

Exercise 4.3

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8)

(iii) (-2, -3), (3, 2), (-1, -8)

Answer

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} \\ &= \frac{1}{2} [1(0-3) - 0(6-4) + 1(18-0)] \\ &= \frac{1}{2} [-3+18] = \frac{15}{2} \text{ square units}\end{aligned}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix} \\ &= \frac{1}{2} [2(1-8) - 7(1-10) + 1(8-10)] \\ &= \frac{1}{2} [2(-7) - 7(-9) + 1(-2)] \\ &= \frac{1}{2} [-14 + 63 - 2] = \frac{1}{2} [-16 + 63] \\ &= \frac{47}{2} \text{ square units}\end{aligned}$$

(iii) The area of the triangle with vertices (-2, -3), (3, 2), (-1, -8) is given by the relation,

$$\begin{aligned}
 \Delta &= \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix} \\
 &= \frac{1}{2} [-2(2+8) + 3(3+1) + 1(-24+2)] \\
 &= \frac{1}{2} [-2(10) + 3(4) + 1(-22)] \\
 &= \frac{1}{2} [-20 + 12 - 22] \\
 &= -\frac{30}{2} = -15
 \end{aligned}$$

Hence, the area of the triangle is $|-15| = 15$ square units.

Question 2:

Show that points

$A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear

Answer

Area of ΔABC is given by the relation,

$$\begin{aligned}
 \Delta &= \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} \\
 &= \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix} \quad (\text{Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1) \\
 &= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} \\
 &= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (\text{Applying } R_3 \rightarrow R_3 + R_2) \\
 &= 0 \quad (\text{All elements of } R_3 \text{ are } 0)
 \end{aligned}$$

Thus, the area of the triangle formed by points A, B, and C is zero.

Hence, the points A, B, and C are collinear.

Question 3:

Find values of k if area of triangle is 4 square units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$ (ii) $(-2, 0), (0, 4), (0, k)$

Answer

We know that the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) is the absolute value of the determinant (Δ), where

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.

$$\therefore \Delta = \pm 4.$$

(i) The area of the triangle with vertices $(k, 0), (4, 0), (0, 2)$ is given by the relation,

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \\ &= \frac{1}{2} [k(0-2) - 0(4-0) + 1(8-0)] \\ &= \frac{1}{2} [-2k + 8] = -k + 4 \end{aligned}$$

$$\therefore -k + 4 = \pm 4$$

When $-k + 4 = -4$, $k = 8$.

When $-k + 4 = 4$, $k = 0$.

Hence, $k = 0, 8$.

(ii) The area of the triangle with vertices $(-2, 0)$, $(0, 4)$, $(0, k)$ is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} \\ &= \frac{1}{2} [-2(4-k)] \\ &= k - 4\end{aligned}$$

$$\therefore k - 4 = \pm 4$$

When $k - 4 = -4$, $k = 0$.

When $k - 4 = 4$, $k = 8$.

Hence, $k = 0, 8$.

Question 4:

(i) Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants

(ii) Find equation of line joining $(3, 1)$ and $(9, 3)$ using determinants

Answer

(i) Let $P(x, y)$ be any point on the line joining points $A(1, 2)$ and $B(3, 6)$. Then, the points A, B , and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\begin{aligned}\therefore \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} &= 0 \\ \Rightarrow \frac{1}{2} [1(6-y) - 2(3-x) + 1(3y-6x)] &= 0 \\ \Rightarrow 6 - y - 6 + 2x + 3y - 6x &= 0 \\ \Rightarrow 2y - 4x &= 0 \\ \Rightarrow y &= 2x\end{aligned}$$

Hence, the equation of the line joining the given points is $y = 2x$.

(ii) Let $P(x, y)$ be any point on the line joining points $A(3, 1)$ and

B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [3(3-y) - 1(9-x) + 1(9y-3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow 6y - 2x = 0$$

$$\Rightarrow x - 3y = 0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$.

Question 5:

If area of triangle is 35 square units with vertices (2, -6), (5, 4), and (k, 4). Then k is

A. 12 **B.** -2 **C.** -12, -2 **D.** 12, -2

Answer

Answer: D

The area of the triangle with vertices (2, -6), (5, 4), and (k, 4) is given by the relation,

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} \\ &= \frac{1}{2} [2(4-4) + 6(5-k) + 1(20-4k)] \\ &= \frac{1}{2} [30 - 6k + 20 - 4k] \\ &= \frac{1}{2} [50 - 10k] \\ &= 25 - 5k \end{aligned}$$

It is given that the area of the triangle is ± 35 .

Therefore, we have:

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5 - k) = \pm 35$$

$$\Rightarrow 5 - k = \pm 7$$

When $5 - k = -7$, $k = 5 + 7 = 12$.

When $5 - k = 7$, $k = 5 - 7 = -2$.

Hence, $k = 12, -2$.

The correct answer is D.