Exercise 3.3

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Answer

L.H.S. = $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$

$$=\text{R.H.S.}$$

Question 2:

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Answer

L.H.S. = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$

$=\text{R.H.S.}$

Question 3:

Prove that $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Answer

L.H.S. = $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$

$=3+2+1=6$

$=\text{R.H.S}$

Question 4:

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Answer

L.H.S = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

Question 5:

Find the value of:

(i) sin 75°

(ii) tan 15°

Answer

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin (x + y) = sin x cos y + cos x sin y]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

Question 6:

Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

Answer

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\begin{bmatrix} \because 2\cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B = \cos(A+B) - \cos(A-B) \end{bmatrix}$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y)$$

$$= \text{R.H.S}$$

Question 7:

Prove that: $$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Answer

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

∴L.H.S. = $$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

Question 8:

Prove that $$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Answer

$$\begin{aligned}
\text{L.H.S.} &= \frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}\\
&=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}\\
&=\frac{-\cos^2 x}{-\sin^2 x}\\
&=\cot^2 x\\
&=\text{R.H.S.}
\end{aligned}$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Answer

L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$$=\sin x\cos x[\tan x+\cot x]$$

$$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

$$=1=\text{R.H.S.}$$

Question 10:

Prove that sin (n + 1)x sin (n + 2)x + cos (n + 1)x cos (n + 2)x = cos x

Answer

L.H.S. = sin (n + 1)x sin(n + 2)x + cos (n + 1)x cos(n + 2)x

$$=\frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x\right]$$

$$=\frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$$

$$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$$

$$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$=\cos(-x)=\cos x=\text{R.H.S.}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Answer

It is known that $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$.

∴L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x$$

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$= -\sqrt{2}\sin x$$

$$= \text{R.H.S.}$$

Question 12:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Answer

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\sin^2 6x - \sin^2 4x$

= $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= $(2\sin 5x \cos x)(2\cos 5x \sin x)$

= $(2\sin 5x \cos 5x)(2\sin x \cos x)$

= $\sin 10x \sin 2x$

= R.H.S.

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer

It is known that

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\cos^2 2x - \cos^2 6x$

= $(\cos 2x + \cos 6x)(\cos 2x - 6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

= $[2\cos 4x\cos 2x]\,[-2\sin 4x(-\sin 2x)]$

= $(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

= $\sin 8x\sin 4x$

= R.H.S.

Question 14:

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x\sin 4x$

Answer

L.H.S. = $\sin 2x + 2\sin 4x + \sin 6x$

= $[\sin 2x + \sin 6x] + 2\sin 4x$

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

$$\left[\because \sin A+\sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

= $2\sin 4x\cos(-2x) + 2\sin 4x$

= $2\sin 4x\cos 2x + 2\sin 4x$

= $2\sin 4x(\cos 2x + 1)$

= $2\sin 4x(2\cos^2 x - 1 + 1)$

= $2\sin 4x(2\cos^2 x)$

$= 4\cos^2 x \sin 4x$

= R.H.S.

Question 15:

Prove that cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Answer

L.H.S = cot 4x (sin 5x + sin 3x)

$$=\frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x\cos x]$$

= 2 cos 4x cos x

R.H.S. = cot x (sin 5x – sin 3x)

$$=\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{\cos x}{\sin x}[2\cos 4x\sin x]$$

= 2 cos 4x. cos x

L.H.S. = R.H.S.

Question 16:

Prove that $\dfrac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\dfrac{\sin 2x}{\cos 10x}$

Answer

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S = $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= \text{R.H.S.}$$

Question 17:

Prove that $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Answer

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x = \text{R.H.S.}$$

Question 18:

Prove that $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan\dfrac{x-y}{2}$

Answer

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Question 19:

Prove that $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Answer

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

Question 20:

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Answer

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \ \cos^2 A - \sin^2 A = \cos 2A$$

∴L.H.S. = $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

Question 21:

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Answer

L.H.S. = $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$=\cot 3x=\text{R.H.S.}$$

Question 22:

Prove that cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x = 1

Answer

L.H.S. = cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x

= cot x cot $2x$ – cot $3x$ (cot $2x$ + cot x)

= cot x cot $2x$ – cot ($2x$ + x) (cot $2x$ + cot x)

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

$$\left[\because \cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$

= cot x cot $2x$ – (cot $2x$ cot x – 1)

= 1 = R.H.S.

Question 23:

Prove that $\tan 4x=\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Answer

It is known that $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$.

∴L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Question 24:

Prove that $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

Answer

L.H.S. = $\cos 4x$

= $\cos 2(2x)$

= $1 - 2\sin^2 2x$ [$\cos 2A = 1 - 2\sin^2 A$]

= $1 - 2(2\sin x\cos x)^2$ [$\sin 2A = 2\sin A\cos A$]

= $1 - 8\sin^2 x\cos^2 x$

= R.H.S.

Question 25:

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Answer

L.H.S. = $\cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$ $[\cos 3A = 4\cos^3 A - 3\cos A]$

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$ $[\cos 2x = 2\cos^2 x - 1]$

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

= R.H.S.