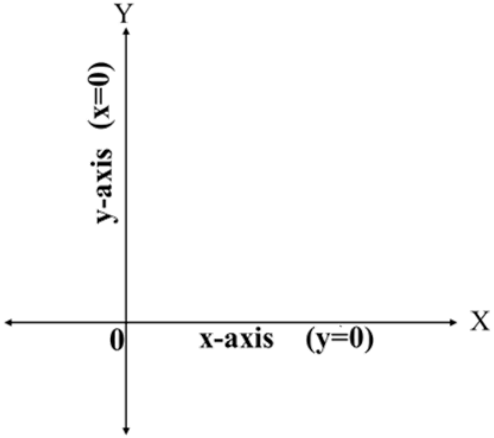
MIND MAP

Straight Line

1. Straight Line

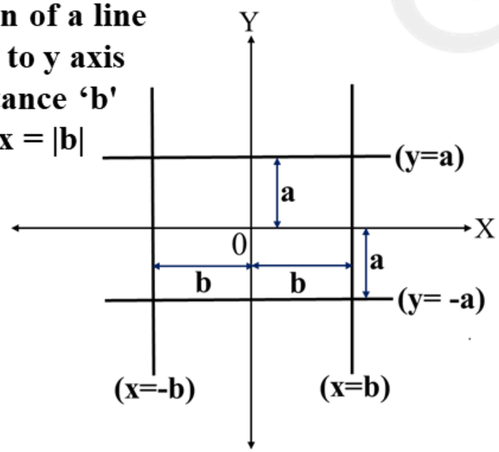
Equation of Co-ordinate axes



Equation of Straight Line Parallel to Axes

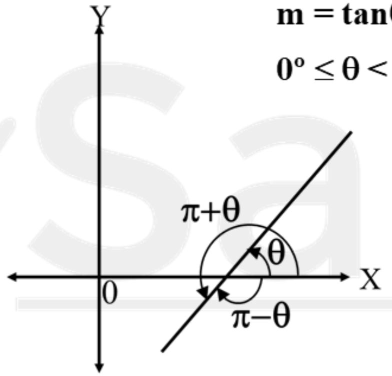
Equation of a line parallel to x axis at a distance 'a' from it $y = |a|$

Equation of a line parallel to y axis at a distance 'b' from it $x = |b|$



2. Slope (Gradient)

If a straight line makes an angle 'θ' with the positive direction of x-axis (in anti clockwise direction), then the slope of the line is denoted by 'm' and defined as

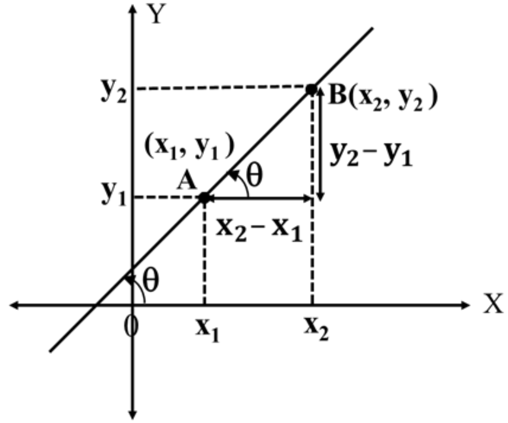


$m = \tan\theta$

$0° \le \theta < 180°, \quad \theta \ne 90°$

Slope of the line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$



Note $m = \tan\theta$

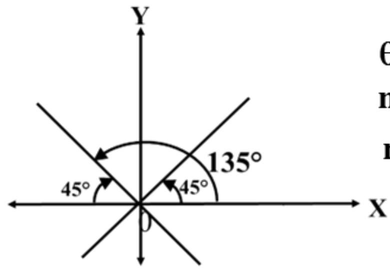
(i) If $\theta = 0^0$ means line is parallel to x-axis $m = 0$

(ii) If $\theta = 90^0$ means line m does not exist is parallel to y - axis.

(iii) For acute angle $(0° < \theta < 90°)$ m is positive

(iv) For obtuse angle $(90° < \theta < 180°)$ m is negative

(v) Line equally inclined to both axes



$\theta = 45°$ or $135°$

$m = \tan\theta$

$m = \pm 1$

Note

(vi) Let m_1 and m_2 be slopes of 2 given lines.

(a) If lines are parallel

$m_1 = m_2$ and vice versa.

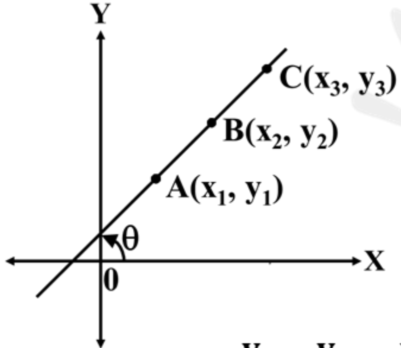
(b) If lines are perpendicular

$m_1 = \tan\theta$

$m_2 = \tan(90 + \theta) = -\cot\theta$

$m_1 . m_2 = -1$ and vice versa.

(vii) Condition for collinearity of three points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$



$m_{AB} = m_{BC} = m_{AC} \Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$

3. Intercept of a straight line on the coordinate axes

OA and OB with proper sign are called the intercepts of the line AB on x and y axis respectively.

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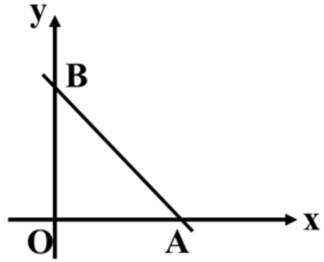
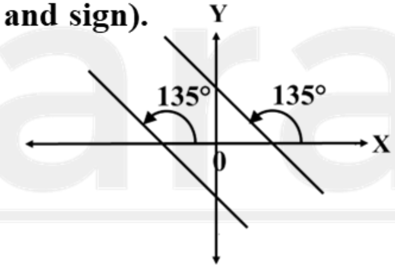
Straight Line



Note

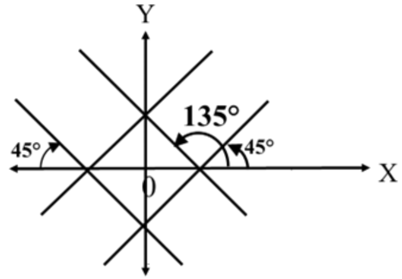
(i) If Line cuts equal non zero intercepts on axes (equal intercept means equal in magnitude and sign).

$\Rightarrow m = -1$



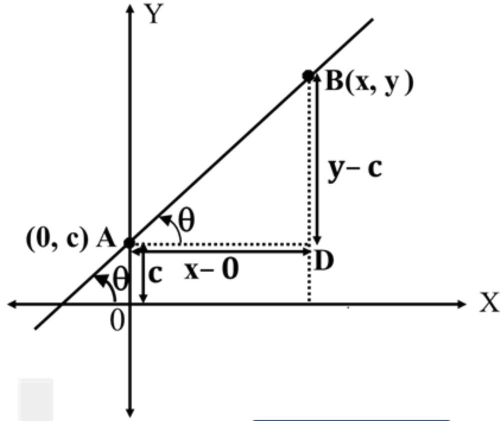
(ii) If Line cuts equal non-zero length on axes.

$\Rightarrow m = \pm 1.$



4. Standard Forms of Equations of Straight Lines

(I) Slope-intercept Form

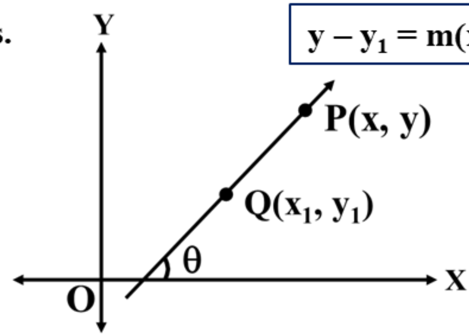


$\tan\theta = m = \frac{y-c}{x-0}$ $\boxed{y = mx + c}$

(II) Point Slope Form

Line passing through point (x_1, y_1) and having the slope = m

$\boxed{y - y_1 = m(x - x_1)}$



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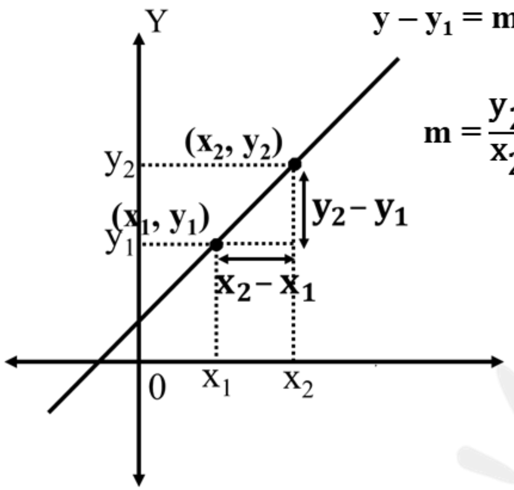
(III) Two Point Form

Line passing through two points (x_1, y_1) and (x_2, y_2)



$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Line passing through two points (x_1, y_1) and (x_2, y_2)

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

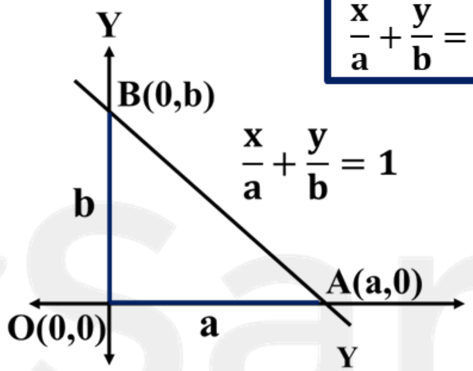
(IV) Determinant form

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(V) Intercept Form

Line making intercepts a & b on x & y-axis respectively

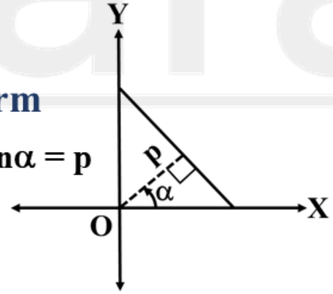
$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



$\frac{x}{a} + \frac{y}{b} = 1$

(VI) Normal Form

$x \cos\alpha + y \sin\alpha = p$

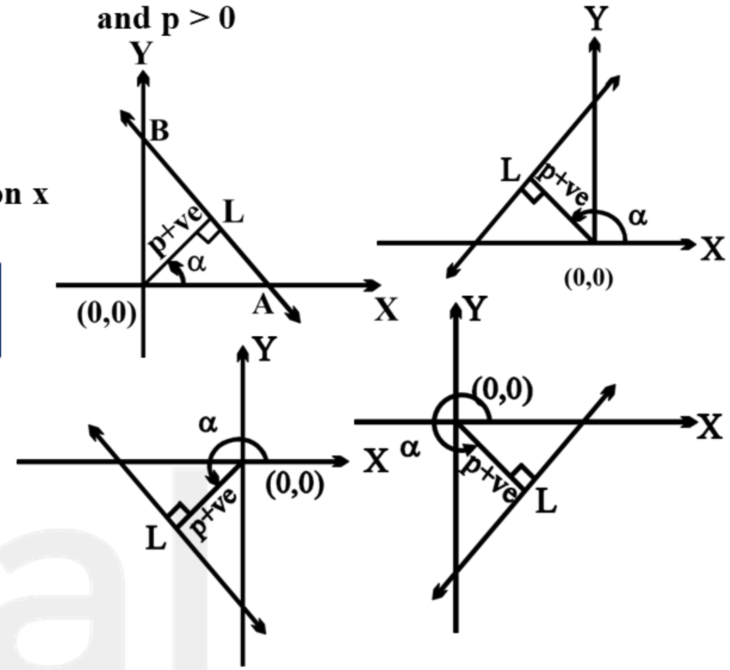


p : length of the perpendicular from the origin to the line

α : angle which this perpendicular to the line makes with positive x axis in anticlockwise direction

The normal form can be identified by checking that

(coefficient of x)^2 + (coefficient of y)^2 = 1 and $p > 0$



$0 \le \alpha < 2\pi$ and p is always positive

(VII) General Form

$Ax + By + C = 0$

Reduction of the general equation to different standard forms

Slope of the line $Ax + By + C = 0$ is $-\frac{A}{B}$

i.e. $-\left(\frac{\text{coefficient of } x}{\text{coefficient of } y}\right)$

y intercept of the line = $-C/B$

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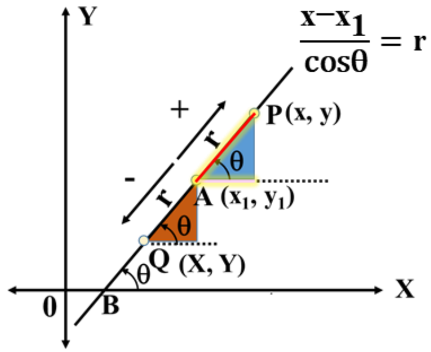
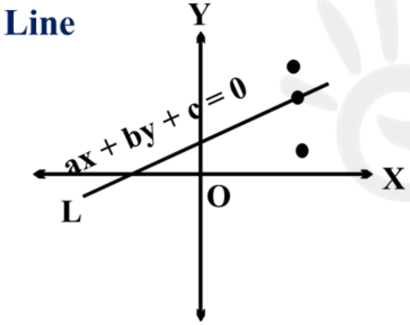
(VIII) Parametric Form

$$\frac{x-x_1}{\cos\theta} = r = \frac{y-y_1}{\sin\theta}$$



5. Point and Line

Position of a Point Relative to a Given Line



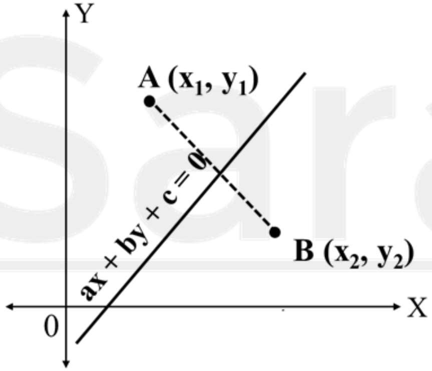
Note

(i) If $L(P_1).L(P_2) > 0$ (Means of same sign) : Points P_1 and P_2 are on same sides of the line $L = 0$

(ii) If $L(P_1).L(P_2) < 0$ (Means of opposite sign) : Points P_1 and P_2 are on opposite sides of the line $L = 0$

(iii) The straight line $ax + by + c = 0$ divides the line joining the points A (x_1, y_1) and B (x_2, y_2) in the ratio – $\frac{ax_1+by1+c}{ax_2+by1+c}$

If $L(x_1,y_1)$ and $L(x_2,y_2)$ are of opposite sign, then division is internal.



6. Collinearity of three given points

Three given points A, B, C are collinear if any one of the following conditions is satisfied.

(i) Area of triangle ABC is zero.

(ii) Slope of AB = slope of BC = slope of AC.

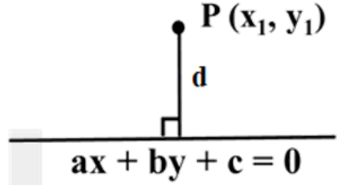
(iii) AC = AB + BC.

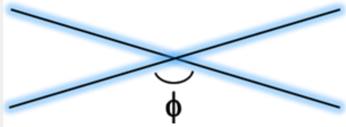
(iv) Equation of line AB, is satisfied by point C.

7. Length of The Perpendicular From a Point on a Line

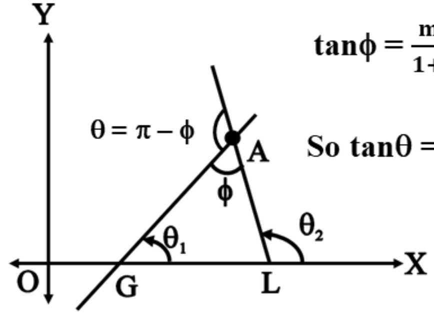
The length of the perpendicular from the point $P(x_1, y_1)$ on the line $ax + by + c = 0$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$





8. The Angle Between Two Straight Lines



$\tan\phi = \frac{m_2-m_1}{1+m_1m_2}$

So $\tan\theta = \frac{m_1-m_2}{1+m_1m_2}$

So acute angle between two lines

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

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Straight Line

Note

(I) If the lines are parallel

$\theta_2 = \theta_1$ and $\phi = 0$

$\tan\phi = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right| = 0$ $\boxed{m_1 = m_2}$

(II) If the lines are perpendicular

$\boxed{m_1 m_2 = -1}$

$a_1x+b_1y+c_1=0$; $a_2x+b_2y+c_2=0$

$m_1 = -\dfrac{a_1}{b_1}$ $m_2 = -\dfrac{a_2}{b_2}$

The lines are -

(i) Parallel $\Leftrightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

(ii) Perpendicular $\Leftrightarrow a_1a_2+b_1b_2=0$

(iii) Coincident $\Leftrightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

9. Eq of Lines Parallel And Perpendicular to a given line

Eq. of line, // to the line $ax + by + c = 0$

$ax + by + \lambda = 0$

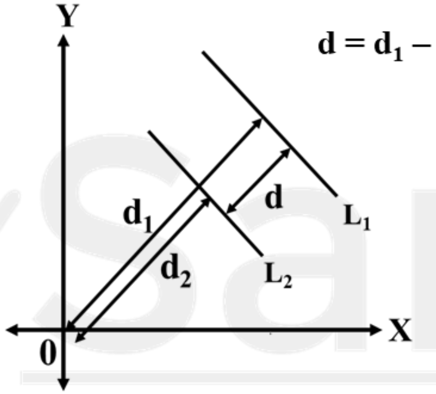
Eq. of line, $\perp$ to the line $ax + by + c = 0$

$bx - ay + \lambda = 0$

10. The Distance Between Two Parallel Lines

L_1: $ax + by + c_1 = 0$ and L_2: $ax + by + c_2 = 0$

$d_1 = \dfrac{|c_1|}{\sqrt{a^2+b^2}}$ $d_2 = \dfrac{|c_2|}{\sqrt{a^2+b^2}}$



$d = d_1 - d_2 = \dfrac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$

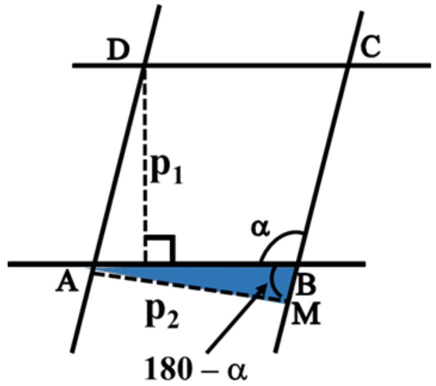
Note

The coefficients of x and y in both the equations should be same while applying this formula

Area of Parallelogram

p_1 and p_2 are the distances between the parallel sides and "α" is the angle between any two non parallel sides.

$$\boxed{\text{Area} = \frac{p_1 p_2}{\sin\alpha}}$$



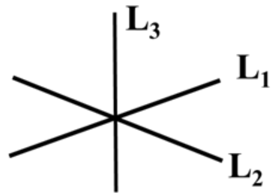
if lines are $y = m_1x + c_1$ & $y = m_1x + d_1$ and $y = m_2x + c_2$ & $y = m_2x + d_2$

$$\text{Area} = \left|\frac{(c_1-d_1)(c_2-d_2)}{(m_1-m_2)}\right|$$

Note

if $p_1 = p_2$ then parallelogram will be a rhombus.

11. Equation of any straight line through the point of intersection of two given straight lines



$L_1 + \lambda L_2 = 0$

λ is a parameter which can be evaluated specifically using given conditions

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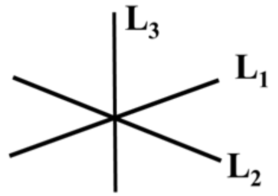
Straight Line

12. Equation of any straight line through the point of intersection of two given straight lines

Conversely, Any line of the form $L_1 + \lambda L_2 = 0$ passes through a fixed point which is the point of intersection of the lines $L_1 = 0$ and $L_2 = 0$.

13. Concurrency of Straight Lines

The condition for Concurrency of three lines



$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$a_3x + b_3y + c_3 = 0$$

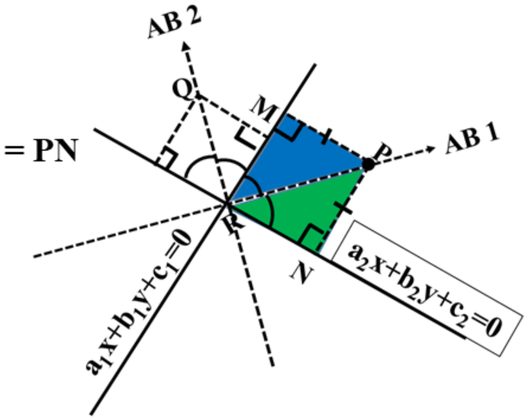
The three lines are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Note

If determinant = 0 then it is not always necessary that lines are concurrent but may be parallel.

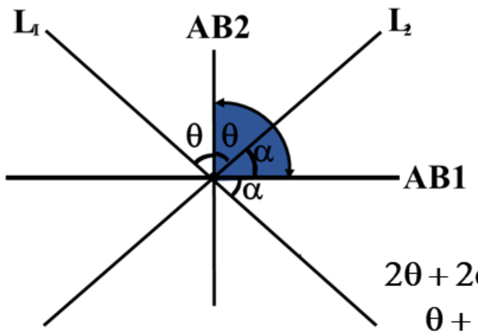
14. Bisectors of The Angles Between Two Given Lines

Angle bisector is the locus of a point which moves in such a way so that its distance from two intersecting lines remains same.



$\Rightarrow PM = PN$

$$\Rightarrow \left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

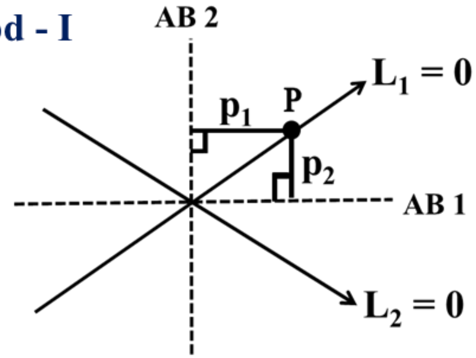


$$2\theta + 2\alpha = 180^\circ$$
$$\theta + \alpha = 90^\circ$$

Whether both given lines are perpendicular or not, but the angle bisectors of these lines will always be mutually perpendicular

15. Discrimination between acute / obtuse angle bisector

Method - I



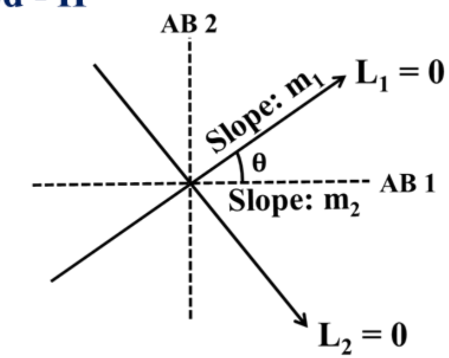
If

$|p_1| > |p_2| \Rightarrow$ AB 2 is obtuse angle bisector

$|p_1| < |p_2| \Rightarrow$ AB 2 is acute angle bisector

$|p_1| = |p_2| \Rightarrow$ the lines are perpendicular

Method - II



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Method - II

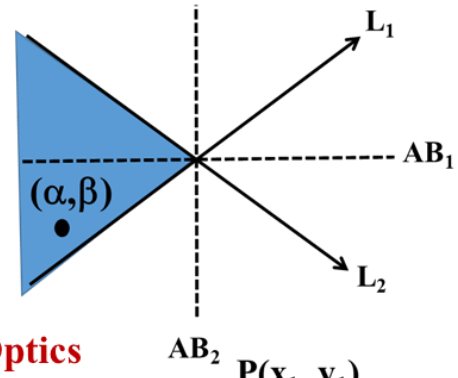
Find $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

If $\tan\theta > 1 \Rightarrow \theta > 45^\circ$

AB 1 is the bisector of the obtuse angle and AB 2 will be the bisector of the acute angle

If $0 < \tan\theta < 1 \Rightarrow \theta < 45^\circ$

AB 1 is the bisector of the acute angle



16. The equation of the bisector of the angle between the two lines containing the point (α, β)

If $a_1\alpha + b_1\beta + c_1$ and $a_2\alpha + b_2\beta + c_2$ are of the same signs, then it is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = +\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Or of opposite signs , then it is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

17. Optics

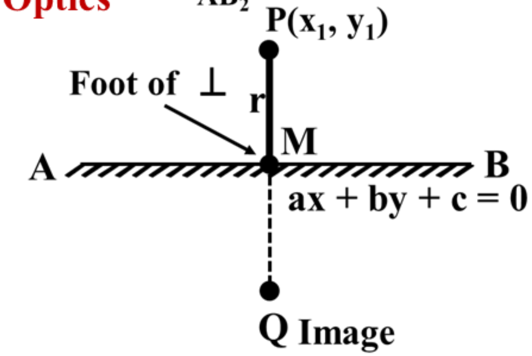


Image Q $\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\frac{ax_1 + by_1 + c}{a^2 + b^2}}$

Foot of Perpendicular M

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

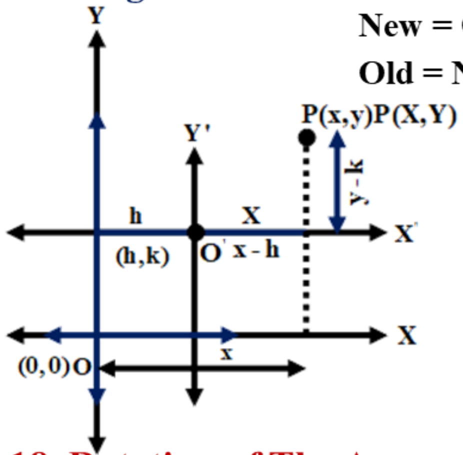
OR Find Mid point of PQ

Shifting of origin without rotating the axes : new axes are parallel to the original ones

$X = x - h \quad Y = y - k$

New = Old – h, k

Old = New + h, k



18. Rotation of The Axes

Rotation of The Axes, Without Changing The Origin Both Systems of Co-ordinates Being Rectangular

$x = x'\cos\theta - y'\sin\theta \quad y = x'\sin\theta + y'\cos\theta$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

It is easy to deduce the equivalent form

$x' = x\cos\theta + y\sin\theta$

$y' = y\cos\theta - x\sin\theta$

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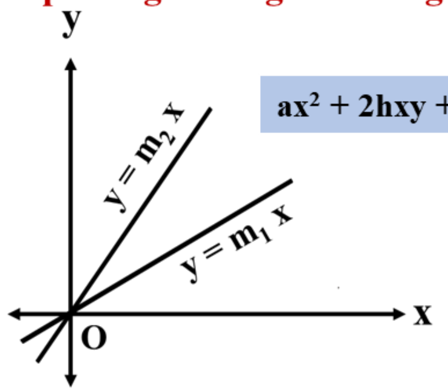
19. Homogeneous equation of 2nd degree represents combined equation of pair of straight lines passing through the origin.



$ax^2 + 2hxy + by^2 = 0$

Note

$ax^2 + 2hxy + by^2 = 0$

$\Rightarrow b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0, \frac{y}{x} = m$

$D = 4h^2 - 4ab = 4(h^2 - ab)$

These line will be

(I) Real and different if $D > 0 \Rightarrow h^2 - ab > 0$

(II) Real and coincident if $D = 0 \Rightarrow h^2 - ab = 0$

(III) Imaginary but with real point of intersection (0, 0) if $D < 0 \Rightarrow h^2 - ab < 0$

(IV) If a and b are of opposite sign
$\Rightarrow ab < 0 \Rightarrow h^2 - ab > 0$
Hence, lines are always real.

$m_1 + m_2 = -\frac{2h}{b}$ $\quad$ $m_1 m_2 = \frac{a}{b}$

20. The Angle Between Two Lines

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

The condition that these lines are

(I) At right angles to each other

$m_1 m_2 = \frac{a}{b} = -1 \Rightarrow a + b = 0$

coefficient of x^2 + coefficient of $y^2 = 0$.

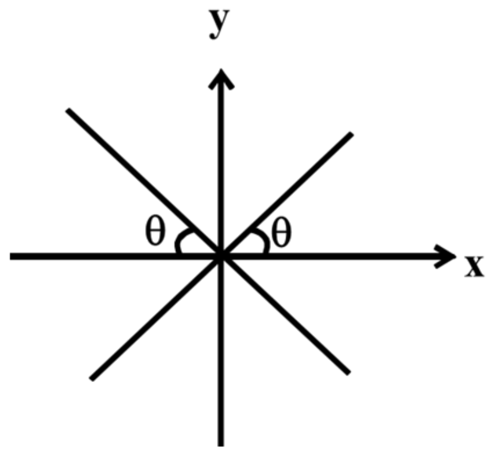
(II) Coincident

$\tan\theta = 0 \quad m_1 = m_2$
$\Rightarrow h^2 - ab = 0 \quad h^2 = ab$

(III) Equally inclined to the x axis

$h = 0$

coefficient of $xy = 0$



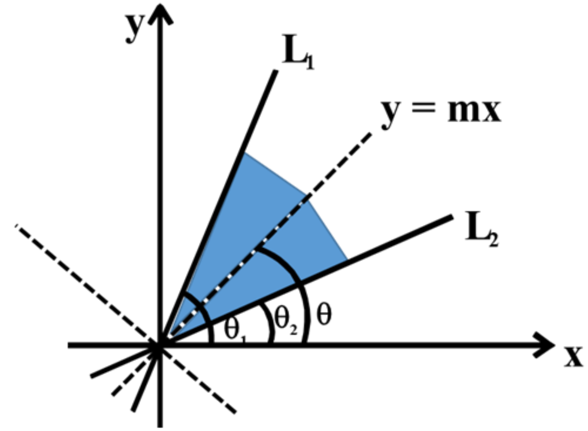
21. The combined equation of angle bisectors between the lines

$ax^2 + 2hxy + by^2 = 0$ is given by

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$a \neq b, h \neq 0$

$ax^2 + 2hxy + by^2 = 0$



22. General equation of degree two.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Represents a pair of straight lines in general if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ called discriminant of equation

This is necessary but not sufficient condition for lines to be real.

The sufficient conditions for lines to be real are $\Delta = 0$ and $h^2 - ab \geq 0$.

MIND MAP

Straight Line

Point of intersection of two lines represented by

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Method-I

Get equation of two straight lines and solve

Method-II

Partially differentiate the equation f(x,4) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\frac{xf}{x} = 0 \Rightarrow 2ax + 2hy + 2g = 0 \quad \ldots \text{(i)}$

$\frac{xf}{xy} = 0 \Rightarrow 2hx + 2by + 2f = 0 \quad \ldots \text{(ii)}$

On solving (i) and (ii) we get required point of intersection of lines

23. (Homogenisation)

Curve: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Line : $lx + my = n$

The given line equation may be written as

$\frac{lx+my}{n} = 1$

$ax^2 + 2hxy + by^2 + (2gx + 2fy)$

$\left(\frac{lx+my}{n}\right) + c\left(\frac{lx+my}{n}\right)^2 = 0 \quad \ldots \text{(i)}$

The points P & Q satisfy the line, curve and eqn …(i)

Equation (i) is homogeneous and of the 2^{nd} degree, it represents 2 st. lines passing through the origin and points P & Q