\cot^{-1}x$	$-\frac{1}{1+x^2},\ x\in R$
$\sec^{-1}x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert>1$

3. Sum/Difference Rule

- $\frac{d}{dx}(kf(x)) = k\cdot\frac{d}{dx}f(x) = kf'(x)$ (where k is any constant)
- $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

4. Product Rule

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

= (I Function × Diff. of II) + (II function × Diff. of I)

Note:

$D(f(x)\cdot g(x)\cdot h(x)) = f'(x)\cdot g(x)\cdot h(x) + f(x)\cdot g'(x)\cdot h(x) + f(x)\cdot g(x)\cdot h'(x)$

Differentiate one function at a time & keep repeating as it is & complete the process for all functions one by one.

This result can be generalised upto the product of 'n' functions.

5. Quotient Rule

$$y = \frac{f(x)}{g(x)};\ \frac{dy}{dx} = D\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).f'(x) - f(x).g'(x)}{g^2(x)}$$

6. Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x)$$

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Methods of Differentiation

7. Logarithmic Differentiation

- A function which is product or quotient of a number of functions.
- A function of the form : $[f(x)]^{g(x)}$

  where, f & $g \rightarrow$ derivable functions.

8. Parametric Differentiation

If $x = f(t)$ & $y = g(t)$, $\Rightarrow \frac{dx}{dt} = f'(t)$

& $\frac{dy}{dt} = g'(t)$ Then, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$

9. Derivative of $f(x)$ w.r.t. $g(x)$

If $u = f(x)$ & $v = g(x)$

Then, derivative of $f(x)$ w.r.t. $g(x)$ is given by

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{f'(x)}{g'(x)}$$

10. Derivative of Implicit Functions

$\phi(x, y) = 0$

- To find dy/dx of implicit functions, we differentiate each term w.r.t. x regarding y as a function of x.

11. Derivative of Homogeneous Eq.

A homogeneous equation of degree n represents 'n' straight lines passing through the origin.

$y = mx$; $\frac{dy}{dx} = \frac{y}{x} = m = \text{constant}$ Hence, $\frac{d^2y}{dx^2} = 0$

12. Derivative of Inverse Functions

If the inverse functions f & g are defined by

$y = f(x)$ & $x = g(y)$ $\Rightarrow g(x) = f^{-1}(x) \Rightarrow f(g(x)) = x$

Differentiate w.r.t. x $\Rightarrow f'(g(x)) \cdot g'(x) = 1$

$$\Rightarrow f'(g(x)) = \frac{1}{g'(x)}$$

13. Derivatives of Inverse Trigo Functions

- For terms of the form $x^2 + a^2$ or $\sqrt{x^2 + a^2}$,
  Put $x = a\tan\theta$ OR $a\cot\theta$.
- For terms of the form $x^2 - a^2$ or $\sqrt{x^2 - a^2}$,
  Put $x = a\sec\theta$ OR $a\,\text{cosec}\theta$.
- For terms of the form $a^2 - x^2$ or $\sqrt{a^2 - x^2}$,
  Put $x = a\sin\theta$ OR $a\cos\theta$.
- If both $\sqrt{a+x}$, $\sqrt{a-x}$ are present,
  then put $x = a\cos 2\theta$.
- For the type $\sqrt{(x-a)(b-x)}$ or $\sqrt{\frac{x-a}{b-x}}$
  put $x = a\cos^2\theta + b\sin^2\theta$.
- For the type $\sqrt{(x-a)(x-b)}$ or $\sqrt{\frac{x-a}{x-b}}$
  put $x = a\sec^2\theta - b\tan^2\theta$.

14. Successive Differentiation

Important Result:

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{\frac{d}{dx}\left(\frac{dx}{dy}\right)}{\frac{dy}{dx}} = \frac{\frac{d}{dx}\left(\frac{1}{\frac{dy}{dx}}\right)}{\frac{dy}{dx}}$$

$$= \frac{\frac{-1}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{d^2y}{dx^2}}{\frac{dy}{dx}} \Rightarrow \frac{d^2x}{dy^2} = \frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

15. Derivative of Determinants

$$F(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ \ell & m & n \end{vmatrix}$$

$$F'(x) = \begin{vmatrix} f' & g' & h' \\ u & v & w \\ \ell & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ \ell & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ \ell' & m' & n' \end{vmatrix}$$