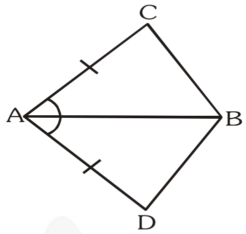
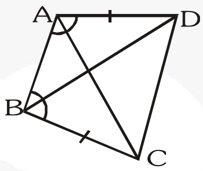
Ex - 7.1

Q1. In quadrilateral ACBD, AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



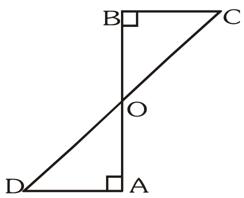
Sol. Given : In quadrilateral ACBD, AC = AD and AB bisect ∠A.
To prove : ΔABC ≅ ΔABD
Proof : In ΔABC and ΔABD
AC = AD (Given)
AB = AB (Common)
∠CAB = ∠DAB (AB bisect ∠A)
∴ ΔABC ≅ ΔABD (by SAS criteria)
BC = BD (by CPCT)

Q2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that
(i) ΔABD ≅ ΔBAC
(ii) BD = AC
(iii) ∠ABD = ∠BAC.



Sol. In ΔABD and ΔBAC,
AD = BC (Given)
∠DAB = ∠CBA (Given)
AB = AB (Common side)
∴ By SAS congruence rule, we have
ΔABD ≅ ΔBAC
Also, by CPCT, we have
BD = AC and ∠ABD = ∠BAC

Q3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Sol. Given : AD and BC are equal perpendiculars to line AB.

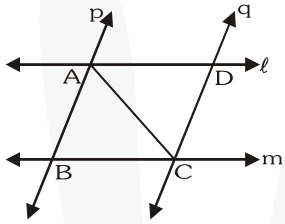
To prove : CD bisect AB

Proof : In ΔOAD and ΔOBC

AD = BC	(Given)
∠OAD = ∠OBC	(Each 90°)
∠AOD = ∠BOC	(Vertically opposite angles)
ΔOAD ≅ ΔOBC	(AAS rule)
OA = OB	(by CPCT)

∴ CD bisect AB.

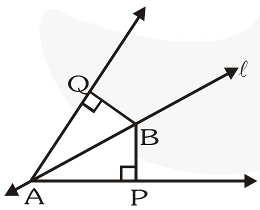
Q4. ℓ and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC ≅ ΔCDA.



Sol. In ΔABC and ΔCDA

∠CAB = ∠ACD	(Pair of alternate angle)
∠BCA = ∠DAC	(Pair of alternate angle)
AC = AC	(Common side)
∴ ΔABC ≅ ΔCDA	(ASA criteria)

Q5. Line ℓ is the bisector of an angle ∠A and B is any point on ℓ. BP and BQ are perpendiculars from B to the arms of ∠A. Show that :



(i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.

Sol. Given : line ℓ is bisector of angle A and B is any point on ℓ. BP and BQ are perpendicular from B to arms of ∠A.

To prove : (i) Δ APB ≅ Δ AQB (ii) BP = BQ.

Proof :

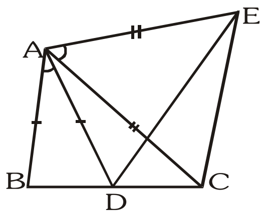
(i) In ΔAPB and ΔAQB

∠BAP = ∠BAQ	(ℓ is bisector)
AB = AB	(common)
∠BPA = ∠BQA	(Each 90°)
∴ Δ APB ≅ Δ AQB	(AAS rule)

(ii) Δ APB ≅ Δ AQB

BP = BQ	(By CPCT)

Q6. In figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



Sol. Given : AC = AE

AB = AD,

∠BAD = ∠EAC

To prove : BC = DE

Proof : In ΔABC and ΔADE

AB = AD (Given)

AC = AE (Given)

∠BAD = ∠EAC

Add ∠DAC to both

⇒ ∠BAD + ∠DAC = ∠DAC + ∠EAC

∠BAC = ∠DAE

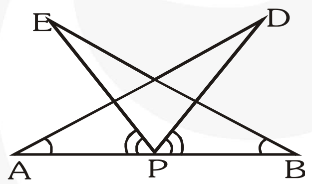
Δ ABC ≅ Δ ADE (SAS rule)

BC = DE (By CPCT)

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that

∠BAD = ∠ABE and ∠EPA = ∠DPB. Show that

(i) ΔDAP ≅ ΔEBP (ii) AD = BE



Sol. ∠EPA = ∠DPB (Given)

⇒ ∠EPA + ∠DPE = ∠DPB + ∠DPE

⇒ ∠APD = ∠BPE ...(1)

Now, in ΔDAP and ΔEBP, we have

AP = PB (∵ P is mid point of AB)

∠PAD = ∠PBE

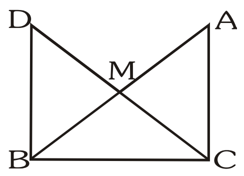
{∵ ∠PAD = ∠BAD, ∠PBE = ∠ABE
and we are given that ∠BAD = ∠ABE}

Also, ∠APD = ∠BPE (By 1)

∴ ΔDAP ≅ ΔEBP (By ASA congruence)

⇒ AD = BE (By CPCT)

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that :



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Sol. (i) In $\Delta AMC \cong \Delta BMD$,

$AM = BM$ ($\because$ M is mid point of AB)

$\angle AMC = \angle BMD$ (Vertically opposite angles)

$CM = DM$ (Given)

$\therefore \Delta AMC \cong \Delta BMD$ (By SAS congruence)

(ii) $\angle AMC = \angle BMD$,

$\Rightarrow \angle ACM = \angle BDM$ (By CPCT)

$\Rightarrow CA \parallel BD$

$\Rightarrow \angle BCA + \angle DBC = 180°$

$\Rightarrow \angle DBC = 90°$ ($\because \angle BCA = 90°$)

(iii) In ΔDBC and ΔACB,

$DB = AC$ ($\because \Delta BMD \cong \Delta AMC$)

$\angle DBC = \angle ACB$ (Each = 90°)

$BC = BC$ (Common side)

$\therefore \Delta DBC \cong \Delta ACB$ (By SAS congruence)

(iv) In $\Delta DBC \cong \Delta ACB \Rightarrow CD = AB$ ...(1)

Also, $\Delta AMC \cong \Delta BMD$

$\Rightarrow CM = DM$

$\Rightarrow CM = DM = \frac{1}{2} CD$

$\Rightarrow CD = 2\ CM$ ...(2)

From (1) and (2),

$2\ CM = AB$

$\Rightarrow CM = \frac{1}{2} AB$