MIND MAP

Trigonometric Equation

1. Equation

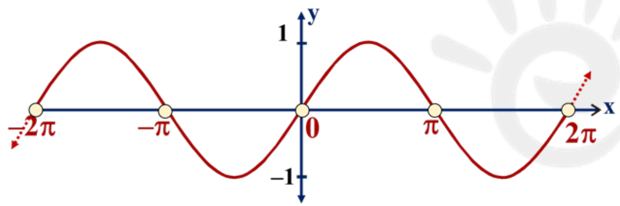
Equation	**Trigonometric Equation**
$x = 0$	$\sin x = 0$
$x^2 - 3x + 2 = 0$	$\sin^2 x - 3\sin x + 2 = 0$
$x + y + z = 0$	$\sin x + \cos y + \tan z = 0$

An equation involving one or more trigonometric ratios of unknown angles is called a trigonometric equation.

2. Solution Of Trigonometric Equation

$\sin x + 1 = 1 \quad \Rightarrow \sin x = 0$

$\Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots..$
$-\pi, -2\pi, -3\pi, \ldots$ (Infinite values)



Principal solution :- The solutions of the trigonometric equation lying in the interval $[0, 2\pi)$.

Particular solution :- The solutions of the trigo. equation lying in the given interval.

General solution	$x = n\pi \;;\; n \in I$
Principal solution (in $[0, 2\pi)$.)	$x = 0, \pi$
Particular solution (in $(0, 5\pi)$.)	$x = \pi, 2\pi, 3\pi, 4\pi$

3. General Solution of Some Trigo. Eqs.

General solution of the equation $\sin\theta = \sin\alpha$

- θ is unknown angle and is assumed to be in radians.
- α is known angle.

$\theta = n\pi + (-1)^n\alpha$ $\quad n \in I$

$\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (V. Imp.)

For deciding α, think of α as numerically smallest angle of any sign '+' or '–'

Note:

- If $\sin\theta = 0$ then $\theta = n\pi$
- If $\sin\theta = 1$ then $\theta = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2}$, $n \in I$

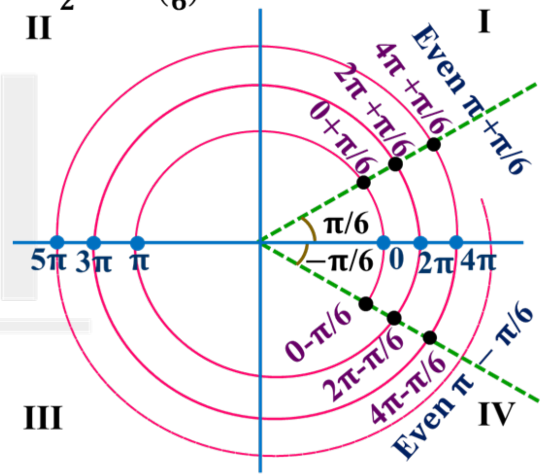
  or $\theta = n\pi + (-1)^n\frac{\pi}{2}$
- If $\sin\theta = -1$ then $\theta = 2n\pi - \frac{\pi}{2} = (4n-1)\frac{\pi}{2}$, $n \in I$

  or $\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

- The equation $\operatorname{cosec}\theta = \operatorname{cosec}\alpha$ is equivalent to $\sin\theta = \sin\alpha$ so these two equation have the same general solution (in their domains).
- General solution of the equation $\sin\theta = C$ exists if $-1 \le C \le 1$.

4. General solution of cos θ = cos α

$\cos\theta = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$



$\theta = 2n\pi \pm \alpha$ $\quad n \in I$ $\quad \alpha \in [0, \pi]$ (V. Imp.)

If $\cos\theta = 0$ then $\theta = 2n\pi \pm \frac{\pi}{2}$ or $\theta = (2n+1)\frac{\pi}{2}$

If $\cos\theta = 1$ then $\theta = 2n\pi$, $n \in I$

If $\cos\theta = -1$ then $\theta = 2n\pi \pm \pi$ or $\theta = (2n+1)\pi$

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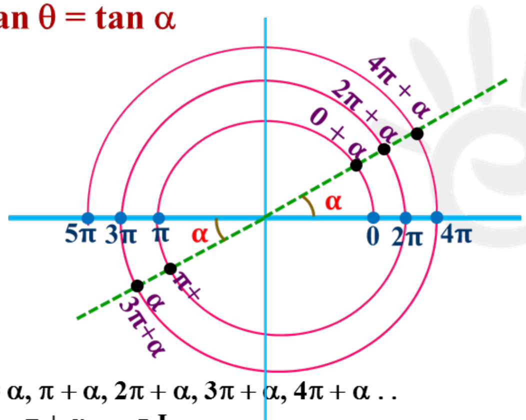
Note

- The equation $\sec\theta = \sec\alpha$ is equivalent to $\cos\theta = \cos\alpha$, so the general solution of these two equations are same (in their domain).
- General solution of the equation $\cos\theta = C$ exists if $-1 \le C \le 1$.

5. General solution of the equation $\tan\theta = \tan\alpha$



$\theta = \alpha, \pi+\alpha, 2\pi+\alpha, 3\pi+\alpha, 4\pi+\alpha \ldots$

$\theta = n\pi + \alpha\ ;\ n \in I$

$\boxed{\theta = n\pi + \alpha}$ $\boxed{n \in I}$ $\boxed{\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$ **(V. Imp.)**

- If $\cot\theta = 0$ then $\theta = (2n+1)\dfrac{\pi}{2}$
- If $\tan\theta = 0$ then $\theta = n\pi$, $n \in I$

- The equation $\cot\theta = \cot\alpha$ is equivalent to $\tan\theta = \tan\alpha$, so these two equations have the same general solution (in their domain).

6. General Solution of Square of the Trigonometric Equations

$\cos^2\theta = \cos^2\alpha$

$\dfrac{1+\cos 2\theta}{2} = \dfrac{1+\cos 2\alpha}{2}$

$\sin^2\theta = \sin^2\alpha$

$\dfrac{1-\cos 2\theta}{2} = \dfrac{1-\cos 2\alpha}{2}$

$\tan^2\theta = \tan^2\alpha$

$\sec^2\theta = \sec^2\alpha$

$\cos^2\theta = \cos^2\alpha$

$\cos 2\theta = \cos 2\alpha$

$2\theta = 2n\pi \pm 2\alpha$

$\boxed{\theta = n\pi \pm \alpha\ ;\ n \in I}$

(Take care of domain)

$\cos^2\theta = \cos^2\alpha$ $\quad$ $\sin^2\theta = \sin^2\alpha$ $\quad$ $\tan^2\theta = \tan^2\alpha$

Hence, in all cases

$\boxed{\theta = n\pi \pm \alpha\ ;\ n \in I}$

(Take care of domain)

7. Strategies For Solving Trigonometric Equations

Type – 1 : Solving Trigonometric Equations by Factorisation

Type – 2 : Solving Trigonometric Equation by reducing it to a Quadratic Equation

Type – 3: Solving Trigo. Eqs. of the form $a\sin\theta + b\cos\theta = c$ By introducing an Auxiliary Argument

Type – 4: Solving Trigo. Eqs. by transforming sum of trigonometric functions into product

Type – 5 : Solving Trigo Eqs. by transforming the product into sum

Type – 6:
Solving Trigo Eqs.by change of variable or by substitution

Type – 7:
Solving Trigo Eqs. with the use of the Boundness of the Functions involved

Type – 8:
Solution of Trigo Eq of the form of $f(x) = \sqrt{\phi(x)}$

Square both side with the condition $f(x) \ge 0$

Type – 9: System of Trigo Equations