Exercise 10.2

Question 1:

Write the equations for the *x* and *y*-axes.

Answer

The *y*-coordinate of every point on the *x*-axis is 0.

Therefore, the equation of the *x*-axis is $y = 0$.

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Therefore, the equation of the *y*-axis is $y = 0$.

Question 2:

Find the equation of the line which passes through the point (–4, 3) with slope $\frac{1}{2}$.

Answer

We know that the equation of the line passing through point (x_0, y_0), whose slope is *m*, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (–4, 3), whose slope is $\frac{1}{2}$, is

$$(y-3)=\frac{1}{2}(x+4)$$
$$2(y-3)=x+4$$
$$2y-6=x+4$$
$$\text{i.e., } x-2y+10=0$$

Question 3:

Find the equation of the line which passes though (0, 0) with slope *m*.

Answer

We know that the equation of the line passing through point (x_0, y_0), whose slope is *m*, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (0, 0), whose slope is *m*,is

$(y - 0) = m(x - 0)$

i.e., $y = mx$

Question 4:

Find the equation of the line which passes though $\left(2,\ 2\sqrt{3}\right)$ and is inclined with the *x*-axis at an angle of 75°.

Answer

The slope of the line that inclines with the *x*-axis at an angle of 75° is

$m = \tan 75°$

$$\Rightarrow m = \tan\left(45° + 30°\right) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

We know that the equation of the line passing through point $\left(x_0, y_0\right)$, whose slope is *m*, is $\left(y - y_0\right) = m\left(x - x_0\right)$.

Thus, if a line passes though $\left(2,\ 2\sqrt{3}\right)$ and inclines with the *x*-axis at an angle of 75°, then the equation of the line is given as

$$\left(y - 2\sqrt{3}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x - 2)$$

$$\left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}+1\right)(x - 2)$$

$$y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) = x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right)$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\sqrt{3} - 4$$

$$\text{i.e., } \left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\left(\sqrt{3}-1\right)$$

Question 5:

Find the equation of the line which intersects the *x*-axis at a distance of 3 units to the left of origin with slope –2.

Answer

It is known that if a line with slope m makes x-intercept d, then the equation of the line is given as

$y = m(x - d)$

For the line intersecting the x-axis at a distance of 3 units to the left of the origin, $d = -3$.

The slope of the line is given as $m = -2$

Thus, the required equation of the given line is

$y = -2[x - (-3)]$

$y = -2x - 6$

i.e., $2x + y + 6 = 0$

Question 6:

Find the equation of the line which intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30° with the positive direction of the x-axis.

Answer

It is known that if a line with slope m makes y-intercept c, then the equation of the line is given as

$y = mx + c$

Here, $c = 2$ and $m = \tan 30° = \frac{1}{\sqrt{3}}$.

Thus, the required equation of the given line is

$$y = \frac{1}{\sqrt{3}}x + 2$$

$$y = \frac{x + 2\sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3}y = x + 2\sqrt{3}$$

$$\text{i.e., } x - \sqrt{3}y + 2\sqrt{3} = 0$$

Question 7:

Find the equation of the line which passes through the points (–1, 1) and (2, –4).

Answer

It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

.

Therefore, the equation of the line passing through the points (–1, 1) and (2, –4) is

$$(y-1) = \frac{-4-1}{2+1}(x+1)$$

$$(y-1) = \frac{-5}{3}(x+1)$$

$$3(y-1) = -5(x+1)$$

$$3y - 3 = -5x - 5$$

$$\text{i.e., } 5x + 3y + 2 = 0$$

Question 8:

Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive *x*-axis is 30°

Answer

If *p* is the length of the normal from the origin to a line and ω is the angle made by the normal with the positive direction of the *x*-axis, then the equation of the line is given by *x*cos ω + *y* sin ω = *p*.

Here, *p* = 5 units and ω = 30°

Thus, the required equation of the given line is

x cos 30° + *y* sin 30° = 5

$$x\frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = 5$$

$$\text{i.e., } \sqrt{3}x + y = 10$$

Question 9:

The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find equation of the median through the vertex R.

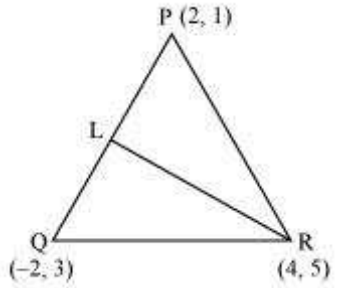
Answer

It is given that the vertices of ΔPQR are P (2, 1), Q (–2, 3), and R (4, 5).

Let RL be the median through vertex R.

Accordingly, L is the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)=(0, 2)$



It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

.

Therefore, the equation of RL can be determined by substituting $(x_1, y_1) = (4, 5)$ and $(x_2, y_2) = (0, 2)$.

Hence, $y-5=\frac{2-5}{0-4}(x-4)$

$$\Rightarrow y-5=\frac{-3}{-4}(x-4)$$
$$\Rightarrow 4(y-5)=3(x-4)$$
$$\Rightarrow 4y-20=3x-12$$
$$\Rightarrow 3x-4y+8=0$$

Thus, the required equation of the median through vertex R is $3x-4y+8=0$.

Question 10:

Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

Answer

The slope of the line joining the points (2, 5) and (–3, 6) is $m=\frac{6-5}{-3-2}=\frac{1}{-5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points (2, 5) and (–3, 6)

$$=-\frac{1}{m}=-\frac{1}{\left(\frac{-1}{5}\right)}=5$$

Now, the equation of the line passing through point (–3, 5), whose slope is 5, is

$$(y-5)=5(x+3)$$
$$y-5=5x+15$$
$$\text{i.e., } 5x-y+20=0$$

Question 11:

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1:*n*. Find the equation of the line.

Answer

According to the section formula, the coordinates of the point that divides the line segment joining the points (1, 0) and (2, 3) in the ratio 1: *n* is given by

$$\left(\frac{n(1)+1(2)}{1+n},\frac{n(0)+1(3)}{1+n}\right)=\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

The slope of the line joining the points (1, 0) and (2, 3) is

$$m=\frac{3-0}{2-1}=3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points (1, 0) and (2, 3) $=-\frac{1}{m}=-\frac{1}{3}$

Now, the equation of the line passing through $\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$ is given by

$$\left(y-\frac{3}{n+1}\right)=\frac{-1}{3}\left(x-\frac{n+2}{n+1}\right)$$
$$\Rightarrow 3\left[(n+1)y-3\right]=-\left[x(n+1)-(n+2)\right]$$
$$\Rightarrow 3(n+1)y-9=-(n+1)x+n+2$$
$$\Rightarrow (1+n)x+3(1+n)y=n+11$$

Question 12:

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Answer

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \ldots \text{(i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that $a = b$.

Accordingly, equation (i) reduces to

$$\frac{x}{a}+\frac{y}{a}=1$$
$$\Rightarrow x+y=a \qquad \ldots \text{(ii)}$$

Since the given line passes through point (2, 3), equation (ii) reduces to

$2 + 3 = a \Rightarrow a = 5$

On substituting the value of a in equation (ii), we obtain

$x + y = 5$, which is the required equation of the line

Question 13:

Find equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Answer

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \ldots \text{(i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that$a + b = 9 \Rightarrow b = 9 - a$ … (ii)

From equations (i) and (ii), we obtain

$$\frac{x}{a}+\frac{y}{9-a}=1 \qquad \ldots(\text{iii})$$

It is given that the line passes through point (2, 2). Therefore, equation (iii) reduces to

$$\frac{2}{a}+\frac{2}{9-a}=1$$
$$\Rightarrow 2\left(\frac{1}{a}+\frac{1}{9-a}\right)=1$$
$$\Rightarrow 2\left(\frac{9-a+a}{a(9-a)}\right)=1$$
$$\Rightarrow \frac{18}{9a-a^2}=1$$
$$\Rightarrow 18=9a-a^2$$
$$\Rightarrow a^2-9a+18=0$$
$$\Rightarrow a^2-6a-3a+18=0$$
$$\Rightarrow a(a-6)-3(a-6)=0$$
$$\Rightarrow (a-6)(a-3)=0$$
$$\Rightarrow a=6 \text{ or } a=3$$

If $a = 6$ and $b = 9 - 6 = 3$, then the equation of the line is

$$\frac{x}{6}+\frac{y}{3}=1 \Rightarrow x+2y-6=0$$

If $a = 3$ and $b = 9 - 3 = 6$, then the equation of the line is

$$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y-6=0$$

Question 14:

Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Answer

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive *x*-axis is $m = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

Now, the equation of the line passing through point (0, 2) and having a slope $-\sqrt{3}$ is $(y-2) = -\sqrt{3}(x-0)$.

$y-2 = -\sqrt{3}x$

i.e., $\sqrt{3}x + y - 2 = 0$

The slope of line parallel to line $\sqrt{3}x + y - 2 = 0$ is $-\sqrt{3}$.

It is given that the line parallel to line $\sqrt{3}x + y - 2 = 0$ crosses the *y*-axis 2 units below the origin i.e., it passes through point (0, –2).

Hence, the equation of the line passing through point (0, –2) and having a slope $-\sqrt{3}$ is

$y-(-2) = -\sqrt{3}(x-0)$

$y+2 = -\sqrt{3}x$

$\sqrt{3}x + y + 2 = 0$

Question 15:

The perpendicular from the origin to a line meets it at the point (– 2, 9), find the equation of the line.

Answer

The slope of the line joining the origin (0, 0) and point (–2, 9) is $m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point (–2, 9) is

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$$

Now, the equation of the line passing through point (–2, 9) and having a slope m_2 is

$$(y-9)=\frac{2}{9}(x+2)$$
$$9y-81=2x+4$$
$$\text{i.e., } 2x-9y+85=0$$

Question 16:

The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

Answer

It is given that when C = 20, the value of L is 124.942, whereas when C = 110, the value of L is 125.134.

Accordingly, points (20, 124.942) and (110, 125.134) satisfy the linear relation between L and C.

Now, assuming C along the *x*-axis and L along the *y*-axis, we have two points i.e., (20, 124.942) and (110, 125.134) in the XY plane.

Therefore, the linear relation between L and C is the equation of the line passing through points (20, 124.942) and (110, 125.134).

$$(\text{L}-124.942)=\frac{125.134-124.942}{110-20}(\text{C}-20)$$
$$\text{L}-124.942=\frac{0.192}{90}(\text{C}-20)$$
$$\text{i.e., L}=\frac{0.192}{90}(\text{C}-20)+124.942, \text{ which is the required linear relation}$$

Question 17:

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

Answer

The relationship between selling price and demand is linear.

Assuming selling price per litre along the *x*-axis and demand along the *y*-axis, we have two points i.e., (14, 980) and (16, 1220) in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points (14, 980) and (16, 1220).

$$y-980=\frac{1220-980}{16-14}(x-14)$$

$$y-980=\frac{240}{2}(x-14)$$

$$y-980=120(x-14)$$

$$\text{i.e., } y=120(x-14)+980$$

When x = Rs 17/litre,

$$y=120(17-14)+980$$

$$\Rightarrow y=120\times 3+980=360+980=1340$$

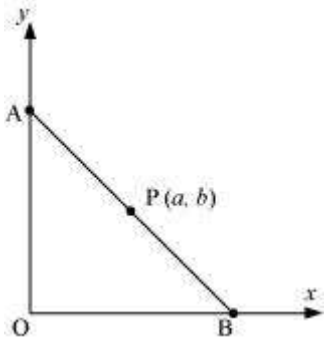
Thus, the owner of the milk store could sell 1340 litres of milk weekly at Rs 17/litre.

Question 18:

P (*a, b*) is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$

Answer

Let AB be the line segment between the axes and let P (*a*, *b*) be its mid-point.



Let the coordinates of A and B be (0, *y*) and (*x*, 0) respectively.

Since P (*a*, *b*) is the mid-point of AB,

$$\left(\frac{0+x}{2},\frac{y+0}{2}\right)=(a,b)$$

$$\Rightarrow\left(\frac{x}{2},\frac{y}{2}\right)=(a,b)$$

$$\Rightarrow\frac{x}{2}=a \text{ and } \frac{y}{2}=b$$

$$\therefore x=2a \text{ and } y=2b$$

Thus, the respective coordinates of A and B are (0, 2*b*) and (2*a*, 0).

The equation of the line passing through points (0, 2*b*) and (2*a*, 0) is

$$(y-2b)=\frac{(0-2b)}{(2a-0)}(x-0)$$

$$y-2b=\frac{-2b}{2a}(x)$$

$$a(y-2b)=-bx$$

$$ay-2ab=-bx$$

$$\text{i.e., } bx+ay=2ab$$

On dividing both sides by *ab*, we obtain

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$$

$$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$$

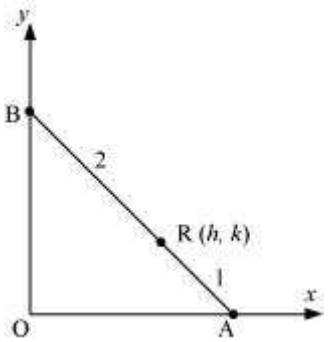
Thus,the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

Question 19:

Point R (*h, k*) divides a line segment between the axes in the ratio 1:2. Find equation of the line.

Answer

Let AB be the line segment between the axes such that point R (*h*, *k*) divides AB in the ratio 1: 2.



Let the respective coordinates of A and B be (x, 0) and (0, y).

Since point R (h, k) divides AB in the ratio 1: 2, according to the section formula,

$$(h,k)=\left(\frac{1\times 0+2\times x}{1+2},\frac{1\times y+2\times 0}{1+2}\right)$$

$$\Rightarrow (h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$$

$$\Rightarrow h=\frac{2x}{3} \text{ and } k=\frac{y}{3}$$

$$\Rightarrow x=\frac{3h}{2} \text{ and } y=3k$$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2},0\right)$ and (0, 3k).

Now, the equation of line AB passing through points $\left(\frac{3h}{2},0\right)$ and (0, 3k) is

$$(y-0)=\frac{3k-0}{0-\frac{3h}{2}}\left(x-\frac{3h}{2}\right)$$

$$y=-\frac{2k}{h}\left(x-\frac{3h}{2}\right)$$

$$hy=-2kx+3hk$$

$$\text{i.e., } 2kx+hy=3hk$$

Thus,the required equation of the line is $2kx + hy = 3hk$

Question 20:

By using the concept of equation of a line, prove that the three points (3, 0),

(–2, –2) and (8, 2) are collinear.

Answer

In order to show that points (3, 0), (–2, –2), and (8, 2) are collinear, it suffices to show that the line passing through points (3, 0) and (–2, –2) also passes through point (8, 2).

The equation of the line passing through points (3, 0) and (–2, –2) is

$$(y-0)=\frac{(-2-0)}{(-2-3)}(x-3)$$

$$y=\frac{-2}{-5}(x-3)$$

$$5y=2x-6$$

$$\text{i.e., } 2x-5y=6$$

It is observed that at $x = 8$ and $y = 2$,

L.H.S. = 2 × 8 – 5 × 2 = 16 – 10 = 6 = R.H.S.

Therefore, the line passing through points (3, 0) and (–2, –2) also passes through point (8, 2). Hence, points (3, 0), (–2, –2), and (8, 2) are collinear.