Ex - 8.1

Q1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Let the four angles of the quadrilateral be
$3x$, $5x$, $9x$ and $13x$.

$\therefore\ 3x + 5x + 9x + 13x = 360°$

$\therefore$ [Sum of all the angles of quadrilateral is 360°]

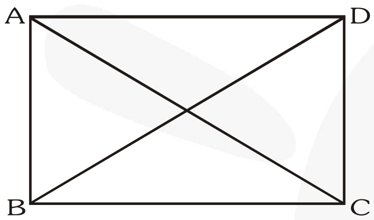
$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence, the angles of the quadrilateral are
$3 \times 12° = 36°$, $5 \times 12° = 60°$, $9 \times 12° = 108°$ and $13 \times 12° = 156°$.

Q2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. **Given :** ABCD is a parallelogram with diagonal
AC = diagonal BD



To prove : ABCD is a rectangle.

Proof : In triangle ABC and ABD,

	AB = AB	[Common]
	AC = BD	[Given]
	AD = BC	[Opp. Sides of a ‖gm]
$\therefore$	$\Delta ABC \cong BAD$	[By SSS congruency]
$\Rightarrow$	$\angle DAB = \angle CBA$	[By C.P.C.T.] ....(i)

[$\because$ AD‖BC and AB cuts them, the sum of the interior angle of the same side of transversal is 180°]

$\angle DAB + \angle CBA = 180°$ .....(ii)

From eq. (i) and (ii), $\angle DAB = \angle CBA = 90°$

Hence, ABCD is a rectangle

Q3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

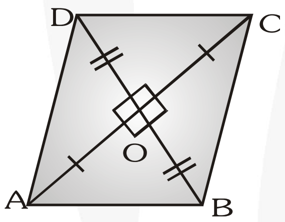
Sol. **Given :** ABCD is a quadrilateral where diagonals AC and BD meet at 0, such that AO = OC, OB = OD and AC ⊥ BD

To Prove : Quadrilateral ABCD is a rhombus,

i.e., AB = BC = CD = DA

Proof : In ΔAOB and ΔAOD,

	OB = OD	[Common]
	AO = AO	[Given]
	∠AOB = ∠AOD	[Each = 90°]
∴	ΔAOB ≅ ΔAOD	[SAS Rule]
∴	AB = AD	[C.P.C.T.]



Similarly, we can prove that

AB = BC ...(i)

BC = CD ...(ii)

CD = AD ...(iii)

From (i), (ii), (iii) and (iv), we obtain

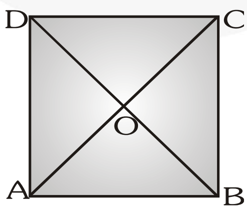
AB = BC = CD = DA

∴ Quadrilateral ABCD is a rhombus.

Q4. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. **Given:** ABCD is a square.

To Prove : (i) AC = BD (ii) AC and BD bisect each other at right angles.



Proof: In ΔABC and ΔBAD,

AB = BA	[Common]
BC = AD	[Opp. sides of square ABCD]
∠ABC = ∠BAD	[Each = 90° (∵ ABCD is a square]

∴ $\quad \Delta ABC \cong \Delta BAD$ [SAS Rule]

∴ $\quad AC = BD$ ... (i) [C.P.C.T.]

In ΔAOD and ΔBOC

$AD = CB$ [Opp. sides of square ABCD]

$\angle OAD = \angle OCB$

[Alternate angles as $AD \parallel BC$ and transversal AC intersects them]

$\angle ODA = \angle OBC$

[Alternate angles as $AD \parallel BC$ and transversal BD intersects them]

$\Delta AOD \cong \Delta BOC$ [ASA Rule]

∴ $\quad OA = OC$ and $OB = OD$ ...(ii) [C.P.C.T.]

So, O is the mid point of AC and BD.

Now, In ΔAOB and ΔCOB

$AB = BC$ [Given]

$OA = OC$ [from (ii)]

$OB = OB$ [Common]

∴ $\quad \Delta AOB \cong \Delta COB$ [By SSS Rule]

∴ $\quad \angle AOB = \angle BOC$ [C.P.C.T]

But $\quad \angle AOB + \angle BOC = 180°$ [Linear pair]

$\angle AOB + \angle AOB = 180°$

[AOB = BOC proved earlier]

⇒ $\quad 2\angle AOB = 180°$

⇒ $\quad \angle AOB = \frac{180°}{2} = 90°$

∴ $\quad \angle AOB = \angle BOC = 90°$

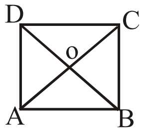
∴ $\quad$ AC and BD bisect each other at right angles.

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Sol. **Given :** The diagonals AC and BD of a quadrilateral ABCD are equal and bisect each other at right angles.

To prove : Quadrilateral ABCD is a square.

Proof :



In ΔAOD and ΔBOC,

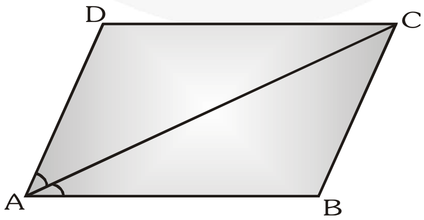
$OA = OC$ [Given]

$OD = OB$ [Given]

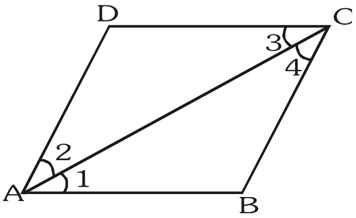
$\angle AOD = \angle COB$ [Vertically Opposite Angles]

∴	$\Delta AOD \cong \Delta BOC$	[SAS Rule]
∴	$AD = BC$	[C.P.C.T.]
	$\angle ODA = \angle OBC$	[C.P.C.T.]
∴	$AD \parallel BC$	
	Now, $AD = CB$ and $AD \parallel CB$	
∴	Quadrilateral ABCD is a ∥ gm.	
	In ΔAOB and ΔAOD,	
	$AO = AO$	[Common]
	$OB = OD$	[Given]
	$\angle AOB = \angle AOD$	[Each = 90° (Given)]
∴	$\Delta AOB \cong \Delta AOD$	[SAS Rule]
∴	$AB = AD$	
	Now,	
∵	ABCD is a parallelogram and $AB = AD$	
∴	ABCD is a rhombus.	
	Again, in ΔABC and ΔBAD,	
	$AC = BD$	[Given]
	$BC = AD$	
	[∵ ABCD is a Rhombus]	
	$AB = BA$	[Common]
∴	$\Delta ABC \cong \Delta BAD$	[SSS rule]
∴	$\angle ABC = \angle BAD$	[C.P.C.T.]
∵	$AD \parallel BC$	
	[Opposite sides of ∥ gm ABCD]	
	and transversal AB intersects them.	
∴	$\angle ABC + \angle BAD = 180°$ [Sum of consecutive interior angles on the same side of the transversal is 180°]	
∴	$\angle ABC = \angle BAD = 90°$	
	Similarly, $\angle BCD = \angle ADC = 90°$	
∴	ABCD is a square.	

Q6. In figure, ABCD is a parallelogram. Diagonal AC bisects $\angle A$. Show that
(i) it bisects $\angle C$ also (ii) ABCD is a rhombus.



Sol. **Given :**



Diagonal AC bisects ∠A of the parallelogram ABCD.

To prove :

(i) AC bisects ∠C

(ii) ABCD is a rhombus

Proof :

(i) Since AB||DC and AC intersects them.

∴ ∠1 = ∠3 [Alternate angles] ...(i)

Similarly ∠2 = ∠4 ...(ii)

But ∠1 = ∠2 [Given] ...(iii)

∴ ∠3 = ∠4 [Using eq. (i), (ii) and (iii)]

Thus AC bisects ∠C.

(ii) ∠2 = ∠3 = ∠4 = ∠1

⇒ AD = CD [Sides opposite to equal angles]

∴ AB = CD = AD = BC

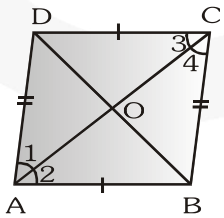
Hence, ABCD is a rhombus.

Q7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Sol. **Given :** ABCD is a rhombus and AC and BD are its diagonal

To prove : (i) Diagonal AC bisects ∠A as well as ∠C.

(ii) Diagonal BD bisect ∠B as well as ∠D.



Proof :

(i) ∴ In ΔABC

AB = BC (sides of Rhombus)

so, ∠2 = ∠4 (Angle opposite to equal sides are equal)

But $\angle 2 = \angle 3$ (Alternate angles as AB || CD)

so, $\angle 2 = \angle 3 = \angle 4$

But $\angle 1 = \angle 4$ (Alternate angles as AD || BC)

so, $\angle 1 = \angle 2 = \angle 3 = \angle 4$ ... (1)

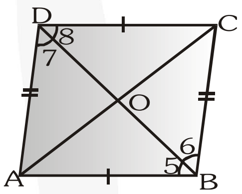
$\angle 1 = \angle 2$ by (1)

so, AC bisect $\angle A$

$\angle 3 = \angle 4$ by (1)

so, AC bisect $\angle C$

(ii) In ΔABD



AB = AD (Sides of Rhombus)

so, $\angle 5 = \angle 7$ (Angle opposite to equal sides are equal)

But $\angle 7 = \angle 6$ (Alternate angle as AD || BC)

so, $\angle 5 = \angle 6 = \angle 7$

$\angle 5 = \angle 8$ (Alternate angle as AB || CD)

so, $\angle 5 = \angle 6 = \angle 7 = \angle 8$ ... (2)

$\angle 5 = \angle 6$ by (2)

so, BD bisect $\angle B$

$\angle 7 = \angle 8$ by (2)

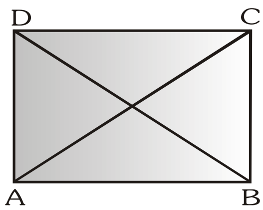
so, BD bisect $\angle D$

Q8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that

(i) ABCD is a square

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. **Given :** ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.



To prove : (i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Proof :

(i) ∵ AB || DC and transversal AC intersects them.

so, ∠BAC = ∠DCA [Alternate angles]

But ∠BAC = ∠DAC [∵ AC bisects ∠A]

∴ ∠ DCA = ∠DAC

⇒ DA = CD

[Sides opposite to equal angles of a triangle are equal]

But AB = CD and DA = BC [Opposite side of a rectangle]

∴ AB = BC = CD = DA

Also ∠A = ∠B = ∠C = ∠D = 90°

[∵ ABCD is a rectangle]

Hence, ABCD is a square

(ii) In Δ BAD and Δ BCD,

BA = BC [∵ ABCD is a square]

AD = CD [∵ ABCD is a square]

BD = BD [Common]

∴ ΔBAD ≅ ΔBCD [By SSS congruence rule]

∴ ∠ ABD = ∠ CBD [By C.P.C.T.]

∠ADB = ∠CDB [By C.P.C.T.]

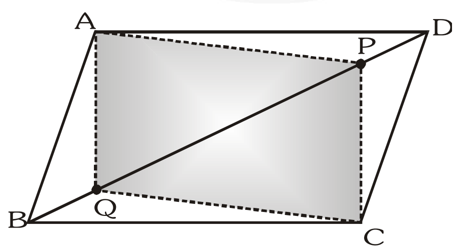
Hence, diagonal BD bisect ∠B as well as ∠D

Q9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that :

(i) ΔAPD ≅ ΔCQB (ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD (iv) AQ = CP

(v) APCQ is a parallelogram



Sol. (i) In ΔAPD and ΔCQB, we have

$DP = BQ$ [Given]

$AD = CB$

[Opposite sides of parallelogram ABCD]

$\angle ADP = \angle CBQ$ [Pair of alternate angles]

$\Rightarrow$ $\Delta APD \cong \Delta CQB$ [SAS congruence criteria]

(ii) Then, by CPCT, we have $AP = CQ$

(iii) We can prove

$\Delta AQB \cong \Delta CPD$ [as we have done in (i)]

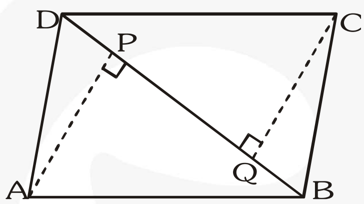
(iv) By CPCT, we have $AQ = CP$

(v) Now, we have $AP = CQ$ and $AQ = CP$

Hence, APCQ is a parallelogram.

Q10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

(i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$



Sol. **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

To prove : (i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$

Proof :

(i) In ΔAPB and ΔCQD,

$AB = CD$ [Opp. side of || gm ABCD]

$\angle ABP = \angle CDQ$ [$\therefore$ AB || DC and transversal BD intersect them]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore \Delta APB \cong \Delta CQD$ [AAS Rule]

(ii) $\therefore AP = CQ$ [C.P.C.T.]

Q11. In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively. Show that :

(i) quadrilateral ABED is a parallelogram

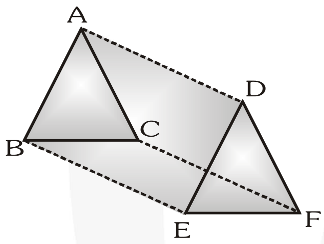
(ii) quadrilateral BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF



Sol. **Given :** AB = DE, AB || DE, BC = EF & BC || EF

To Prove

(i) ABED is a parallelogram.

(ii) BEFC is parallelogram.

(iii) AD || CF and AD = CF

(iv) ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF

Proof

(i) In Δ ABC and Δ DEF

AB = DE [Given]

and AB || DE [Given]

∴ ABED is a parallelogram

(ii) In Δ ABC and Δ DEF

BC = EF [Given]

and BC = EF [Given]

∴ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

∴ AD || BE and AD = BE ....(i)

Also, BEFC is a parallelogram

∴ CF || BE and CF = BE ....(ii)

From (i) and (ii), we get

∴	AD ‖ CF and AD = CF	
(iv)	As AD ‖ CF and AD = CF	
⇒	ACFD is a parallelogram.	
(v)	As ACFD is a parallelogram.	
∴	AC = DF	
(vi)	In Δ ABC and Δ DEF,	
	AB = DE	[Given]
	BC = EF	[Given]
	AC = DF	[Proved]
∴	ΔABC ≅ ΔDEF	[By SSS congruency]

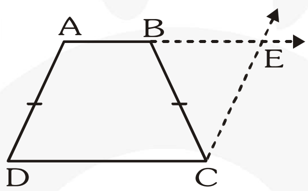
Q12. ABCD is a trapezium in which AB‖CD and AD = BC. Show that (fig)

(i) ∠A = ∠B

(ii) ∠C = ∠D

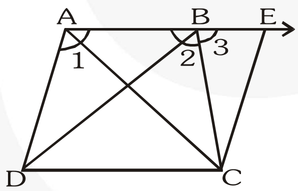
(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD



Sol. **Given :** ABCD is a trapezium.

AB ‖ CD and AD = BC



To Prove :

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) Δ ABC ≅ Δ BAD

(iv) Diagonal AC = Diagonal BD

Construction : Draw CE ‖ AD and extend AB to intersect CE at E.

Proof :

(i)	As AECD is a parallelogram.	
	[By construction]	
∴	AD = EC	
	But AD = BC	[Given]
∴	BC = EC	

$\Rightarrow$ $\angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now, $\angle 1 + \angle 4 = 180°$ [Interior angles]

and $\angle 2 + \angle 3 = 180°$ [Linear pair]

$\Rightarrow$ $\angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow$ $\angle 1 = \angle 2$ [$\because \angle 3 = \angle 4$]

$\Rightarrow$ $\angle A = \angle B$

(ii) $\angle 3 = \angle BCD$ [Alternate interior angles]

$\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [Δ BCE is an isosceles triangle]

$\therefore$ $\angle BCD = \angle ADC$

$\therefore$ $\angle C = \angle D$

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

$\angle 1 = \angle 2$ [Proved]

AD = BC [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.]