

Therefore, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for the sets A, B, and C.

Exercise 1.4

Question 1:

Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$

$B = \{x: x \text{ is a natural number less than } 6\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

Answer

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

$X \cup Y = \{1, 2, 3, 5\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

$$A \cup B = \{a, b, c, e, i, o, u\}$$

$$\text{(iii) } A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \dots\}$$

$$B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12 \dots\}$$

$$\therefore A \cup B = \{x: x = 1, 2, 3, 4, 5 \text{ or a multiple of } 3\}$$

$$\text{(iv) } A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\therefore A \cup B = \{x: x \in \mathbb{N} \text{ and } 1 < x < 10\}$$

$$\text{(v) } A = \{1, 2, 3\}, B = \Phi$$

$$A \cup B = \{1, 2, 3\}$$

Question 2:

Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Answer

Here, $A = \{a, b\}$ and $B = \{a, b, c\}$

Yes, $A \subset B$.

$$A \cup B = \{a, b, c\} = B$$

Question 3:

If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Answer

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

Question 4:

If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

$$\text{(i) } A \cup B$$

$$\text{(ii) } A \cup C$$

$$\text{(iii) } B \cup C$$

$$\text{(iv) } B \cup D$$

$$\text{(v) } A \cup B \cup C$$

$$\text{(vi) } A \cup B \cup D$$

(vii) $B \cup C \cup D$

Answer

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Question 5:

Find the intersection of each pair of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$

$B = \{x: x \text{ is a natural number less than } 6\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

Answer

(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

$X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

$A \cap B = \{a\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \dots\}$

$B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5\}$

$\therefore A \cap B = \{3\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

$A \cap B = \Phi$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

$$A \cap B = \Phi$$

Question 6:

If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

- (i) $A \cap B$
- (ii) $B \cap C$
- (iii) $A \cap C \cap D$
- (iv) $A \cap C$
- (v) $B \cap D$
- (vi) $A \cap (B \cup C)$
- (vii) $A \cap D$
- (viii) $A \cap (B \cup D)$
- (ix) $(A \cap B) \cap (B \cup C)$
- (x) $(A \cup D) \cap (B \cup C)$

Answer

- (i) $A \cap B = \{7, 9, 11\}$
- (ii) $B \cap C = \{11, 13\}$
- (iii) $A \cap C \cap D = \{A \cap C\} \cap D = \{11\} \cap \{15, 17\} = \Phi$
- (iv) $A \cap C = \{11\}$
- (v) $B \cap D = \Phi$
- (vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 $= \{7, 9, 11\} \cup \{11\} = \{7, 9, 11\}$
- (vii) $A \cap D = \Phi$
- (viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$
 $= \{7, 9, 11\} \cup \Phi = \{7, 9, 11\}$
- (ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$
- (x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$
 $= \{7, 9, 11, 15\}$

Question 7:

If $A = \{x: x \text{ is a natural number}\}$, $B = \{x: x \text{ is an even natural number}\}$

$C = \{x: x \text{ is an odd natural number}\}$ and $D = \{x: x \text{ is a prime number}\}$, find

- (i) $A \cap B$
- (ii) $A \cap C$
- (iii) $A \cap D$
- (iv) $B \cap C$
- (v) $B \cap D$
- (vi) $C \cap D$

Answer

$$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4, 5 \dots\}$$

$$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8 \dots\}$$

$$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9 \dots\}$$

$$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7 \dots\}$$

$$\text{(i) } A \cap B = \{x: x \text{ is an even natural number}\} = B$$

$$\text{(ii) } A \cap C = \{x: x \text{ is an odd natural number}\} = C$$

$$\text{(iii) } A \cap D = \{x: x \text{ is a prime number}\} = D$$

$$\text{(iv) } B \cap C = \Phi$$

$$\text{(v) } B \cap D = \{2\}$$

$$\text{(vi) } C \cap D = \{x: x \text{ is odd prime number}\}$$

Question 8:

Which of the following pairs of sets are disjoint

$$\text{(i) } \{1, 2, 3, 4\} \text{ and } \{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$$

$$\text{(ii) } \{a, e, i, o, u\} \text{ and } \{c, d, e, f\}$$

$$\text{(iii) } \{x: x \text{ is an even integer}\} \text{ and } \{x: x \text{ is an odd integer}\}$$

Answer

$$\text{(i) } \{1, 2, 3, 4\}$$

$$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$$

$$\text{Now, } \{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$$

Therefore, this pair of sets is not disjoint.

$$\text{(ii) } \{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\}$$

Therefore, $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ are not disjoint.

$$\text{(iii) } \{x: x \text{ is an even integer}\} \cap \{x: x \text{ is an odd integer}\} = \Phi$$

Therefore, this pair of sets is disjoint.

Question 9:

If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$,
 $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$; find

- (i) $A - B$
- (ii) $A - C$
- (iii) $A - D$
- (iv) $B - A$
- (v) $C - A$
- (vi) $D - A$
- (vii) $B - C$
- (viii) $B - D$
- (ix) $C - B$
- (x) $D - B$
- (xi) $C - D$
- (xii) $D - C$

Answer

- (i) $A - B = \{3, 6, 9, 15, 18, 21\}$
- (ii) $A - C = \{3, 9, 15, 18, 21\}$
- (iii) $A - D = \{3, 6, 9, 12, 18, 21\}$
- (iv) $B - A = \{4, 8, 16, 20\}$
- (v) $C - A = \{2, 4, 8, 10, 14, 16\}$
- (vi) $D - A = \{5, 10, 20\}$
- (vii) $B - C = \{20\}$
- (viii) $B - D = \{4, 8, 12, 16\}$
- (ix) $C - B = \{2, 6, 10, 14\}$
- (x) $D - B = \{5, 10, 15\}$
- (xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$
- (xii) $D - C = \{5, 15, 20\}$

Question 10:

If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

- (i) $X - Y$
- (ii) $Y - X$
- (iii) $X \cap Y$

Answer

- (i) $X - Y = \{a, c\}$
- (ii) $Y - X = \{f, g\}$
- (iii) $X \cap Y = \{b, d\}$

Question 11:

If **R** is the set of real numbers and **Q** is the set of rational numbers, then what is **R - Q**?

Answer

R: set of real numbers

Q: set of rational numbers

Therefore, $R - Q$ is a set of irrational numbers.

Question 12:

State whether each of the following statement is true or false. Justify your answer.

- (i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.
- (ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.
- (iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.
- (iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Answer

(i) False

As $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

$\Rightarrow \{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

As $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

$\Rightarrow \{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

As $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$

(iv) True

As $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$