MIND MAP

Point

1. Distance Formula

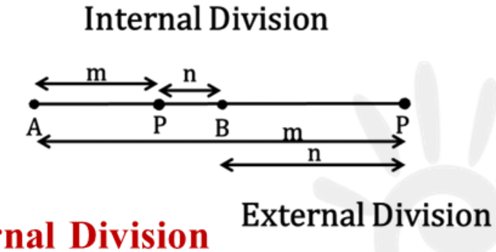
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance of Point A (x_1, y_1) from origin

$= \sqrt{x_1^2 + y_1^2}$

2. Section Formula

The coordinates of point $P(x, y)$ dividing the line joining the point $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$



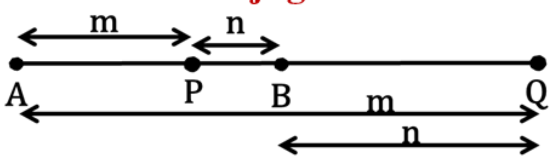
3. Internal Division

$$\frac{AP}{PB} = \frac{m}{n} \quad P \equiv \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

4. External Division

$$\frac{AP}{PB} = \frac{m}{n} \quad P \equiv \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$$

5. Harmonic Conjugate



If P divides AB internally in the ratio m : n and Q divides AB externally in the same ratio m : n P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically : $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$

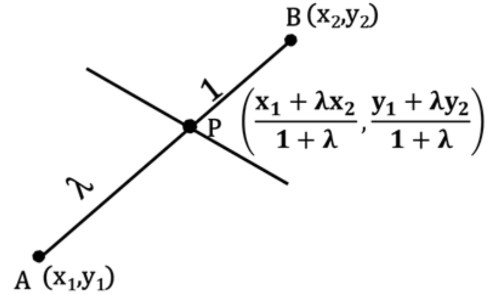
i.e. AP, AB & AQ are in H.P.

Line joining (x_1, y_1) and (x_2, y_2) is divided by

Line $ax + by + c = 0$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $\lambda : 1$ then

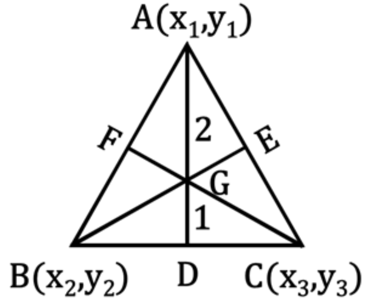
$$\lambda = -\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$$

If ratio is positive, line divides internally, if ratio is negative, line divides externally.



6. Centres of Triangle

Co-ordinates of centroid

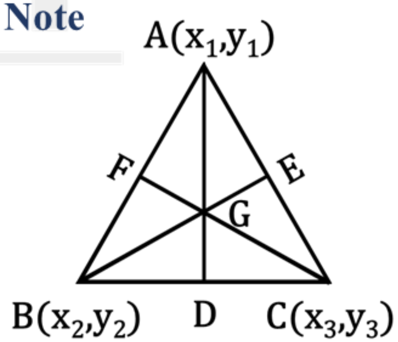


Point of intersection of the medians is the centroid.

Centroid divides the medians in the ratio of 2:1

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Note



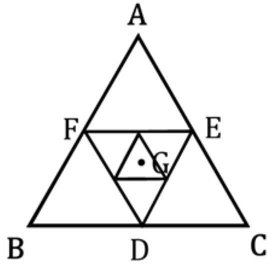
(i) Area of ΔGAB = Area of ΔGAC

= Area of $\Delta GBC = \frac{1}{3}$ Area of ΔABC

MIND MAP

Point

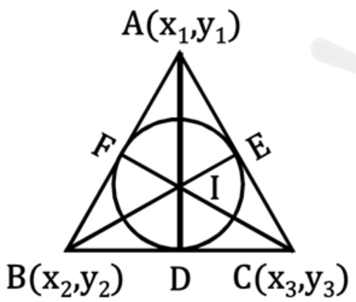
Co-ordinates of centroid



(ii) Centroid of ΔABC is centroid of triangle formed by joining the points dividing the three sides in same ratio.

(iii) Centroid always lies inside the triangle

Co-ordinates of In-centre



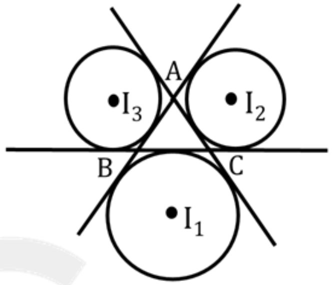
Point of intersection of Internal angle bisectors

Angle bisector divides the opposite sides in the ratio of sides containing it

In-centre divides the angle bisectors internally in the ratio $(b + c) : a$, $(a + b) : c$, $(c + a) : b$ respectively

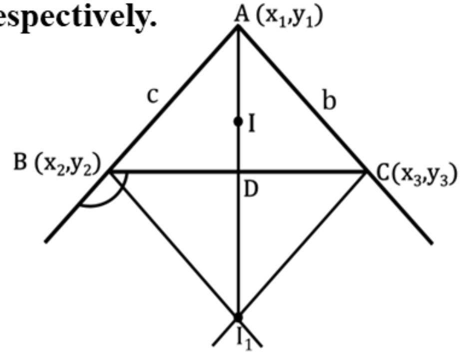
$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Co-ordinates of Ex-centres



The centre of the circle which touches side BC and the extended portions of sides AB and AC is called the ex-centre of ΔABC with respect to the vertex A. It is denoted by I_1

Similarly ex-centers of ΔABC with respect to vertices B and C are denoted by I_2 and I_3 respectively.



As I divides AD internally in the ratio $(b + c) : a$

I_1 divides AD externally in the same ratio $(b + c) : a$

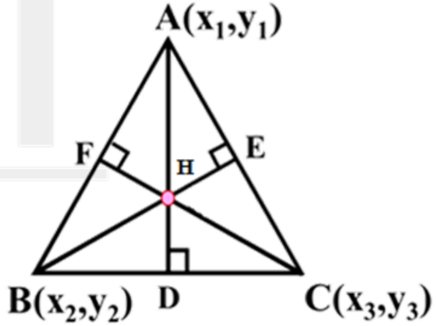
I and I_1 are harmonic conjugates of each other

$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right)$$

$$I_3 \equiv \left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

7. Coordinates of Orthocentre



P.O.I. of perpendiculars drawn from vertices on opposite sides (altitudes) of a triangle

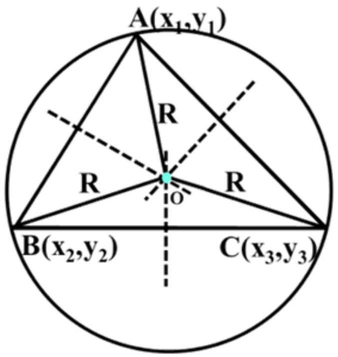
$$\left(\frac{x_1\tan A + x_2\tan B + x_3\tan C}{\tan A + \tan B + \tan C}, \frac{y_1\tan A + y_2\tan B + y_3\tan C}{\tan A + \tan B + \tan C}\right)$$

Can also be obtained by solving the equation of any two altitudes

MIND MAP

Point

8. Coordinates of Circumcentre



P.O.I. intersection of perpendicular bisectors of the sides of a triangle.

It is a centre of a circle passing through the vertices of a triangle and of radius R

$$\left(\frac{x_1\sin 2A + x_2\sin 2B + x_3\sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1\sin 2A + y_2\sin 2B + y_3\sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

Remarks

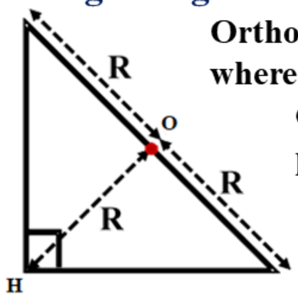
Incentre and centroid always lie inside the triangle.

Circumcentre & orthocentre lie inside for acute angle triangle and outside for obtuse angle triangle.

In equilateral triangle, centroid, incentre, orthocentre and circumcentre coincides.

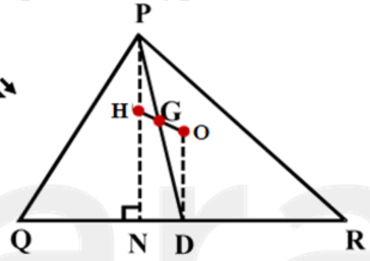
In an isosceles triangle centroid, orthocentre, incentre, circumcentre lie on the same line.

In right angled triangle



Orthocentre H is the point where right angle is formed

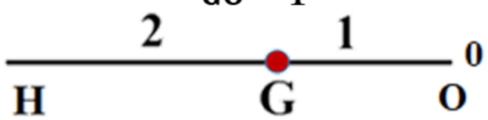
Circumcentre O is mid point of hypotenuse



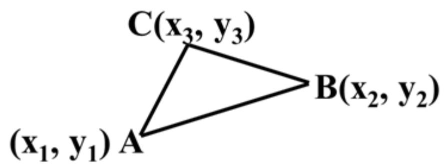
In any triangle

Orthocentre, centroid and circumcentre are always collinear

Centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1 i.e. $\frac{HG}{GO} = \frac{2}{1}$ internally



9. Area of Triangle



$$= \left|\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)]\right|$$

$$= \frac{1}{2}\left\|\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}\right\|$$

Conditions For Collinearity Of Three Given Points

Three points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are collinear if Area of triangle ABC is zero

$$\Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

10. Locus

The path traced out by a moving point under certain geometrical conditions is called it's locus.

Equation of Path is called equation of Locus

Procedure for finding the equation of the locus of a point

i) Assume coordinates of the point P (h, k)

ii) From the given geometrical conditions generate a relation in h , k and known quantities.

iii) Simplify the obtained relation.

iv) Replace h by x, and k by y, in the simplified relation. The resulting equation would be the equation of the locus of P.