

Exercise 2.1

Question 1:

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$. Then $\sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of \sin^{-1} is

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$.

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

Question 2:

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$. Then, $\cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of \cos^{-1} is

$[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

Question 3:

Find the principal value of $\operatorname{cosec}^{-1}(2)$

Answer

Let $\operatorname{cosec}^{-1}(2) = y$. Then, $\operatorname{cosec} y = 2 = \operatorname{cosec}\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $\tan^{-1}(-\sqrt{3})$

Answer

Let $\tan^{-1}(-\sqrt{3}) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of \tan^{-1} is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}(\sqrt{3})$ is $-\frac{\pi}{3}$.

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

We know that the range of the principal value branch of \cos^{-1} is

$[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Answer

$$\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right).$$

Let $\tan^{-1}(-1) = y$. Then,

We know that the range of the principal value branch of \tan^{-1} is

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \tan\left(-\frac{\pi}{4}\right) = -1.$$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Answer

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y. \text{ Then, } \sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \sec^{-1} is

$$[0, \pi] - \left\{\frac{\pi}{2}\right\} \text{ and } \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}.$$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $\cot^{-1}(\sqrt{3})$

Answer

$$\text{Let } \cot^{-1}(\sqrt{3}) = y. \text{ Then, } \cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \cot^{-1} is $(0, \pi)$ and

$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}.$$

Therefore, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y. \text{ Then, } \cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right).$$

We know that the range of the principal value branch of \cos^{-1} is $[0, \pi]$ and

$$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}.$$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$

Answer

$$\text{Let } \operatorname{cosec}^{-1}(-\sqrt{2}) = y. \text{ Then, } \operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right).$$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\} \text{ and } \operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}.$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

Question 11:

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

Find the value of

Answer

$$\text{Let } \tan^{-1}(1) = x. \text{ Then, } \tan x = 1 = \tan \frac{\pi}{4}.$$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = z. \text{ Then, } \sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

Question 12:

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

Find the value of

Answer

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y. \text{ Then, } \sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Question 13:

Find the value of y if $\sin^{-1} x = y$, then

(A) $0 \leq y \leq \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of \sin^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Question 14:

Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Answer

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan \frac{\pi}{3}$.

We know that the range of the principal value branch of \tan^{-1} is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec \frac{2\pi}{3}$.

We know that the range of the principal value branch of \sec^{-1} is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$$

$$\text{Hence, } \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$