Ex - 1.1

Q1. Use Euclid's division algorithm to find the HCF of :
(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 225.

Sol. (i) 135 and 225.
Start with the larger integer, that is, 225. Apply the division lemma to 225 and 135, to get.
$225 = 135 \times 1 + 90$
Since the remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to get
$135 = 90 \times 1 + 45$
We consider the new divisior 90 and the new remainder 45, and apply the division lemma to get
$90 = 45 \times 2 + 0$
The remainder has now become zero, so our procedure stops.
Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

(ii) 196 and 38220
Start with the larger integer, that is, 38220. Apply the division lemma to 38220 and 196, to get.
$38220 = 196 \times 195 + 0$
Remainder at this stage is zero, so our procedure stops.
So, HCF of 196 and 38220 is 196.

(iii) 867 and 225
Start with the larger integer, that is, 867. Apply the division lemma to 867 and 225, to get.
$867 = 225 \times 3 + 192$
Since the remainder $192 \neq 0$, we apply the division lemma to 225 and 192 to get
$225 = 192 \times 1 + 33$
Since the remainder $33 \neq 0$, we apply the division lemma to 33 and 27 to get
$192 = 33 \times 5 + 27$
Since the remainder $27 \neq 0$, we apply the division lemma to 27 and 6 to get
$33 = 27 \times 1 + 6$
Since the remainder $6 \neq 0$, we apply the division lemma to 6 and 3 to get
$27 = 6 \times 4 + 3$
Since the remainder $3 \neq 0$, we apply the division lemma to 6 and 3 to get
$6 = 3 \times 2 + 0$
Now, remainder at this stage is zero, so our procedure stops.
So, HCF of 867 and 225 is 3.

Q2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$ or $6q + 5$, where q is some integer.

Sol. Let us start with taking a, where a is any positive odd integer. We apply the division algorithm, with a and $b = 6$. Since $0 \leq r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5. That is, a can be 6q or $6q + 1$, or $6q + 2$, or $6q + 3$, or $6q + 4$, or $6q + 5$, where q is the quotient. However, since a is odd, we do not consider the cases 6q, $6q + 2$ and

6q + 4 (since all the three are divisible by 2). Therefore, any positive odd integer is of the form 6q + 1 or 6q + 3 or 6q + 5.

Q3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. 616 and 32

$616 = 32 \times 19 + 8$

$32 = 4 \times 8$

HCF of (616, 32) = 8

Q4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let a be any odd positive integer. We apply the division lemma with a and b = 3. Since $0 \leq r < 3$, the possible remainders are 0, 1 and 2. That is, a can be 3q, or 3q + 1, or 3q + 2, where q is the quotient.

Now, $(3q)^2 = 9q^2$

which can be written in the form 3m, since 9 is divisible by 3.

Again, $(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

which can be written in the form 3m + 1 since $9q^2 + 6q$, i.e., $3(3q^2 + 2q)$ is divisible by 3.

Lastly, $(3q + 2)^2 = 9q^2 + 12q + 4$

$= (9q^2 + 12q + 3) + 1$

$= 3(3q^2 + 4q + 1) + 1$

which can be written in the form 3m + 1, since
$9q^2 + 12q + 3$, i.e., $3(3q^2 + 4q + 1)$ is divisible by 3.

Therefore, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Q5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Sol. Any positive integer is of the form

3q, 3q + 1, 3q + 2

Case1: Let, n = 3q

Cube of this will be

$n^3 = 27q^3$

$n^3 = 9(3q^3)$

So, $n^3 = 9m$, where $m = 3q^3$

Case2: n = 3q + 1

So, $n^3 = (3q + 1)^3$

$n^3 = 27q^3 + 1 + 27q^2 + 9q$

$= 9(3q^3 + 3q^2 + q) + 1$

$n^3 = 9m + 1$, where $m = 3q^3 + 3q^2 + q$

Case3: $n = 3q + 2$

So, $n^3 = (3q + 2)^3$

$= 27q^3 + 54q^2 + 36q + 8$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$n^3 = 9m + 8$, where $m = 3q^3 + 6q^2 + 4q$

So, it means cube of positive integer is of the form 9m, 9m + 1 and 9m + 8.