Exercise 7.6

Question 1:

x sin *x*

Answer

Let $I = \int x \sin x \, dx$

Taking *x* as first function and sin *x* as second function and integrating by parts, we obtain

$$I = x \int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$
$$= x(-\cos x) - \int 1 \cdot (-\cos x) dx$$
$$= -x \cos x + \sin x + C$$

Question 2:

$x \sin 3x$

Answer

Let $I = \int x \sin 3x \, dx$

Taking *x* as first function and sin 3*x* as second function and integrating by parts, we obtain

$$I = x \int \sin 3x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x \, dx \right\}$$
$$= x\left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$
$$= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \, dx$$
$$= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C$$

Question 3:

$x^2 e^x$

Answer

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2 \int e^x\, dx - \int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\}dx$$

$$= x^2 e^x - \int 2x \cdot e^x dx$$

$$= x^2 e^x - 2\int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2\left[x \cdot \int e^x dx - \int\left\{\left(\frac{d}{dx}x\right) \cdot \int e^x dx\right\}dx\right]$$

$$= x^2 e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2 e^x - 2\left[xe^x - e^x\right]$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x\left(x^2 - 2x + 2\right) + C$$

Question 4:

$x \log x$

Answer

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x\; dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x\; dx\right\}dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

Question 5:

$x \log 2x$

Answer

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\ dx - \int \left\{ \left(\frac{d}{dx} 2\log x \right) \int x\ dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

Question 6:

$x^2 \log x$

Answer

Let $I = \int x^2 \log x\ dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\ dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Question 7:

$x\sin^{-1}x$

Answer

Let $I = \int x\sin^{-1}x\ dx$

Taking $\sin^{-1}x$ as first function and *x* as second function and integrating by parts, we obtain

$$I = \sin^{-1}x\int x\ dx - \int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int x\ dx\right\}dx$$

$$= \sin^{-1}x\left(\frac{x^2}{2}\right) - \int\frac{1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\left\{\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\left\{\int\sqrt{1-x^2}dx - \int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x\right\} + C$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1}x - \frac{1}{2}\sin^{-1}x + C$$

$$= \frac{1}{4}\left(2x^2-1\right)\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$$

Question 8:

$x\tan^{-1}x$

Answer

Let $I = \int x\tan^{-1}x\ dx$

Taking $\tan^{-1} x$ as first function and *x* as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\tan^{-1} x\right)\int x\,dx\right\}dx$$

$$= \tan^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\cdot\frac{x^2}{2}\,dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + \mathrm{C}$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + \mathrm{C}$$

Question 9:

$x\cos^{-1} x$

Answer

Let $I = \int x\cos^{-1} x\,dx$

Taking $\cos^{-1} x$ as first function and *x* as second function and integrating by parts, we obtain

$$I=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$=\cos^{-1}x\frac{x^2}{2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}+\left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\sqrt{1-x^2}dx-\frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}x \qquad \ldots(1)$$

where, $I_1=\int\sqrt{1-x^2}dx$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{\int\sqrt{1-x^2}\,dx+\int\frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{I_1+\cos^{-1}x\right\}$$

$$\Rightarrow 2I_1=x\sqrt{1-x^2}-\cos^{-1}x$$

$$\therefore I_1=\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I=\frac{x\cos^{-1}x}{2}-\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x\right)-\frac{1}{2}\cos^{-1}x$$

$$=\frac{\left(2x^2-1\right)}{4}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+\mathrm{C}$$

Question 10:

$$\left(\sin^{-1} x\right)^2$$

Answer

Let $I = \int\left(\sin^{-1} x\right)^2 \cdot 1\, dx$

Taking $\left(\sin^{-1} x\right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \left(\sin^{-1} x\right) \int 1\, dx - \int\left\{\frac{d}{dx}\left(\sin^{-1} x\right)^2 \cdot \int 1 \cdot dx\right\} dx$$

$$= \left(\sin^{-1} x\right)^2 \cdot x - \int \frac{2\sin^{-1} x}{\sqrt{1-x^2}} \cdot x\, dx$$

$$= x\left(\sin^{-1} x\right)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}}\right) dx$$

$$= x\left(\sin^{-1} x\right)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}\, dx - \int\left\{\left(\frac{d}{dx}\sin^{-1} x\right) \int \frac{-2x}{\sqrt{1-x^2}} dx\right\} dx\right]$$

$$= x\left(\sin^{-1} x\right)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx\right]$$

$$= x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2}\,\sin^{-1} x - \int 2\, dx$$

$$= x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2}\,\sin^{-1} x - 2x + \mathrm{C}$$

Question 11:

$$\frac{x\cos^{-1} x}{\sqrt{1-x^2}}$$

Answer

Let $I = \int \frac{x\cos^{-1} x}{\sqrt{1-x^2}}\, dx$

$$I = \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int\left\{\left(\frac{d}{dx}\cos^{-1} x\right)\int \frac{-2x}{\sqrt{1-x^2}} dx\right\} dx\right]$$

$$= \frac{-1}{2}\left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + \int 2\, dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + 2x\right] + \mathrm{C}$$

$$= -\left[\sqrt{1-x^2}\cos^{-1} x + x\right] + \mathrm{C}$$

Question 12:

$x \sec^2 x$

Answer

Let $I = \int x \sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x\int \sec^2 x\, dx - \int\left\{\left\{\frac{d}{dx}x\right\}\int \sec^2 x\, dx\right\} dx$$

$$= x \tan x - \int 1 \cdot \tan x dx$$

$$= x \tan x + \log\left|\cos x\right| + \mathrm{C}$$

Question 13:

$\tan^{-1} x$

Answer

Let $I = \int 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x\, dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log\left|1+x^2\right| + \mathrm{C}$$

$$= x \tan^{-1} x - \frac{1}{2} \log\left(1+x^2\right) + \mathrm{C}$$

Question 14:

$x(\log x)^2$

Answer

$I = \int x (\log x)^2 dx$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\log x)^2 \int x\, dx - \int \left[\left\{ \left(\frac{d}{dx} \log x \right)^2 \right\} \int x\, dx \right] dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x\, dx$$

Again integrating by parts, we obtain

$$I = \frac{x^2}{2}(\log x)^2 - \left[\log x \int x\,dx - \int \left\{ \left(\frac{d}{dx}\log x\right) \int x\,dx \right\} dx \right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

Question 15:

$(x^2+1)\log x$

Answer

Let $I = \int (x^2+1)\log x\,dx = \int x^2 \log x\,dx + \int \log x\,dx$

Let $I = I_1 + I_2$ … (1)

Where, $I_1 = \int x^2 \log x\,dx$ and $I_2 = \int \log x\,dx$

$I_1 = \int x^2 \log x dx$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx}\log x\right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3}\log x - \frac{1}{3}\left(\int x^2 dx\right)$$

$$= \frac{x^3}{3}\log x - \frac{x^3}{9} + C_1 \qquad \ldots(2)$$

$I_2 = \int \log x\,dx$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2 \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

Question 16:

$e^x (\sin x + \cos x)$

Answer

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

□ $f'(x) = \cos x$

□ $I = \int e^x \{ f(x) + f'(x) \} dx$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

Question 17:

$$\frac{xe^x}{(1+x)^2}$$

Answer

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Let $f(x) = \frac{1}{1+x}$ □ $f'(x) = \frac{-1}{(1+x)^2}$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + \mathrm{C}$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + \mathrm{C}$$

Question 18:

$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$

Answer

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad \ldots(1)$$

Let $\tan\frac{x}{2}=f(x)$ $\square$ $f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\left\{f(x)+f'(x)\right\}dx=e^x f(x)+\mathrm{C}$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+\mathrm{C}$$

Question 19:

$e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Answer

Let $I=\int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Also, let $\frac{1}{x}=f(x)$ □ $f'(x)=\frac{-1}{x^2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+\mathrm{C}$

$\therefore I=\frac{e^x}{x}+\mathrm{C}$

Question 20:

$\frac{(x-3)e^x}{(x-1)^3}$

Answer

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

Let $f(x)=\frac{1}{(x-1)^2}$ □ $f'(x)=\frac{-2}{(x-1)^3}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+\mathrm{C}$

$$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx=\frac{e^x}{(x-1)^2}+\mathrm{C}$$

Question 21:

$e^{2x}\sin x$

Answer

Let $I = \int e^{2x} \sin x \, dx \qquad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \qquad \text{[From (1)]}$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} \left[2 \sin x - \cos x \right] + C$$

Question 22:

$$\sin^{-1} \left(\frac{2x}{1 + x^2} \right)$$

Answer

Let $x = \tan \theta$ □ $dx = \sec^2 \theta \, d\theta$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta)$ = 2θ

□ $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta \cdot \sec^2\theta\, d\theta = 2\int \theta \cdot \sec^2\theta\, d\theta$

Integrating by parts, we obtain

$$2\left[\theta \cdot \int \sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int \sec^2\theta d\theta\right\}d\theta\right]$$

$$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$$

$$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$$

Question 23:

$\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{3}e^{x^2} + C$

Answer

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t$ □ $3x^2 dx = dt$

$\Rightarrow I = \frac{1}{3}\int e^{t}\,dt$

$= \frac{1}{3}\left(e^{t}\right) + C$

$= \frac{1}{3}e^{x^{3}} + C$

Hence, the correct Answer is A.

Question 24:

$\int e^{x}\sec x\left(1+\tan x\right)dx$ equals

(A) $e^{x}\cos x + C$ (B) $e^{x}\sec x + C$

(C) $e^{x}\sin x + C$ (D) $e^{x}\tan x + C$

Answer

$\int e^{x}\sec x\left(1+\tan x\right)dx$

Let $I = \int e^{x}\sec x\left(1+\tan x\right)dx = \int e^{x}\left(\sec x + \sec x\tan x\right)dx$

Also, let $\sec x = f\left(x\right)$ □ $\sec x\tan x = f'\left(x\right)$

It is known that, $\int e^{x}\left\{f\left(x\right)+f'\left(x\right)\right\}dx = e^{x}f\left(x\right)+C$

$\therefore I = e^{x}\sec x + C$

Hence, the correct Answer is B.