Ex - 2.4

Q1. Determine which of the following polynomials, (x + 1) is a factor of :

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol. (i) $x^3 + x^2 + x + 1$

Let $p(x) = x^3 + x^2 + x + 1$

The zero of x + 1 is – 1

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

By Factor theorem x + 1 is a factor of p(x).

(ii) $x^4 + x^3 + x^2 + x + 1$

Let $p(x) = x^4 + x^3 + x^2 + x + 1$

The zero of x + 1 is –1

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 \neq 0$

By Factor theorem x + 1 is not a factor of p(x)

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Zero of x + 1 is –1

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0$

By Factor theorem x + 1 is not a factor of p(x)

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

zero of x + 1 is –1

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$

By Factor theorem, x + 1 is not a factor of p(x).

Q2. Use the factor theorem to determine whether g(x) is a factor of p(x) in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$; $g(x) = x - 3$

Sol. (i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

$g(x) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Zero of g(x) is –1

Now, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

$\therefore$ By factor theorem, g(x) is a factor of p(x).

(ii) Let $p(x) = x^3 + 3x^2 + 3x + 1$,

$g(x) = x + 2$

$g(x) = 0 \Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

$\therefore$ Zero of g(x) is –2

Now, $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1 = -1$

$\therefore$ By Factor theorem, g(x) is not a factor of p(x)

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

$g(x) = 0$

$\Rightarrow x - 3 = 0 \Rightarrow x = 3$

$\therefore$ Zero of g(x) = 3

Now $p(3) = 3^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

$\therefore$ By Factor theorem, g(x) is a factor of p(x).

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Sol. (i) $p(x) = x^2 + x + k$

If x – 1 is a factor of p(x), then p(1) = 0

$\Rightarrow (1)^2 + (1) + k = 0$

$\Rightarrow 1 + 1 + k = 0$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

If (x – 1) is a factor of p(x) then p(1) = 0

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$k = -(2 + \sqrt{2})$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

If (x – 1) is a factor of p(x) then p(1) = 0

$k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

If $(x-1)$ is a factor of $p(x)$ then $p(1) = 0$

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$2k = 3$

$k = 3/2$

Q4. Factorise :

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (3x - 2)(2x + 3)$

(iv) $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (x + 1)(3x - 4)$

Q5. Factorise :

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Sol. (i) $x^3 - 2x^2 - x + 2$

Let $p(x) = x^3 - 2x^2 - x + 2$

By trial, we find that

$p(1) = (1)^3 - 2(1)^2 - (1) + 2$

$= 1 - 2 - 1 + 2 = 0$

$\therefore$ By factor Theorem, $(x - 1)$ is a factor of $p(x)$.

Now, $x^3 - 2x^2 - x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x - 2)(x + 1)$

(ii) $x^3 - 3x^2 - 9x - 5$

Let $p(x) = x^3 - 3x^2 - 9x - 5$

By trial, we find

$p(-1) = (-1)^3 -3(-1)^2 -9(-1) - 5$

$= -1-3 + 9 - 5 = 0$

$\therefore$ By Factor Theorem, $x - (-1)$ or $x + 1$ is factor of $p(x)$

Now, $x^3 - 3x^2 - 9x - 5$

$= x^2(x + 1) - 4x(x + 1) - 5(x + 1)$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)\{x(x - 5) + 1(x - 5)\}$

$= (x + 1)^2(x - 5)$

(iii) $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

By trial, we find

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20 = 0$

$\therefore$ By Factor theorem, $x - (-1)$, $x + 1$ is a factor of $p(x)$

$x^3 + 13x^2 + 32x + 20$

$= x^2(x + 1) + 12(x)(x + 1) + 20(x + 1)$

$= (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 2x + 10x + 20)$

$= (x + 1)\{x(x + 2) + 10(x + 2)\}$

$= (x + 1)(x + 2)(x + 10)$

(iv) $2y^3 + y^2 - 2y - 1$

$p(y) = 2y^3 + y^2 - 2y - 1$

By trial, we find that

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1 = 0$

$\therefore$ By Factor theorem, $(y - 1)$ is a factor of $p(y)$

$2y^3 + y^2 - 2y - 1$

$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)\{2y(y + 1) + 1(y + 1)\}$

$= (y - 1)(2y + 1)(y + 1)$