MIND MAP

Probability

1. Random Experiments

An experiment is called random experiment if

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcome in advance.

2. Trial

Each performance of the random experiment is called a trial.

3. Out Comes And Sample Space

A possible result of a random experiment is called its outcome.

And the set of all possible outcomes of a random experiment is called the sample space associated with it (denoted by S).

Note:

(1) Each element of the sample space is called a sample point.

OR

Each outcome of the random experiment is also called sample point.

(2) A sample space having finite number of sample points, is called "Discrete sample space"

4. Event

Occurrence of particular outcomes of an experiment under a situation is known as an event.

In other words any subset E of a sample space S is called an event.

Note:

The maximum number of events which can be associated with an experiment is 2^n, where n is the number of elements in the sample space.

i.e., ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

5. Impossible and Sure Events

Experiment : Rolling a die

Sample Space : Set of outcomes : {1, 2, 3, 4, 5, 6}

Event 'E' : The no. appearing on the die is 7". $= \phi$

❖ ϕ is called an Impossible Event .

Event 'F' : The number turns up is odd or even. $= S$

❖ S, i.e., the whole sample space is called the Sure Event.

6. Simple Event

❖ If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

❖ Which cannot be further split.

❖ In a sample space containing n distinct elements, there are exactly n simple events.

7. Compound Event

If an event has more than one sample point, it is called a compound event.

8. Three Most Important Events

1. Equally Likely Events
2. Mutually Exclusive Events
3. Exhaustive Events

9. Equally Likely Events

Events are said to be equally likely when no particular event have preference to occur in relation to the other event.

Note:

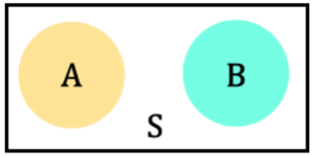
Events which are equally likely are also known as "Equiprobable“.

MIND MAP

Probability

10. Mutually Exclusive/ Disjoint/Incompatible Events

- Two events A and B are said to be mutually exclusive events if their simultaneous occurrence is impossible
- i.e. both the events can not occur together.



$A \cap B = \phi$

11. Exhaustive Events

Let there be an experiment.

Sample space : S

Events : $E_1, E_2, \ldots\ldots E_n$ and if

$$E_1 \cup E_2 \cup E_3 \cup \ldots\ldots \cup E_n = \bigcup_{i=1}^{n} E_i = S$$

Then, $E_1, E_2, \ldots\ldots, E_n$ are called exhaustive events.

In other words, events $E_1, E_2, \ldots\ldots, E_n$ are said to be exhaustive if atleast one of them necessarily occurs whenever the experiment is performed.

Note:

If, $E_i \cap E_j = \phi \quad \text{for } i \neq j$

i.e., events E_i & E_j are pairwise disjoint and

$$\bigcup_{i=1}^{n} E_i = S$$

Then, events $E_1, E_2, \ldots\ldots, E_n$ are called mutually exclusive and exhaustive events.

12. Probability

If an experiment has

Total no. of outcomes : (m+ n)

'm' outcomes are in favour of an event 'A'

'n' outcomes are not in favour of the event

Then,

The probability of occurrence of the event 'A' is defined by :

$$P(A) = \frac{m}{m+n} = \frac{\text{No. of favourable out comes}}{\text{Total no. of out comes}}$$

If $P(A) = 0$ Event is impossible

$P(A) = 1$ Event is sure $(0 \leq P(A) \leq 1)$

Note:

(i) More is the probability of an event, more are chances of its happening.

(ii) $P(\phi) = 0$ & $P(S) = 1$

i.e. nothing outside sample space can occur.

(iii) Probability of non−occurrence of A

$$P(\bar{A}) \text{ or } P(A') \text{ or } P(A^C) = \frac{n}{m+n}$$

$$= 1 - P(A)$$

13. Odds In Favour And odds Against of An Event

Introduction

If an experiment A has

Total no. of outcomes : (m+ n) → in favour of A (m), in against of A (n)

Then,

$$\text{Odds in favour of event A} = \frac{m}{n} = \frac{\text{No. of outcomes in favour of event A}}{\text{No. of outcomes not in favour of event A}}$$

MIND MAP

Probability

Introduction

Odds in against of event $A = \frac{n}{m}$

$= \frac{\text{No. of outcomes not in favour of event A}}{\text{No. of outcomes in favour of event A}}$

Note:

If $P(A) = \frac{a}{b}$

Then,

(i) Odds in favour of event $A = a : b - a$

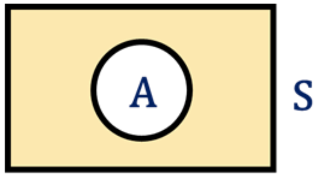
(ii) Odds against of event $A = b - a : a$

14. Algebra of Events

❑ Complementary Event

❖ For every event A, there corresponds another event A' or $\bar{A}$ called the complementary event to A.

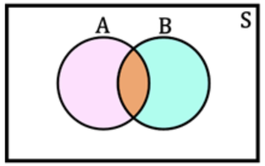
❖ It is also called the event 'not A'.



A' or $\bar{A}$

❑ The Event 'A or B' ($A \cup B$)

The union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both.

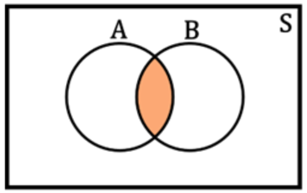


$A \cup B$

❑ The Event 'A and B' ($A \cap B$)

❖ The intersection of two sets ($A \cap B$) is the set of those elements which are common to both A and B.

❖ i.e., which belong to both 'A and B'.



$A \cap B$

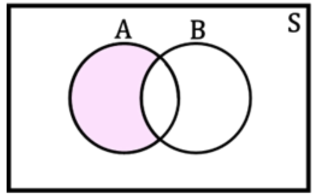
Note:

The set $A \cap B = \{(6, 5), (6, 6)\}$ may represent the event ' the score on the first throw is six and the sum of the scores is at least 11.'

❑ The Event 'A But Not B'

A – B is the set of all those elements which are in A but not in B.

$\therefore A - B = A \cap B'$ OR $A \cap \bar{B}$



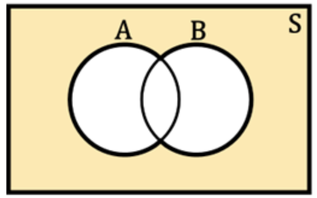
$A \cap \bar{B}$

❑ The Event "Neither A Nor B"

❖ The set of the elements which are neither in set A nor in set B.

❖ i.e. $S - (A \cup B)$

❖ It is denoted by $(\bar{A} \cap \bar{B})$.



$\bar{A} \cap \bar{B}$

Note:

If A & B are two subsets of a universal set U, then

$$\left.\begin{array}{l}
\text{(i)}\ \overline{A \cup B} = \bar{A} \cap \bar{B} \\
\quad (A \cup B)^c = A^c \cap B^c \\
\text{(ii)}\ \overline{A \cap B} = \bar{A} \cup \bar{B} \\
\quad (A \cap B)^c = A^c \cup B^c
\end{array}\right\} \text{De Morgan's Law}$$

MIND MAP

Probability

Note:

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) $(A / B) \longrightarrow$ read as A minus B

$$P(A / B) = P(A) - P(A \cap B)$$

15. Addition Theorem of Probability

Addition theorem

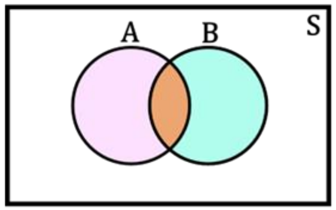
Let, there be an experiment &
If A and B are two events associated with it.

Then,

- $P(A \cup B)$ is called the sum of the probabilities of all the sample points in $(A \cup B)$ OR
- P(Occurrence of atleast one of the events from A and B)

  OR
- P(A or B or both)

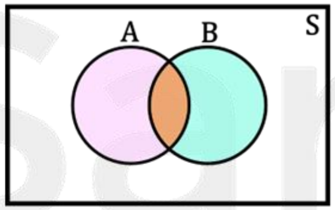
  OR
- $P(A + B)$



$A \cup B$

Thus,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(\bar{A} \cap B)$$
$$= P(B) + P(A \cap \bar{B})$$
$$= 1 - P(\bar{A} \cap \bar{B})$$
$$= 1 - P(\overline{A \cup B})$$



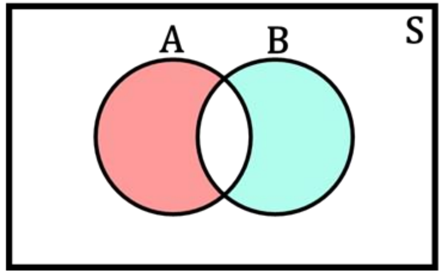
$A \cup B$

Note:

(i) P(occurrence of exactly one of the events)

or P(A or B but not both)

$$\Rightarrow P(A \cap \bar{B}) + P(\bar{A} \cap B)$$
$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - 2P(A \cap B)$$



Note:

(ii) If A and B are mutually exclusive events then :

$$P(A \cup B) = P(A) + P(B) \quad \{\because P(A \cap B) = 0\}$$

(iii) If A and B are exhaustive events then :

$$P(A \cup B) = 1$$

(iv) If E_1, E_2 .... E_n are mutually exclusive & exhaustive, then :

$$P(E_1 \cup E_2 \ldots\ldots\ldots \cup E_n) = 1$$
$$\Rightarrow P(E_1) + P(E_2) \ldots\ldots\ldots\ldots + P(E_n) = 1$$

(v) $P(A \cap B) \le P(A)$

OR

$$P(B) \le P(A \cup B) \le P(A) + P(B)$$

MIND MAP

Probability

16. Conditional Probability

$P\left(\frac{A}{B}\right) =$ Probability of occurrence of A when B has already occurred

$$\therefore P\left(\frac{A}{B}\right) = \frac{\text{Number of cases favourable to } A \cap B}{\text{Number of cases favourable to } B}$$

$$P\left(\frac{A}{B}\right) = \frac{\dfrac{\text{No. of cases favourable to } A \cap B}{\text{No. of cases in the sample space}}}{\dfrac{\text{No. of cases favourable to } B}{\text{No. of cases in the sample space}}}$$

$$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \quad (P(B) \neq 0)$$

Similarly,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \quad (P(A) \neq 0)$$

17. Multiplication Theorem

$$\left.\begin{aligned} P(A \cap B) &= P(A)\, P(B / A) \\ &= P(B)\, P(A / B) \end{aligned}\right\} \begin{aligned} &P(A) \neq 0 \ \& \\ &P(B) \neq 0 \end{aligned}$$

This result is known as the multiplication theorem of probability.

Generalized Form:

Let there be 'm' events :

$A_1, A_2, \ldots\ldots, A_m$ Such that ,

$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_m) \neq 0$

Then,

$$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_m) = P(A_1)\, P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1A_2}\right) P\left(\frac{A_4}{A_1A_2A_3}\right) \ldots P\left(\frac{A_m}{A_1A_2\ldots A_{m-1}}\right)$$

Where $A_1A_2\ldots A_{m-1} = A_1 \cap A_2 \cap \ldots A_{m-1}$

18. Dependent & Independent Events

Let there be an experiment with,

Events : A and B

Events A and B are said to be independent if occurrences or non occurrence of one does not affect the probability of occurrence or non-occurrence of the other.

If A & B are Independent then,

$$P\left(\frac{A}{B}\right) = P(A) \quad \& \quad P\left(\frac{B}{A}\right) = P(B)$$

And in this case Multiplication Theorem :

$P(A \cap B) = P(A) \cdot P(B)$

Note:

(i) Dependent / Independent events come from two different experiments while mutually exclusive events come form the same experiment.

(ii) $P(A \cap B) = P(A) \,.\, P(B)$

is generally used as a defining equation for independent events.

(iii) If A and B are independent then,

$P(A \cup B) = P(A) + P(B) - P(A) \,.\, P(B)$

(iv) If A and B are independent then,

$(\bar{A} \text{ and } \bar{B})$ $(A \text{ and } \bar{B})$ $(\bar{A} \text{ and } B)$

are also independent.

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Probability

19. Difference Between 'Independent' And 'Mutually Exclusive' Events

(1) Mutually exclusive events are defined in terms of events

Whereas,

Independent events is defined in terms of probabilities of events.

(2) Mutually exclusive events can not have a point in common

i.e. $A \cap B = \phi$ & $P(A \cap B) = 0$

Whereas,

Independent events can have a point in common.

Experiment : A die is rolled

Event A : Getting a multiple of 3

$A = \{3, 6\}$ $P(A) = 1/3$

Event B : Getting an even face

$B = \{2, 4, 6\}$ $P(B) = 1/2$

$P(A \cap B) = 1/6$

$\Rightarrow P(A \cap B) = P(A) . P(B)$

Hence, Independent Events but not mutually exclusive.

20. Addition Theorem for three events

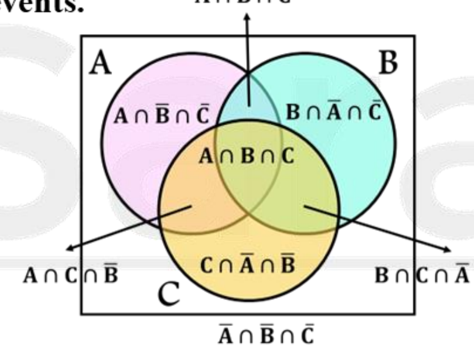
If A, B and C are three events then

$P(A \cup B \cup C)$:

Sum of probabilities of all the sample points in $(A \cup B \cup C)$

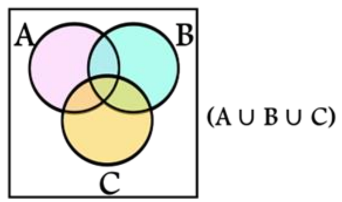
OR

Probability of occurrence of atleast one of the events.



(1) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$- P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$



21. Total Probability Theorem

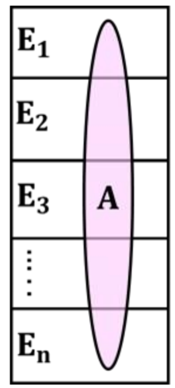
Let there be a random experiment.

$E_1, E_2, \ldots\ldots, E_n$ be 'n'

Mutually Exclusive

& Exhaustive Events

With Non-zero Probabilities.



Now,

$A \longrightarrow$ any arbitrary event of the sample space of the above random experiment

With $P(A) > 0$

Then, the P(A) can be given as :

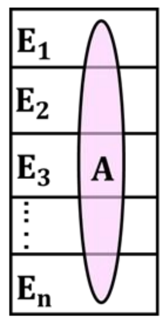
$P(A) =$

$$P(A \cap E_1) + P(A \cap E_2) + \ldots\ldots\ldots + P(A \cap E_n)$$

$$= P(E_1) \cdot P\left(A/E_1\right) + P(E_2) \cdot P\left(A/E_2\right)$$

$$+ \ldots + P(E_n) \cdot P\left(A/E_n\right)$$

$$\therefore P(A) = \sum_{i=1}^{n} P(E_i) P\left(A/E_i\right)$$



MIND MAP

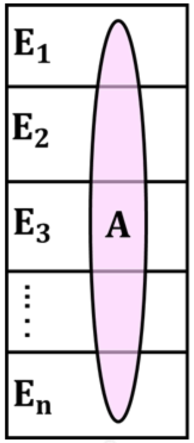
Probability

22. Bayes Theorem

Let, there be 'n' M. E. and exhaustive events

$E_1, E_2, E_3, \ldots\ldots E_n$

Event A : can occur with any of the events $E_1, E_2, E_3, \ldots\ldots E_n$



Given : Event A has already occurred

Then,

P (A has occurred with event E_i)

$= P\left({}^{E_i}/_{A}\right)$ {conditional probability}

Thus, from conditional probability

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i).P(A/E_i)}{P(A)}$$

(using multiplication theorem)

And by total probability theorem

$$P(A) = P(E_1)\ P\left(\frac{A}{E_1}\right) + P(E_2)\ P\left(\frac{A}{E_2}\right) + \ldots\ldots\ldots + P(E_n)\ P\left(\frac{A}{E_n}\right)$$

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i).P(A/E_i)}{P(A)}$$

$$\therefore P\left(\frac{E_i}{A}\right) = \frac{P(E_i).P(A/E_i)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + \ldots\ldots + P(E_n)\,P\left(\frac{A}{E_n}\right)}$$

23. Probabilities Through Statistical (Stochastic) Tree Diagram

These tree diagrams are generally drawn by economists and give a simple approach to solve a problem.

Mathematical Expectation

If 'P' represents a person's chance of success in any venture

'M' represents the sum of money which he will receive in case of success

Then, the sum of money denoted by 'P·M' is called his expectation.

24. Geometrical Probability (Continuous Sample Space)

Probability Distribution

(i) Discrete Random Variables

❖ A random variable is called a Discrete Random Variable if it can take only finitely many values.

(ii) Continuous Random Variables

❖ A random variable is called a Continuous Random Variable if it can take any value between certain limits.

25. Probability Distribution of A discrete Random Variable

Let there be a Discrete random variable : x

Assuming Values : $x_1, x_2, x_3, \ldots\ldots, x_n$

Corresponding to the various outcomes of a Random Experiment.

If,

$P(\text{occurrence of } x = x_i) = P(x_i) = p_i$

$(1 \le i \le n)$

Such that, $p_1 + p_2 + p_3 + \ldots\ldots + p_n = 1$

Then the function,

$P(x_i) = P_i\ (1 \le i \le n)$

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Probability

25. Probability Distribution of A discrete Random Variable

is called the probability function of the random variable x.

And the set,

$\{P(x_1),\ P(x_2),\ P(x_3),\ \ldots\ldots,\ P(x_n)\}$

is called the probability distribution of x.

26. Mean and Variance of a Probability Distribution

(i) Mean

Randam Variable (x_i)	Probability (p_i)	$p_i x_i$
x_1	p_1	$p_1 x_1$
x_2	p_2	$p_2 x_2$
⋮	⋮	⋮
⋮	⋮	⋮
x_n	p_n	$p_n x_n$

Then, Mean $(\mu) = \frac{\sum_{i=1}^{n} p_i x_i}{\sum_{i=1}^{n} p_i} = \sum_{i=1}^{n} p_i x_i$

$\because \left\{\sum_{i=1}^{n} p_i = 1\right\}$

(ii) Variance

$$\sigma^2 = \sum_{i=1}^{n} p_i (x_i - \mu)^2$$

$$= \sum_{i=1}^{n} (p_i x_i^2 - 2\mu p_i x_i + p_i \mu^2)$$

$$= \sum_{i=1}^{n} p_i x_i^2 - 2\mu \sum_{i=1}^{n} p_i x_i + \mu^2 \sum_{i=1}^{n} p_i$$

$$= \sum_{i=1}^{n} p_i x_i^2 - \mu^2$$

(iii) Standard Deviation

$$S.D. = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$$

27. Bernoulli trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes Success or Failure

(iv) The probability of success remains the same in each trial.

Experiment

↓

n independent trials (Bernoulli's trials)

↓

If each of these trials have two possible outcomes

Success	$p + q = 1$	Failure
$P(S) = p$		$P(F) = q$

Then, $P(r \text{ successes}) = {}^{n}C_r\, p^r q^{n-r}$

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Probability

28. Binomial Probability Distribution

Let an experiment has n independent trials.

Possible Outcomes : Success or Failure

Random Variable : getting no. of successes in the experiment

Then,

P(exactly r-successes) = $P(x = r)$

$= {}^nC_r\, p^r \cdot q^{n-r}$

Where,

$p = P(\text{success})$ $q = P(\text{failure})$

29. Mean of BPD of a Random Variable

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	${}^nC_0 p^0 q^n$	$0 \times {}^nC_0 p^0 q^n$	$0^2 . {}^nC_0 p^0 q^n$
1	${}^nC_1 p^1 q^{n-1}$	$1 \times {}^nC_1 p^1 q^{n-1}$	$1^2 . {}^nC_1 p^1 q^{n-1}$
2	${}^nC_2 p^2 q^{n-2}$	⋮	⋮
3	${}^nC_3 p^3 q^{n-3}$	⋮	⋮
⋮	⋮	⋮	⋮
r	${}^nC_r p^r q^{n-r}$	$r \times {}^nC_r p^r . q^{n-r}$	$r^2 . {}^nC_r p^r q^{n-r}$

$$\text{Mean } (\mu) = \sum p_i x_i = \sum_{r=0}^{n} r . {}^nC_r . p^r . q^{n-r}$$

$$= \sum_{r=0}^{n} r \frac{n}{r} . {}^{n-1}C_{r-1} . p^r . q^{n-r}$$

$$= p . n \sum_{r=1}^{n} {}^{n-1}C_{r-1} . p^{r-1} . q^{n-r}$$

$$\text{Mean } (\mu) = p . n \sum_{r=1}^{n} {}^{n-1}C_{r-1} . p^{r-1} . q^{n-r}$$

$$= np\,[{}^{n-1}C_0 . p^0 q^{n-1} + {}^{n-1}C_1 . p^1 q^{n-2} + \ldots\ldots + {}^{n-1}C_{n-1} p^{n-1} q^0]$$

$$= np\,(p+q)^{n-1} = np$$

30. Variance of BPD of a Random Variable

$$\sigma^2 = \sum p_i x_i^2 - \mu^2$$

$$\sum p_i x_i^2 = \sum_{r=0}^{n} r^2 . {}^nC_r . p^r q^{n-r}$$

$$= \sum_{r=0}^{n} r^2 . \frac{n}{r} . {}^{n-1}C_{r-1} . p^r . q^{n-r}$$

$$= \sum_{r=0}^{n} n.(r - 1 + 1) . {}^{n-1}C_{r-1} . p^r . q^{n-r}$$

$$\sigma^2 = \sum p_i x_i^2 - \mu^2 \qquad \sum p_i x_i^2 =$$

$$\sum_{r=0}^{n} n.(r - 1 + 1) . {}^{n-1}C_{r-1} . p^r . q^{n-r}$$

$$= n\left[\sum_{r=0}^{n} (r-1) . {}^{n-1}C_{r-1} . p^r . q^{n-r} + \sum_{r=0}^{n} {}^{n-1}C_{r-1} . p^r . q^{n-r}\right]$$

$$= p^2 n . (n-1) \sum_{r=2}^{n} {}^{n-2}C_{r-2} . p^{r-2} . q^{n-r} + pn \sum_{r=1}^{n} {}^{n-1}C_{r-1} . p^{r-1} . q^{n-r}$$

$$= p^2 . n\,(n-1)\,(p+q)^{n-2} + pn\,.\,(p+q)^{n-1}$$

$$\sum p_i x_i^2 = p^2 . n\,(n-1)\,(p+q)^{n-2} + pn\,.\,(p+q)^{n-1}$$

$$= p^2 . n\,(n-1) + pn \quad (\because p + q = 1)$$

And, $(\mu) = np$

$$\therefore\ \sigma^2 = \sum p_i x_i^2 - \mu^2 = p^2 n^2 - p^2 n + pn - n^2 p^2$$

$$= pn\,(1-p) = npq$$

Standard Deviation of BPD of a Random Variable

We know that,

Positive value of square root of variance is called standard deviation.

$$\therefore\ SD = \sqrt{\sigma^2} = \sqrt{npq}$$

BPD of a Random Variable

Mean $(\mu) = np$ $\quad \sigma^2 = npq$

$SD = \sqrt{\sigma^2} = \sqrt{npq}$