

## Exercise 3.1

## Question 1:

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

In the matrix  $A$ , write:

- (i) The order of the matrix (ii) The number of elements,  
 (iii) Write the elements  $a_{13}$ ,  $a_{21}$ ,  $a_{33}$ ,  $a_{24}$ ,  $a_{23}$

Answer

**(i)** In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is  $3 \times 4$ .

**(ii)** Since the order of the matrix is  $3 \times 4$ , there are  $3 \times 4 = 12$  elements in it.

**(iii)**  $a_{13} = 19$ ,  $a_{21} = 35$ ,  $a_{33} = -5$ ,  $a_{24} = 12$ ,  $a_{23} = \frac{5}{2}$

## Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Answer

We know that if a matrix is of the order  $m \times n$ , it has  $mn$  elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are:  $(1, 24)$ ,  $(24, 1)$ ,  $(2, 12)$ ,  $(12, 2)$ ,  $(3, 8)$ ,  $(8, 3)$ ,  $(4, 6)$ , and  $(6, 4)$

Hence, the possible orders of a matrix having 24 elements are:

$1 \times 24$ ,  $24 \times 1$ ,  $2 \times 12$ ,  $12 \times 2$ ,  $3 \times 8$ ,  $8 \times 3$ ,  $4 \times 6$ , and  $6 \times 4$

$(1, 13)$  and  $(13, 1)$  are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are  $1 \times 13$  and  $13 \times 1$ .

## Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer

We know that if a matrix is of the order  $m \times n$ , it has  $mn$  elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6), and (6, 3)

Hence, the possible orders of a matrix having 18 elements are:

$1 \times 18$ ,  $18 \times 1$ ,  $2 \times 9$ ,  $9 \times 2$ ,  $3 \times 6$ , and  $6 \times 3$

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are  $1 \times 5$  and  $5 \times 1$ .

#### Question 5:

Construct a  $3 \times 4$  matrix, whose elements are given by

$$(i) \ a_{ij} = \frac{1}{2}|-3i + j| \quad (ii) \ a_{ij} = 2i - j$$

Answer

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

In general, a  $3 \times 4$  matrix is given by

$$(i) \ a_{ij} = \frac{1}{2}|-3i + j|, \ i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

$$\therefore a_{11} = \frac{1}{2}|-3 \times 1 + 1| = \frac{1}{2}|-3 + 1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{1}{2}|-6 + 1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9 + 1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3 + 2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6 + 2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9 + 2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = \frac{1}{2}|-3 + 3| = 0$$

$$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{1}{2}|-6 + 3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = \frac{1}{2}|-9 + 3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}|-3 + 4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}|-3 \times 2 + 4| = \frac{1}{2}|-6 + 4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{1}{2}|-9 + 4| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

Therefore, the required matrix is

(ii)  $a_{ij} = 2i - j$ ,  $i = 1, 2, 3$  and  $j = 1, 2, 3, 4$

$$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

$$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$$

$$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$$

$$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$$

$$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$$

$$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$$

$$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Therefore, the required matrix is

### Question 6:

Find the value of  $x$ ,  $y$ , and  $z$  from the following equation:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Answer

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x = 1, y = 4, \text{ and } z = 3$$

$$\text{(ii)} \quad \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y = 6, xy = 8, 5 + z = 5$$

$$\text{Now, } 5 + z = 5 \Rightarrow z = 0$$

We know that:

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$\Rightarrow (x - y)^2 = 36 - 32 = 4$$

$$\Rightarrow x - y = \pm 2$$

Now, when  $x - y = 2$  and  $x + y = 6$ , we get  $x = 4$  and  $y = 2$

When  $x - y = -2$  and  $x + y = 6$ , we get  $x = 2$  and  $y = 4$

$\therefore x = 4, y = 2, \text{ and } z = 0$  or  $x = 2, y = 4, \text{ and } z = 0$

$$\text{(iii)} \quad \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y + z = 9 \dots (1)$$

$$x + z = 5 \dots (2)$$

$$y + z = 7 \dots (3)$$

From (1) and (2), we have:

$$y + 5 = 9$$

$$\Rightarrow y = 4$$

Then, from (3), we have:

$$4 + z = 7$$

$$\Rightarrow z = 3$$

$$\therefore x + z = 5$$

$$\Rightarrow x = 2$$

$\therefore x = 2, y = 4, \text{ and } z = 3$

**Question 7:**

Find the value of  $a$ ,  $b$ ,  $c$ , and  $d$  from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Answer

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$a - b = -1 \dots (1)$$

$$2a - b = 0 \dots (2)$$

$$2a + c = 5 \dots (3)$$

$$3c + d = 13 \dots (4)$$

From (2), we have:

$$b = 2a$$

Then, from (1), we have:

$$a - 2a = -1$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

Now, from (3), we have:

$$2 \times 1 + c = 5$$

$$\Rightarrow c = 3$$

From (4) we have:

$$3 \times 3 + d = 13$$

$$\Rightarrow 9 + d = 13 \Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3, \text{ and } d = 4$$

**Question 8:**

$A = [a_{ij}]_{m \times n}$  is a square matrix, if

**(A)**  $m < n$

**(B)**  $m > n$

(C)  $m = n$

(D) None of these

Answer

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore,  $A = [a_{ij}]_{m \times n}$  is a square matrix, if  $m = n$ .

**Question 9:**

Which of the given values of  $x$  and  $y$  make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A)  $x = \frac{-1}{3}, y = 7$

(B) Not possible to find

(C)  $y = 7, x = \frac{-2}{3}$

(D)  $x = \frac{-1}{3}, y = \frac{-2}{3}$

Answer

The correct answer is B.

It is given that  $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x+7=0 \Rightarrow x = -\frac{7}{3}$$

$$5 = y-2 \Rightarrow y = 7$$

$$y+1=8 \Rightarrow y = 7$$

$$2-3x=4 \Rightarrow x = -\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of  $x$ , which is not possible.

Hence, it is not possible to find the values of  $x$  and  $y$  for which the given matrices are equal.

**Question 10:**

The number of all possible matrices of order  $3 \times 3$  with each entry 0 or 1 is:

- (A) 27
- (B) 18
- (C) 81
- (D) 512

Answer

The correct answer is D.

The given matrix of the order  $3 \times 3$  has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is  $2^9$   
 $= 512$