MIND MAP

SETS

Set

- A set is a collection of well defined objects which are distinct from each other.
- Sets are generally denoted by capital letters A, B, C, ... etc. and the elements of the sets by a, b, c .... etc.
- If ‘a’ is an element of a set A, then we write ‘a’ $\in$ A and say ‘a’ belongs to A.
- If ‘a’ does not belong to A, then we write ‘a’ $\notin$ A.

Some Important Number Sets

N = Set of all natural numbers

= {1, 2, 3, 4, ...}

W = Set of all whole numbers

= {0, 1, 2, 3, ...}

Z or I = Set of all integers

= {.... –3, –2, –1, 0, 1, 2, 3, ...}

Methods to Write a Set

(i) Roster Method:

In this method, a set is described by listing elements, separated by commas and enclose them by curly braces.

(ii) Set Builder From

In this case, we write down a property or rule ‘p’, which gives us all the elements of the set

A = {x : P(x)}

Types of Sets

❑ Null Set OR Empty Set OR Void Set

A set having no element in it is called an empty set or a null set or void set.

It is denoted by ϕ or { }.

- A set consisting of at least one element is called a non-empty set or a non-void set.

❑ Singleton Set

A set consisting of a single element is called a singleton set.

❑ Finite Set

A set which has only finite number of elements is called a finite set.

Order of a Finite Set:

- The number of elements in a finite set is called the order of the set A.
- It is denoted O(A) or n(A).
- It is also called cardinal number of the set.

❑ Infinite Set

A set which has an infinite number of elements is called an infinite set.

❑ Equal Sets

- Two sets A and B are said to be equal, if every element of A is a member of B, & every element of B is a member of A.
- If sets A and B are equal, we write A = B, & A and B are not equal, then A $\neq$ B.

❑ Equivalent Sets

Two finite sets A and B are equivalent, if their number of elements are same.

i.e., n(A) = n(B)

Note:

Equal sets are always equivalent but equivalent sets may not always be equal.

❑ Subsets

Let A & B be two sets, if every element of A is an element of B, then A is called a subset of B.

If A is a subset of B. we write $A \subseteq B$.

❑ Proper Subset

If A is a subset of B and $A \neq B$, then A is a proper subset of B,

& we write $A \subset B$.

✓ Every set is a subset of itself; i.e., $A \subseteq A$ for all A.

✓ Empty set ϕ is a subset of every set.

✓ Clearly, $N \subset W \subset Z \subset Q \subset R \subset C$.

✓ The total number of subsets of a finite set containing n elements is 2^n.

❑ Universal Set

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by U.

Note:

All sets are contained in the universal set.

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❑ Power Set

✓ If $A = \phi$, then P(A) has one element.

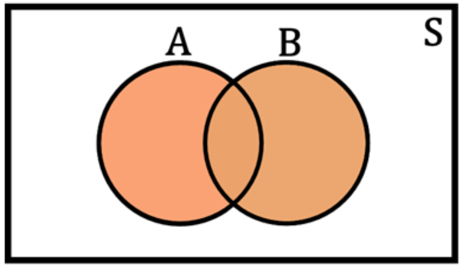
✓ Power set of a given set is always non – empty.

Some Operations on Sets

(i) Union of sets:

It is represented by $A \cup B$ or $A + B$ & contain all those elements which are in A or in B or in both.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$



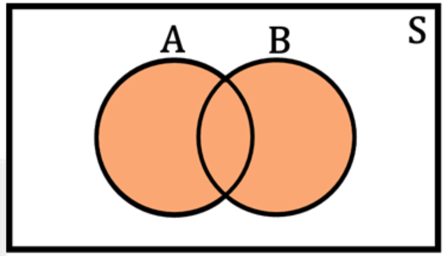
$A \cup B$

(i) Union of sets:

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Example:

If $A = \{1, 2, 3\}$ & $B = \{2, 3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$

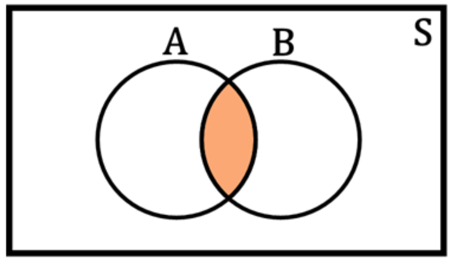


$A \cup B$

(ii) Intersection of sets:

It is represented by $A \cap B$ & contains all those elements which are common in A and B both.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$



$A \cap B$

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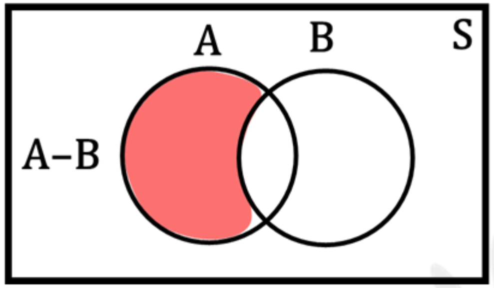
(iii) (a) Difference of sets:

- It is represented by A - B & contain all those elements which are in A but not in B.

$\{x : x \in A \text{ and } x \notin B\}$

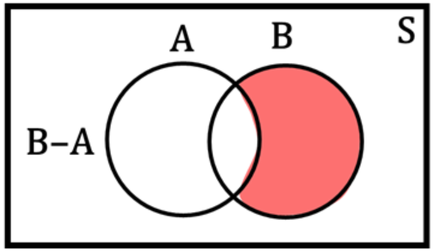


(iii) (b) Difference of sets:

OR

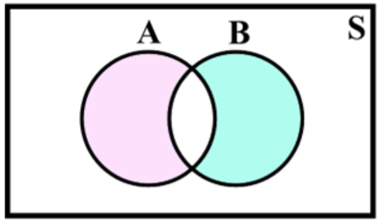
- It is represented by B - A & contain all those elements which are in B not in A.

$B - A = \{x : x \in B \text{ and } x \notin A\}$



(iv) Symmetric Difference of Sets:

$A \Delta B = (A - B) \cup (B - A)$

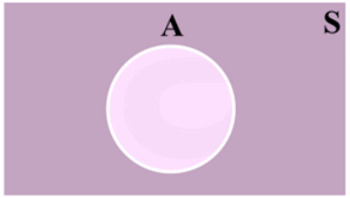


$A \Delta B$

(v) Complement of a set:

It is denoted by A'.

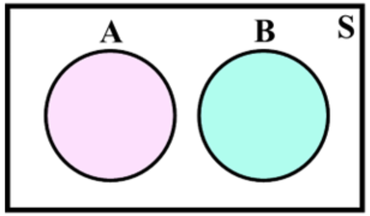
$A' = \{x : x \notin A \text{ but } x \in U\} = U - A$



A'

(vi) Disjoint Sets

If $A \cap B = \phi$, then A, B are disjoint.



Disjoint Sets

Note:

$A \cap A' = \phi \quad \therefore$ A, A' are disjoint.

Properties of Sets

(i) Commutative Laws

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

(ii) Associative Laws

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) Distributive Laws

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) De-Morgan Laws

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

(v) $A \cap \phi = \phi$; $A \cap U = A$

$A \cup \phi = A$; $A \cup U = U$

(vi) $A \cap B \subseteq A$; $A \cap B \subseteq B$

(vii) $A \subseteq A \cup B$; $B \subseteq A \cup B$

(viii) $A \subseteq B \Rightarrow A \cap B = A$

(ix) $A \subseteq B \Rightarrow A \cup B = B$

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Note :

✓ $(A')' = A$

✓ $A \subseteq B \Leftrightarrow B' \subseteq A'$

If A and B are any two sets, then

✓ $A - B = A \cap B'$

✓ $B - A = B \cap A'$

✓ $A - B = A \Leftrightarrow A \cap B = \phi$

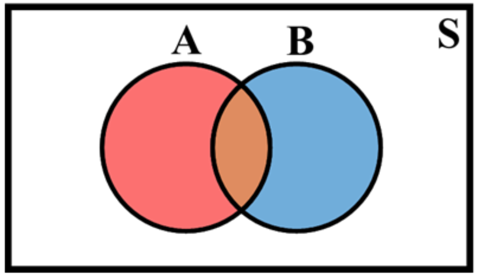
✓ $(A - B) \cup B = A \cup B$

✓ $(A - B) \cap B = \phi$

Some Important Results on Number of Elements in Sets

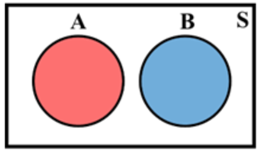
If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



(ii) $n(A \cup B) = n(A) + n(B)$

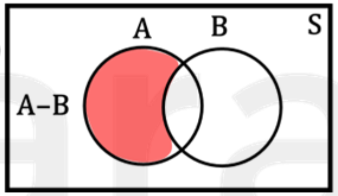
$\Leftrightarrow$ A, B are disjoint non-void sets.



Disjoint Sets

(iii) $n(A - B) = n(A) - n(A \cap B)$

i.e., $n(A - B) + n(A \cap B) = n(A)$



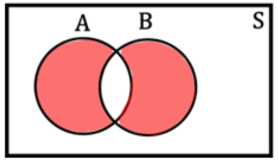
(iv) $n(A \Delta B)$ = No. of elements which belongs to exactly one of A or B

$= n((A - B) \cup (B - A)) = n(A - B) + n(B - A)$

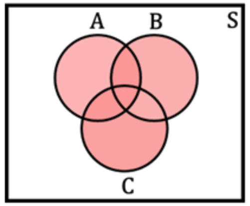
[$\because$ $(A - B)$ and $(B - A)$ are disjoint]

$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$

$= n(A) + n(B) - 2n(A \cap B)$



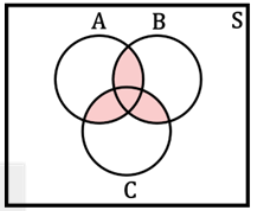
$n(A \Delta B)$



$n(A \cup B \cup C)$

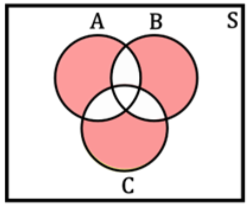
(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C =



$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) Number of elements in exactly one of the sets A, B, C =



$n(A) + n(B) + n(C) - 2n(A \cap B) + 3n(A \cap B \cap C) - 2n(B \cap C) - 2n(A \cap C)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$