Exercise 12.2

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4) (iv) (2, –1, 3) and (–2, 1, 3)

Answer

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

(iii) Distance between points (–1, 3, –4) and (1, –3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, –1, 3) and (–2, 1, 3)

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Question 2:

Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Answer

Let points (–2, 3, 5), (1, 2, 3), and (7, 0, –1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$\begin{aligned} PQ &= \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} \\ &= \sqrt{(3)^2 + (-1)^2 + (-2)^2} \\ &= \sqrt{9+1+4} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\ &= \sqrt{(6)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{36+4+16} \\ &= \sqrt{56} \\ &= 2\sqrt{14} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} \\ &= \sqrt{(9)^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81+9+36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Answer

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (−1, 6, 6), and (−4, 9, 6) be denoted by A, B, and C respectively.

$$\begin{aligned}AB &= \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}\\ &= \sqrt{(-1)^2+(-1)^2+(-4)^2}\\ &= \sqrt{1+1+16} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}\\ &= \sqrt{(-3)^2+(3)^2+(0)^2}\\ &= \sqrt{9+9} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2}\\ &= \sqrt{(4)^2+(-2)^2+(4)^2}\\ &= \sqrt{16+4+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

$$\text{Now, } AB^2+BC^2 = \left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2 = 18+18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (−1, 2, 1), (1, −2, 5), (4, −7, 8), and (2, −3, 4) be denoted by A, B, C, and D respectively.

$$\begin{aligned}AB &= \sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}\\ &= \sqrt{4+16+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}\\ &= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}\\ &= \sqrt{4+16+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-1-2)^2+(2+3)^2+(1-4)^2}\\ &= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Answer

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, –1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-1)^2+(y-2)^2+(z-3)^2 = (x-3)^2+(y-2)^2+(z+1)^2$$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (–4, 0, 0) is equal to 10.

Answer

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$

$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$

$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.