# MIND MAP

# Permutation & Combination

## 1. Notation of Factorial & its Algebra

The continued product of first n, natural number is called as "n factorial" and denoted by n! or $\lfloor n$.

$n! = 1.2.3.4. \ldots\ldots .(n-1).n$

$3! = 1.2.3 = 6$

$4! = 1.2.3.4 = 24$

### Special Results

- $0! = 1$

**Exponent of prime number p in (n!).**

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots\ldots + \left[\frac{n}{p^s}\right]$$

where 's' is the largest positive integer such that

$p^s \le n < p^{s+1}$

## 2. Permutation

Permutation means arrangement of things in a definite order which may be alike or different taken some or all at a time.

e.g. Selecting 11 players of cricket team out of 16 and deciding their bating order.

### Theorem-:

Number of permutations of 'n' distinct things taken 'r' at a time

$0 \le r \le n$

$${}^nP_r = P(n, r) = A_r^n = \frac{n!}{(n-r)!}$$

$${}^nP_n = P(n, n) = A_n^n = n!$$

Combination is selection of one or more things out of 'n' things which may be alike or different, taken some or all at a time

## 3. Combination

Combination/selection/collection/committee refers to the situation where order of occurrence of the event is not important.

### Theorem

Number of combination/selection of 'n' distinct things taken 'r' at a time

$0 \le r \le n$

$${}^nC_r = C(n, r) = \binom{n}{r} = \frac{n!}{(r)!\ (n-r)!}$$

### Important Results

(i) ${}^nC_0 = 1$ Selection of None

(ii) ${}^nC_n = 1$ Selection of All

(iii) ${}^nC_r = {}^nC_{n-r}$

No. of selection = No. of rejection

$\Rightarrow {}^{10}C_3 = {}^{10}C_7$

So, ${}^nC_r = {}^nC_{n-r}$

(iv) If ${}^nC_x = {}^nC_y$

$\Rightarrow x = y$ OR ${}^nC_x = {}^nC_{n-y}$

$\Rightarrow x + y = n$

(v) ${}^nP_r = {}^nC_r .r!$ First select, then arrange

(vi) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

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## 4. Formation of Groups & their distribution

Number of ways in which (m + n) different things can be divided into two groups containing ‘m’ & ‘n’ things.

$$\begin{tikzcd} & m+n \arrow[dl] \arrow[dr] & \\ m & & n \end{tikzcd}$$

(i) $m \neq n$; then number of ways $= \dfrac{(m+n)!}{m!\,n!}$

(ii) If $m = n$,

Then number of ways $= \dfrac{(n+n)!}{n!\,n!\,2!} = \dfrac{2n!}{n!\,n!\,2!}$

similarly , (m + n + p) different things can be divided into 3 unequal groups ($m \neq n \neq p$) of m, n & p things

$$= \frac{(m+n+p)!}{m!\,n!\,p!}$$

## 5. Total Number of Combinations

(1) Number of ways of selection of one or more things out of ‘n’ different things

One thing can be selected in $^nC_1$ ways

Two things can be selected in $^nC_2$ ways

Three things can be selected in $^nC_3$ ways

:

‘n’ things can be selected in $^nC_n$ ways

$\therefore$ Total ways $= {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2^n - 1$

## 6. Combination of things which are NOT all different

(1) Number of ways of selection of ‘r’ identical things out of n identical things = 1

(2) The number of ways of selecting zero or more things out of n identical things = n + 1

(3) Number of ways of selecting one or more things (or at least one thing) out of n identical things = n

(4) Number of ways of selecting one or more things out of which

‘p’ are alike of one kind,

‘q’ are alike of second kind,

‘r’ are alike of third kind,

& remaining ‘s’ are different is

$= (p+1)(q+1)(r+1)\,2^S - 1$

## 7. Problems Based on Number Theory

### Number of Divisors & Their Sum

(a) Number of divisors

$N = 2^p \times 3^q \times 5^r \ldots\ldots$

The Number is divisible by $1, 2, 2^2, 2^3 \ldots., 2^p$ Hence, (p + 1) divisors and so on….

Number of divisors of N

$= (p+1)(q+1)(r+1) \ldots.$

(b) Proper divisors :

All the divisors excluding N are called proper divisors.

= Total divisors – 1

(c) No. of ways in which N can be resolved as a product of 2 divisors:

Let total no. of divisors are “n”

$$= \begin{cases} \dfrac{n}{2}; & \text{If n is even} \\ \dfrac{n-1}{2} + 1; & \text{If n is odd} \end{cases}$$

## 8. Permutation of Alike Objects taken all at a Time

Number of permutation of ‘n’ things in which

- ‘p’ are alike of one kind
- ‘q’ are alike of another kind
- ‘r’ are all different kinds

is $= \dfrac{n!}{p!\,q!}$

## 9. Circular Permutation

### Theorem-I:

The no. of circular permutations of n distinct objects is :$(n-1)!$

### Theorem-II:

If anticlockwise and clockwise are considered to be same total number of circular permutation are given by :

$$\frac{(n-1)!}{2}$$

If we arrange flowers or garland beads in a neckless then there is no distinction between clockwise & anticlockwise direction.

### Theorem-III:

The no. of circular permutation of n distinct objects taken r at a time is :

${}^nC_r \cdot (r-1)!$

### Note

If we have n different things taken r at a time in form of a garland or neckless, required no. of arrangements $= \frac{{}^nC_r \times (r-1)!}{2}$

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## 10. Geometric Problems

If There are n points in a plane, no three of which are collinear, then

(a) Number of straight lines :

No. of ways in which we can select any two points $= {}^nC_2$.

(b) Number of triangles

: Number of ways in which we can select any 3 points. $= {}^nC_3$

(c) Number of diagonals in polygon

: No. of vertices = No. of sides

${}^nC_2$ = No. of sides + No. of diagonals

## Distribution of Alike Objects

Total number of ways in which 'n' identical coins can be distributed among 'r' persons so that each person may get any number of coins is :

$${}^{n+r-1}C_{r-1} = \frac{(n+r-1)!}{(r-1)!\,(n)!}$$

### Type – I

1. Let a homogeneous expression with degree 'n' in 'r' variables be :

$$(x_1 + x_2 + x_3 \ldots\ldots + x_r)^n$$

Then,

Number of different (dissimilar) terms in the expansion is equivalent to distribution of 'n' identical coins among 'r' persons $= {}^{n+r-1}C_{r-1}$

### Type – II

Let,

$$x_1 + x_2 + x_3 + \ldots\ldots x_r = n$$

Then,

Number of non-negative integral solutions of the above equation is $= {}^{n+r-1}C_{r-1}$

## 11. Derangement

If n things are arranged in such a way that none of them occupies its original place, is called Derangement.

$D_n =$

$$n!\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + (-1)^n \frac{1}{n!}\right]$$