

Exercise 11.1**Question 1:**

If a line makes angles 90° , 135° , 45° with x , y and z -axes respectively, find its direction cosines.

Answer

Let direction cosines of the line be l , m , and n .

$$l = \cos 90^\circ = 0$$

$$m = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$, and $\frac{1}{\sqrt{2}}$.

Question 2:

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Answer

Let the direction cosines of the line make an angle a with each of the coordinate axes.

$$\therefore l = \cos a, m = \cos a, n = \cos a$$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow 3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes,

are $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}},$ and $\pm \frac{1}{\sqrt{3}}.$

Question 3:

If a line has the direction ratios $-18, 12, -4,$ then what are its direction cosines?

Answer

If a line has direction ratios of $-18, 12,$ and $-4,$ then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}$$

i.e., $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

Thus, the direction cosines are $-\frac{9}{11}, \frac{6}{11},$ and $\frac{-2}{11}.$

Question 4:

Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

Answer

The given points are A $(2, 3, 4),$ B $(-1, -2, 1),$ and C $(5, 8, 7).$

It is known that the direction ratios of line joining the points, (x_1, y_1, z_1) and $(x_2, y_2, z_2),$ are given by, $x_2 - x_1, y_2 - y_1,$ and $z_2 - z_1.$

The direction ratios of AB are $(-1 - 2), (-2 - 3),$ and $(1 - 4)$ i.e., $-3, -5,$ and $-3.$

The direction ratios of BC are $(5 - (-1)), (8 - (-2)),$ and $(7 - 1)$ i.e., $6, 10,$ and $6.$

It can be seen that the direction ratios of BC are -2 times that of AB i.e., they are proportional.

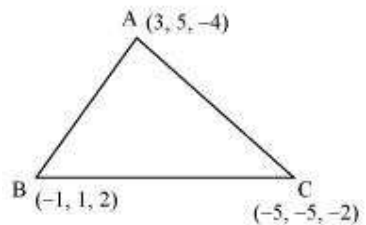
Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B, and C are collinear.

Question 5:

Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4)$, $(-1, 1, 2)$ and $(-5, -5, -2)$

Answer

The vertices of ΔABC are $A(3, 5, -4)$, $B(-1, 1, 2)$, and $C(-5, -5, -2)$.



The direction ratios of side AB are $(-1 - 3)$, $(1 - 5)$, and $(2 - (-4))$ i.e., -4 , -4 , and 6 .

$$\begin{aligned} \text{Then, } \sqrt{(-4)^2 + (-4)^2 + (6)^2} &= \sqrt{16 + 16 + 36} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

Therefore, the direction cosines of AB are

$$\begin{aligned} \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{6}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}} \\ \frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}} \\ \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \end{aligned}$$

The direction ratios of BC are $(-5 - (-1))$, $(-5 - 1)$, and $(-2 - 2)$ i.e., -4 , -6 , and -4 .

Therefore, the direction cosines of BC are

$$\begin{aligned} \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-6}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} \\ \frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}} \end{aligned}$$

The direction ratios of CA are $(-5 - 3)$, $(-5 - 5)$, and $(-2 - (-4))$ i.e., -8 , -10 , and 2 .

Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{-5}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{2}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}$$
$$\frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}$$