Exercise 7.1

Question 1:

sin 2*x*

Answer

The anti derivative of sin 2*x* is a function of *x* whose derivative is sin 2*x*.It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

Question 2:

Cos 3*x*

Answer

The anti derivative of cos 3*x* is a function of *x* whose derivative is cos 3*x*.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$.

Question 3:

e^{2x}

Answer

The anti derivative of e^{2x} is the function of *x* whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$$

$$\Rightarrow e^{2x}=\frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$

$$\therefore e^{2x}=\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

Question 4:

$(ax+b)^2$

Answer

The anti derivative of $(ax+b)^2$ is the function of *x* whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3=3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2=\frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2=\frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

Question 5:

$\sin 2x-4e^{3x}$

Answer

The anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is the function of *x* whose derivative is $\left(\sin 2x-4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)=\sin 2x-4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)$.

Question 6:

$$\int\left(4e^{3x}+1\right)dx$$

Answer

$$\int\left(4e^{3x}+1\right)dx$$

$$=4\int e^{3x}dx+\int 1dx$$

$$=4\left(\frac{e^{3x}}{3}\right)+x+C$$

$$=\frac{4}{3}e^{3x}+x+C$$

Question 7:

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$

Answer

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$

$$=\int\left(x^2-1\right)dx$$

$$=\int x^2dx-\int 1dx$$

$$=\frac{x^3}{3}-x+C$$

Question 8:

$$\int\left(ax^2+bx+c\right)dx$$

Answer

$$\int\left(ax^2+bx+c\right)dx$$

$$= a\int x^2dx + b\int xdx + c\int 1.dx$$

$$= a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$

$$= \frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

Question 9:

$$\int\left(2x^2+e^x\right)dx$$

Answer

$$\int\left(2x^2+e^x\right)dx$$

$$= 2\int x^2dx + \int e^x dx$$

$$= 2\left(\frac{x^3}{3}\right)+e^x+C$$

$$= \frac{2}{3}x^3+e^x+C$$

Question 10:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$

Answer

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$

$$= \int\left(x+\frac{1}{x}-2\right)dx$$

$$= \int xdx + \int\frac{1}{x}dx - 2\int 1.dx$$

$$= \frac{x^2}{2}+\log|x|-2x+C$$

Question 11:

$$\int \frac{x^3+5x^2-4}{x^2}dx$$

Answer

$$\int \frac{x^3+5x^2-4}{x^2}dx$$

$$= \int\left(x+5-4x^{-2}\right)dx$$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$$

$$= \frac{x^2}{2}+5x-4\left(\frac{x^{-1}}{-1}\right)+C$$

$$= \frac{x^2}{2}+5x+\frac{4}{x}+C$$

Question 12:

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

Answer

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

Question 13:

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

Answer

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

On dividing, we obtain

$$= \int (x^2 + 1) dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

Question 14:

$$\int (1 - x)\sqrt{x} dx$$

Answer

$$\int (1 - x)\sqrt{x} dx$$

$$= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx$$

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$$

Question 15:

$$\int \sqrt{x} \left(3x^2 + 2x + 3 \right) dx$$

Answer

$$\int \sqrt{x}\left(3x^2+2x+3\right)dx$$

$$= \int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$$

$$= 3\int x^{\frac{5}{2}}dx+2\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right)+2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)+3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\text{C}$$

$$= \frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+\text{C}$$

Question 16:

$$\int\left(2x-3\cos x+e^x\right)dx$$

Answer

$$\int\left(2x-3\cos x+e^x\right)dx$$

$$= 2\int x dx-3\int \cos x dx+\int e^x dx$$

$$= \frac{2x^2}{2}-3\left(\sin x\right)+e^x+\text{C}$$

$$= x^2-3\sin x+e^x+\text{C}$$

Question 17:

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

Answer

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

$$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$$

$$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

Question 18:

$$\int \sec x(\sec x + \tan x) dx$$

Answer

$$\int \sec x(\sec x + \tan x) dx$$

$$= \int(\sec^2 x + \sec x \tan x) dx$$

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

Question 19:

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

Answer

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int \left(\sec^2 x - 1\right) dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

Question 20:

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

Answer

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

Question 21:

The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$ equals

(A) $\frac{1}{3} x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ **(B)** $\frac{2}{3} x^{\frac{2}{3}} + \frac{1}{2} x^2 + C$

(C) $\frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ **(D)** $\frac{3}{2} x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{1}{2}} + C$

Answer

$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

$= \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

Hence, the correct Answer is C.

Question 22:

If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that *f*(2) = 0, then *f*(*x*) is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ **(B)** $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ **(D)** $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Answer

It is given that,

$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

∴Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}\, dx$

$f(x) = 4\int x^3 dx - 3\int (x^{-4}) dx$

$f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$

$\therefore f(x) = x^4 + \frac{1}{x^3} + C$

Also,

$f(2) = 0$

$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$

$\Rightarrow 16 + \frac{1}{8} + C = 0$

$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$

$\Rightarrow C = \frac{-129}{8}$

$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

Hence, the correct Answer is A.